Foundation course - PHYSICS

Lecture 7-2: Harmonic motion

- simple harmonic motion
- velocity and acceleration of oscillations
- energy of oscillations
- pendulums
- damped and forced oscillations

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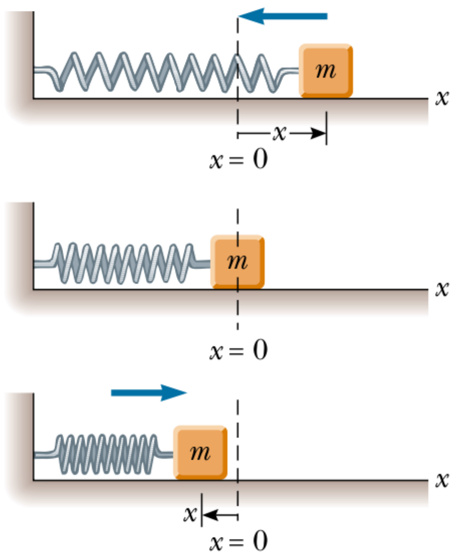
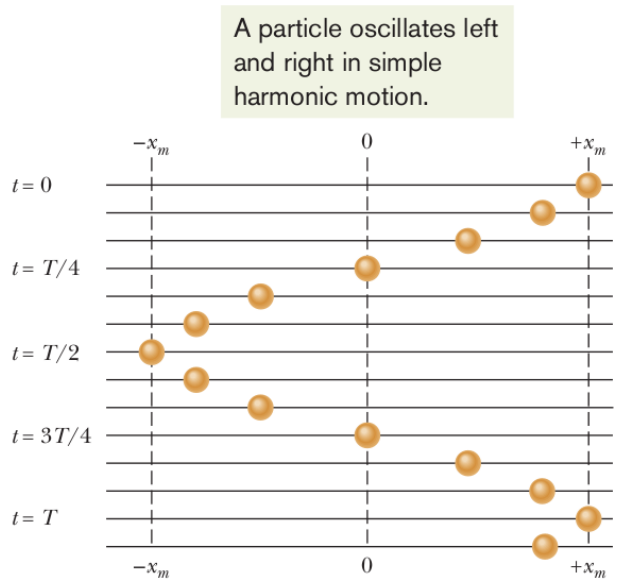
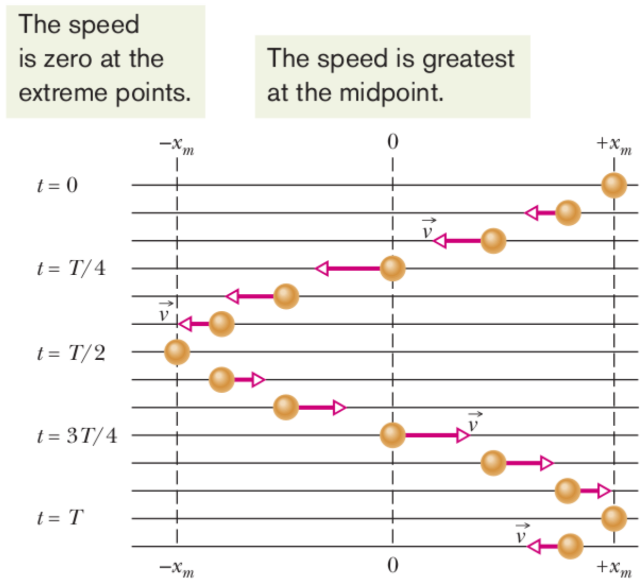
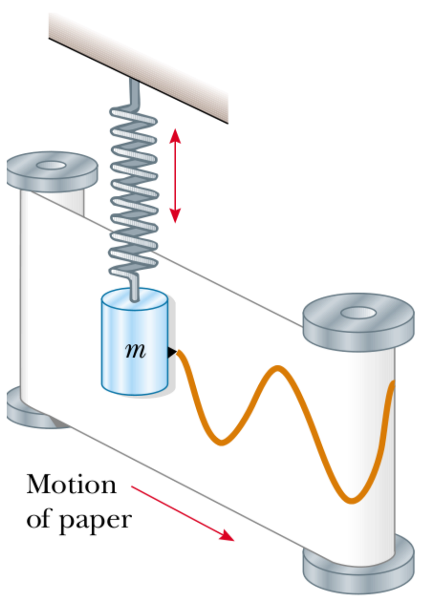
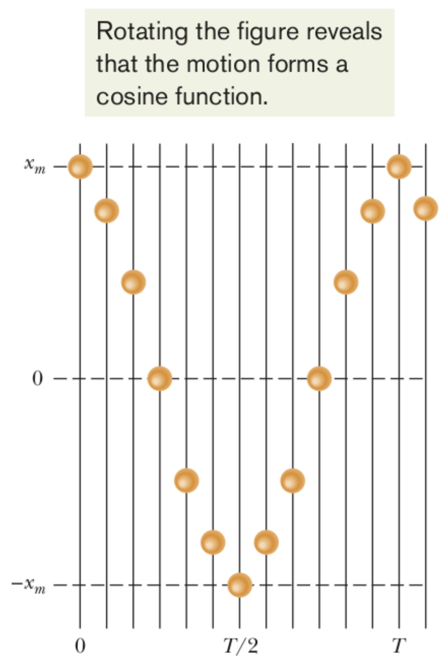
Simple harmonic motion

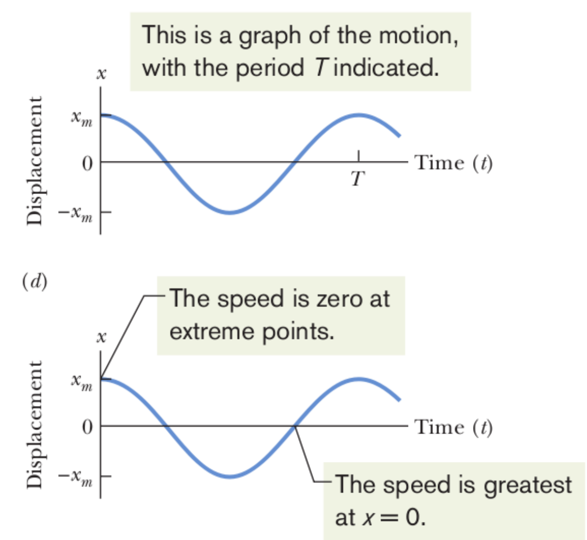












Simple harmonic motion

Frequency *f*:

- number of oscillations that are completed each second

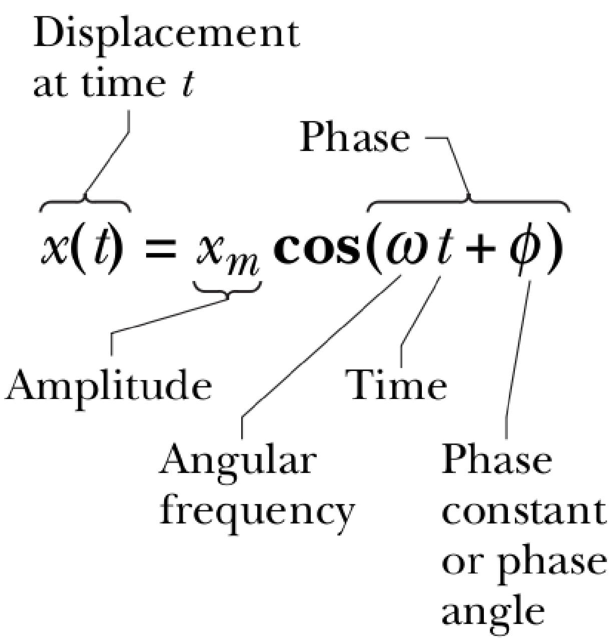
$$f = \frac{1}{T}$$

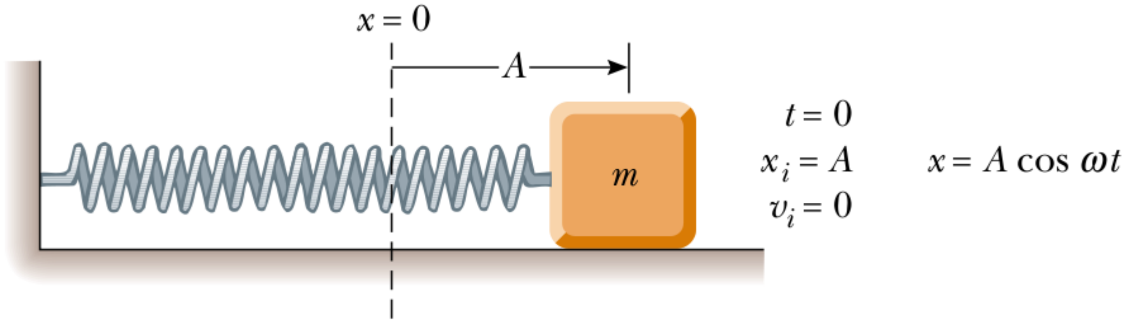
Period *T*:

- time for one complete oscillation (or cycle)

Displacement *x*:

$$x(t) = x_m \cos(\omega t + \phi)$$





After one period *T* the displacement *x* must return to its initial value:
x(t) = *x(t* + *T)* for all *t*

Angular frequency *ω*:

$$\omega t + 2\pi = \omega(t + T)$$

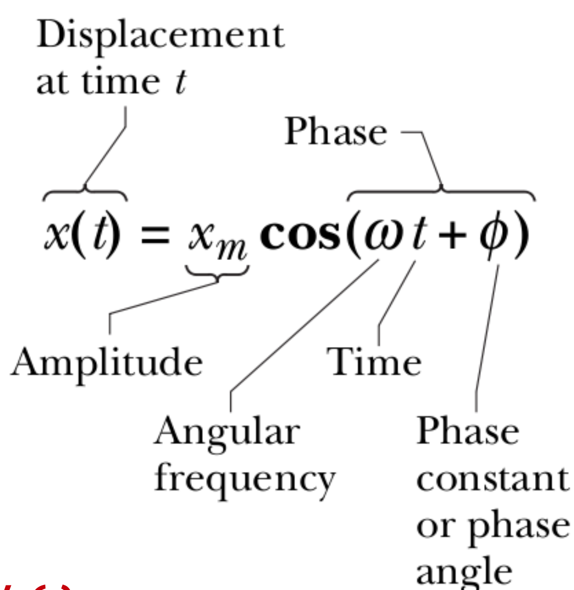
$$\omega T = 2\pi$$

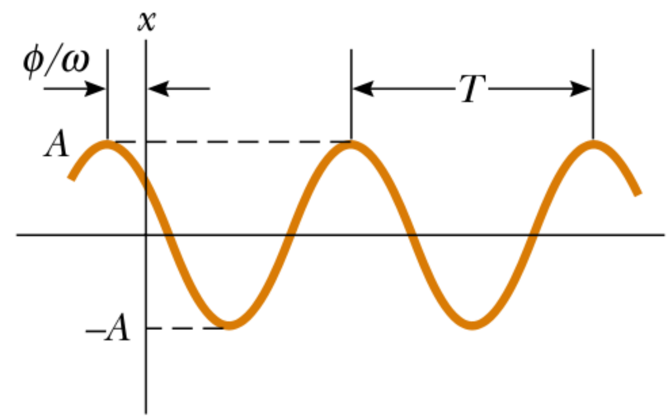
$$\omega = \frac{2\pi}{T} = 2\pi f$$

Simple harmonic motion

Amplitude x_m:

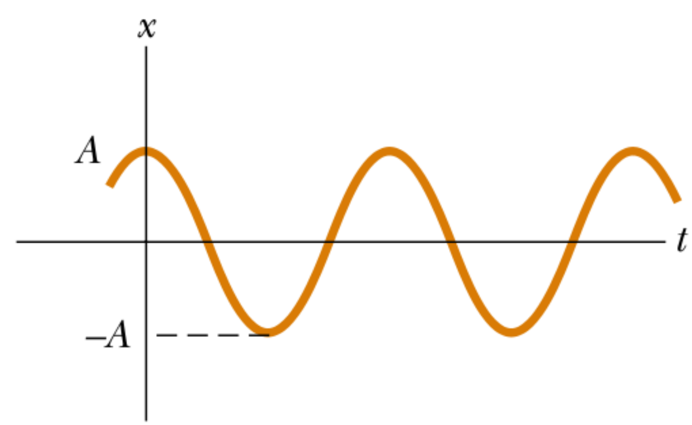
- maximum displacement
- positive constant whose value depends on how the motion was started
- displacement x varies between $\pm x_m$ (cosine varies between ± 1)





Phase $(\omega t + \Phi)$:

- angular frequency ω
- **phase constant (phase angle) Φ:**
  - depends on the displacement and velocity of the particle at time $t = 0$



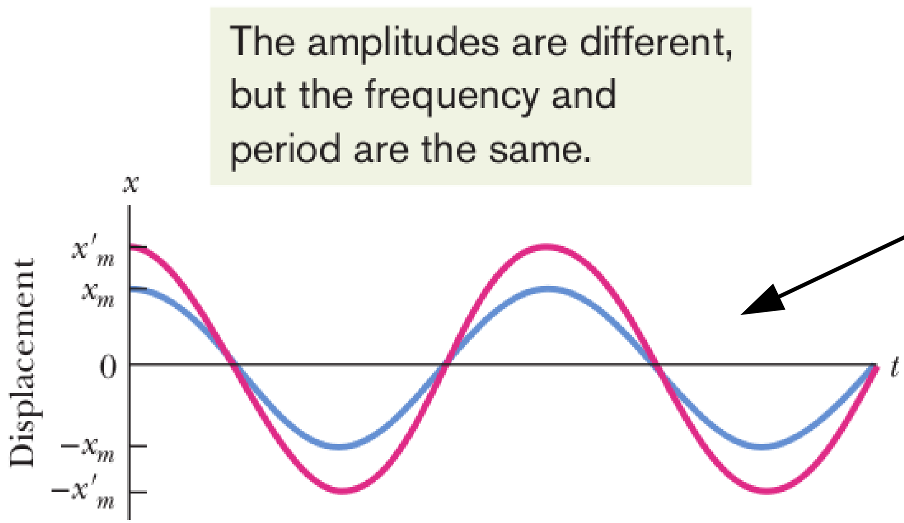
Simple harmonic motion

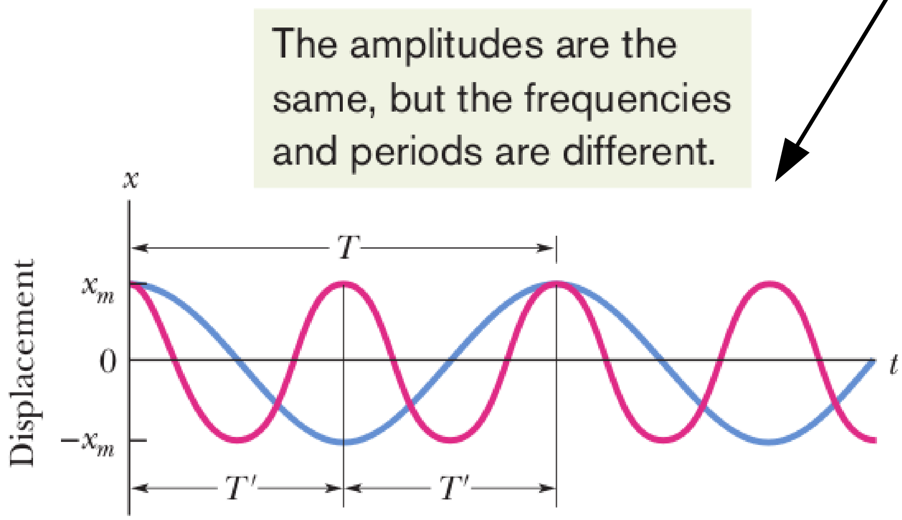
Blue curve: $\Phi = 0$ in all three graphs

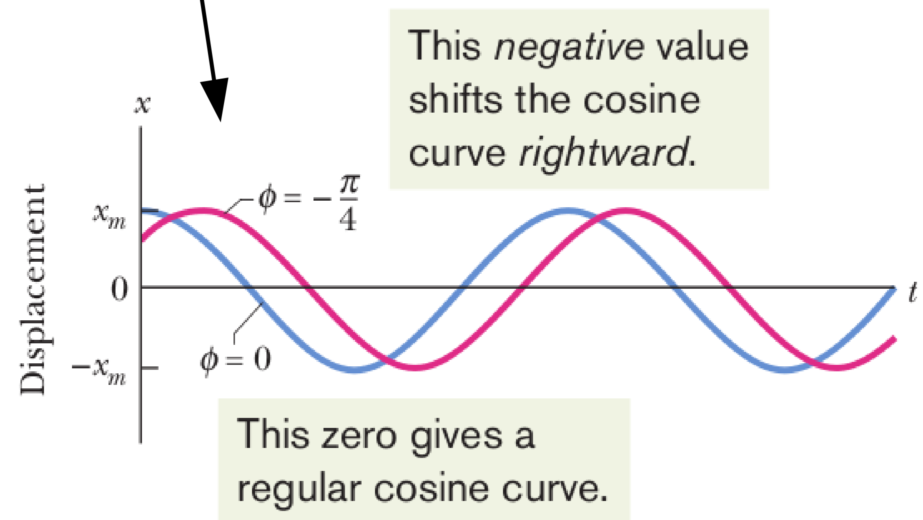
Difference in amplitude: $x'_m > x_m$

Difference in period: $T' = T/2$

Difference in phase angle: $\Phi = -\pi/4$







Example:

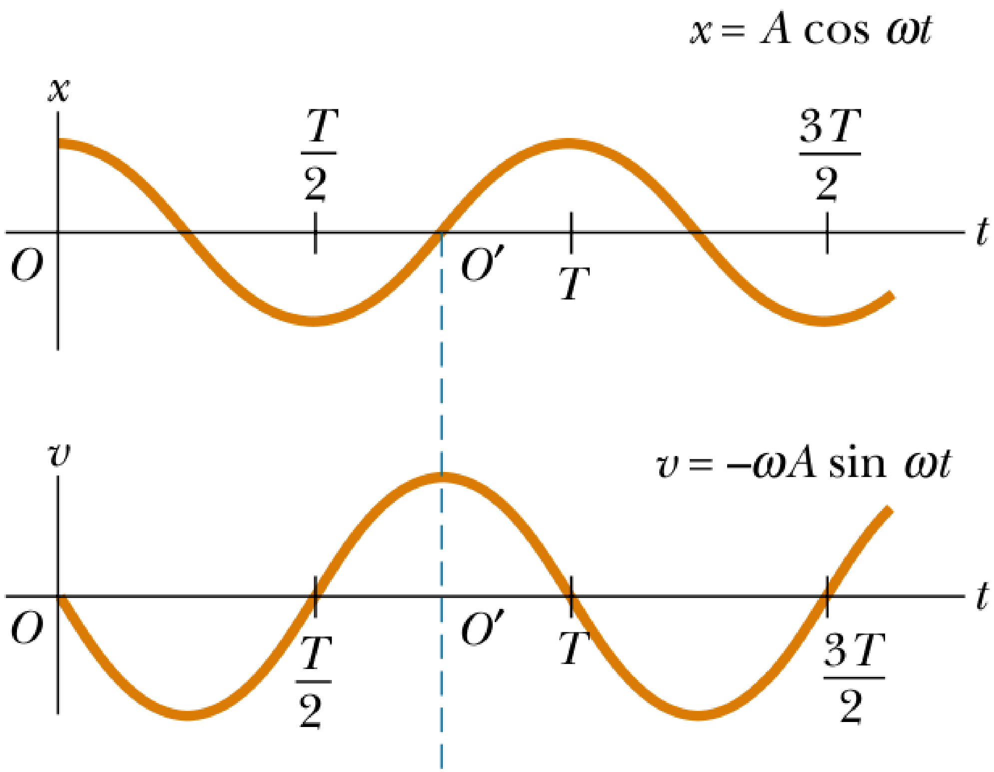
A particle undergoing simple harmonic oscillation of period T is at $-x_m$ at time $t = 0$. Is it at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$ when

(a) $t = 2.00T$,

(b) $t = 3.50T$,

(c) $t = 5.25T$?

Velocity



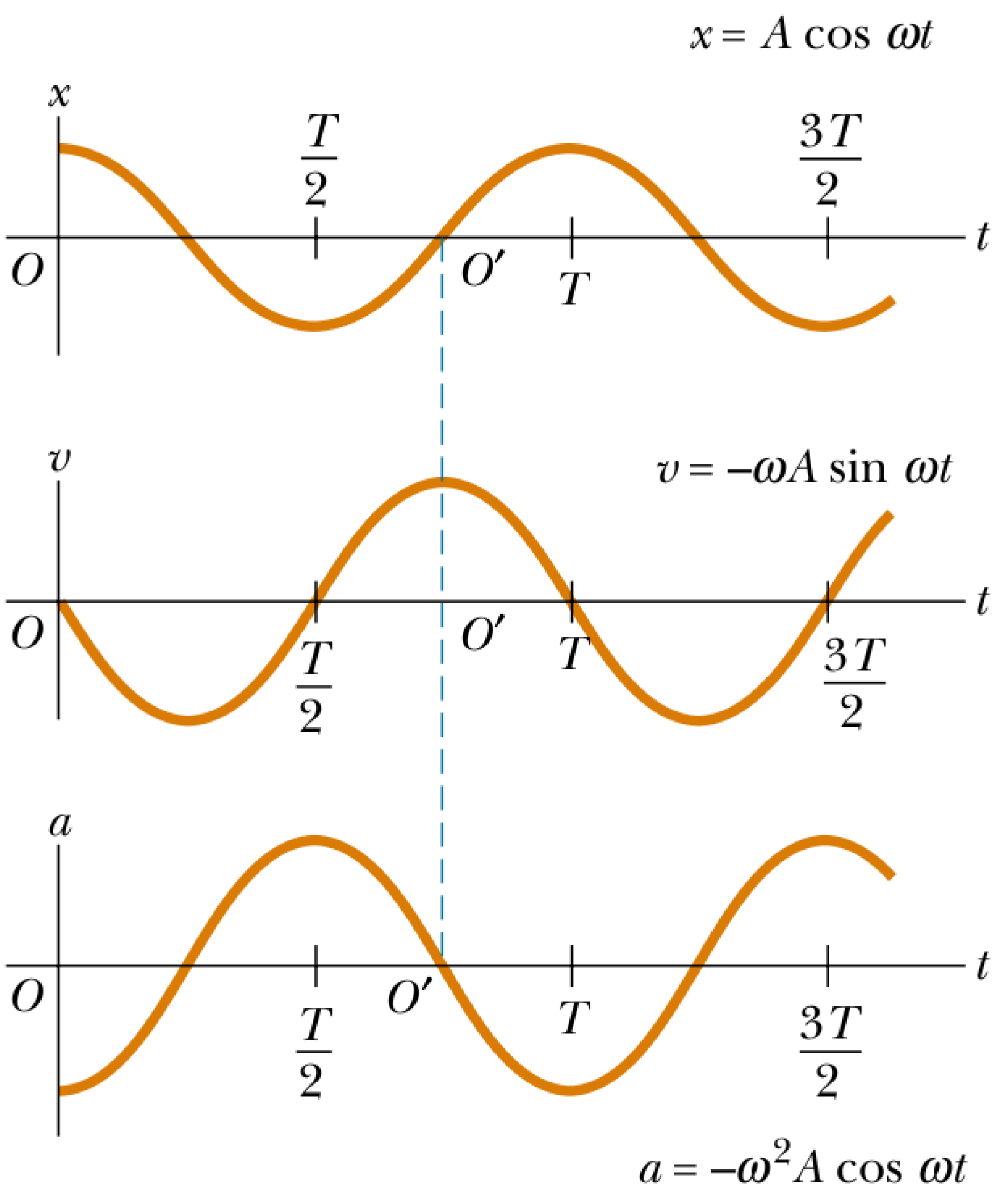
Displacement *x*:

$$x(t) = x_m \cos(\omega t + \phi)$$

Velocity: $v_x(t) = \frac{dx}{dt}$

$$v_x(t) = -\omega x_m \sin(\omega t + \phi)$$

Acceleration



Displacement *x*:

$$x(t) = x_m \cos(\omega t + \phi)$$

Velocity:

$$v_x(t) = -\omega x_m \sin(\omega t + \phi)$$

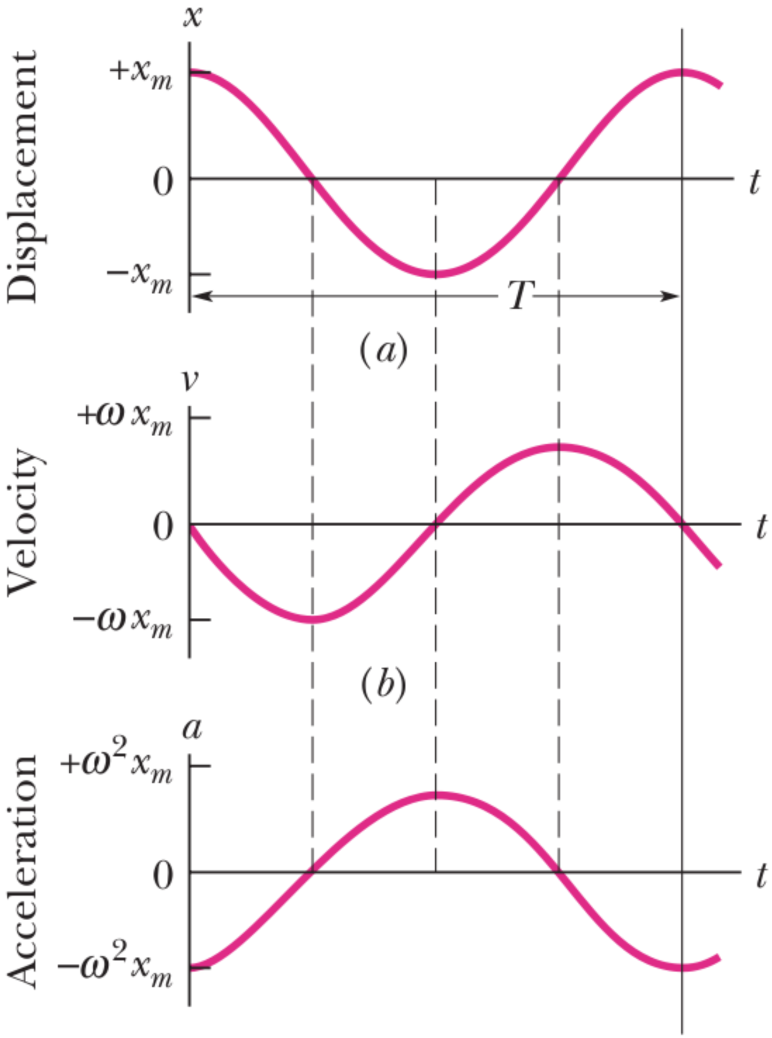
Acceleration: $a_x(t) = \dfrac{dv_x}{dt}$

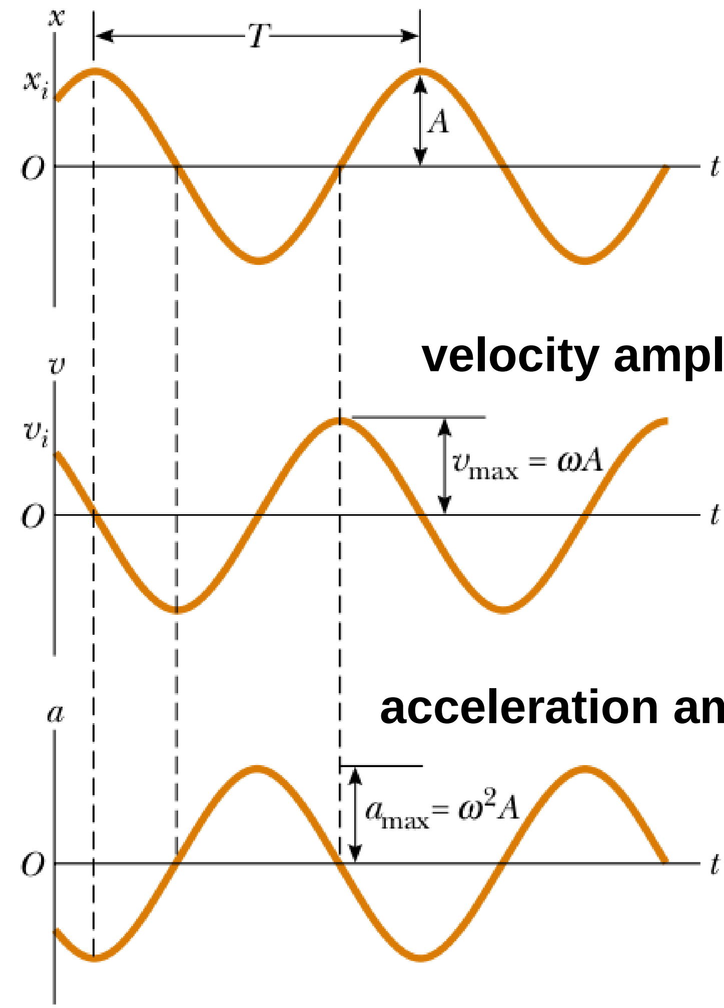
$$a_x(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$a_x(t) = -\omega^2 x(t)$$

Acceleration is proportional to the displacement but opposite in sign

Velocity and acceleration





velocity amplitude

acceleration amplitude

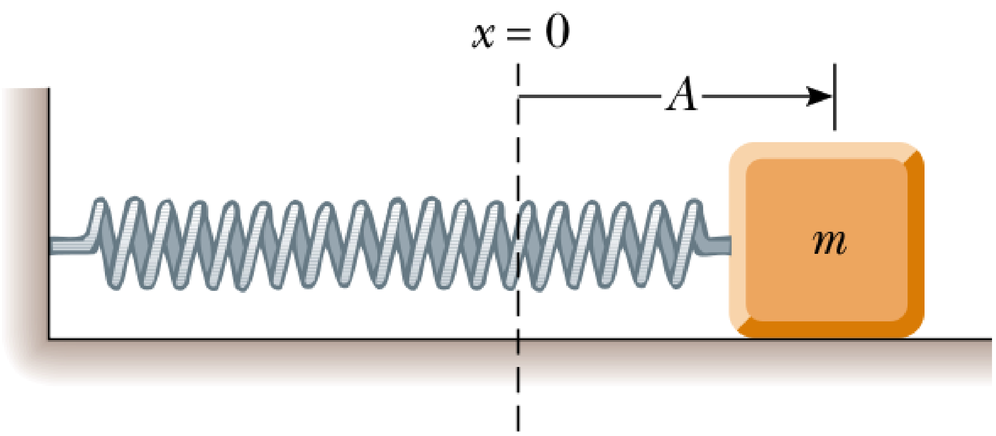
The force law for harmonic motion

$$a_x = -\omega^2 x$$

$$F_x = ma = -m\omega^2 x$$

$$F_x = -kx$$

$$k = m\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$



Linear harmonic oscillator

F_x is proportional to x rather than to some other power of x

Example:

Which of the following relationships between the force F on a particle and the particle's position x implies simple harmonic oscillation:

(a) $F = -5x$,

(b) $F = -400x^2$,

(c) $F = 10x$,

(d) $F = 3x^2$?

Example: simple harmonic oscillator

A block whose mass *m* is 680 g is fastened to a spring whose spring constant *k* is 65 N/m. The block is pulled a distance *x* = 11 cm from its equilibrium position at *x* = 0 on a frictionless surface and released from rest at t = 0.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

(b) What is the amplitude of the oscillation?

(c) What is the maximum speed v_m of the block and where is the block when it has this speed?

(d) What is the magnitude a_m of the maximum acceleration of the block?

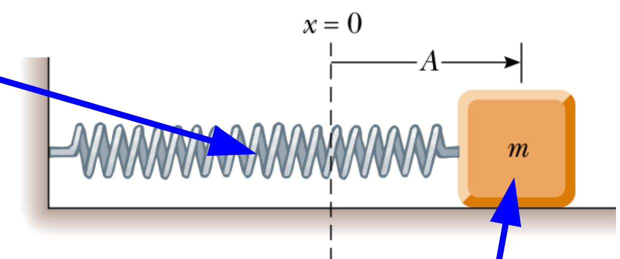
(e) What is the phase constant Φ for the motion?

(f) What is the displacement function *x(t)* for the spring – block system?

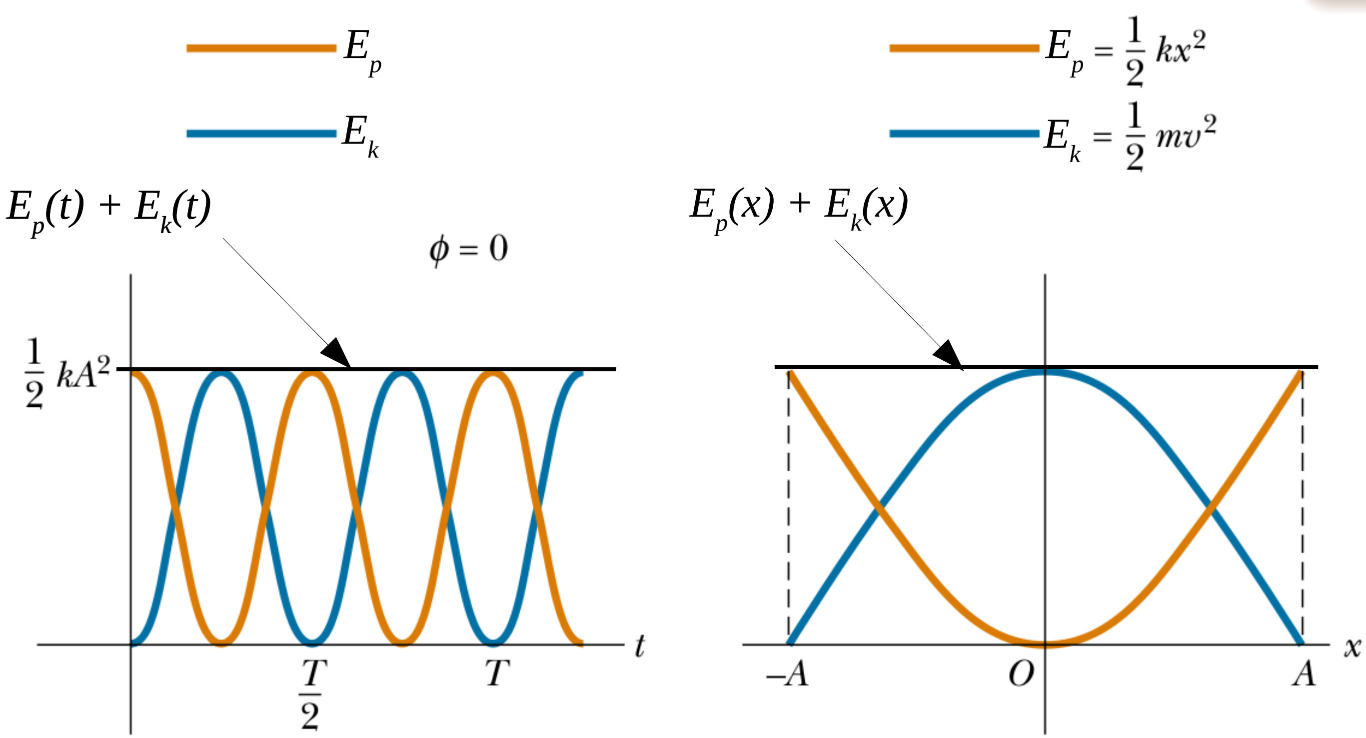
Energy of harmonic oscillator

The energy of a linear oscillator transfers back and forth between **kinetic energy E_k** and **potential energy E_p**, while the sum of E_k and E_p (= mechanical energy E of the oscillator) remains constant

Potential energy is entirely associated with the spring



Kinetic energy is entirely associated with the block



Energy of harmonic oscillator

Potential energy E_p:

$$E_p = \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

Kinetic energy E_k:

$$E_k = \frac{1}{2} m v^2 = \frac{1}{2} k x_m^2 \sin^2(\omega t + \phi)$$

we used substitution $\omega^2 = \frac{k}{m}$

Mechanical energy E:

$$E = E_p + E_k = \frac{1}{2} k x_m^2$$

The mechanical energy of a linear oscillator is constant and independent of time

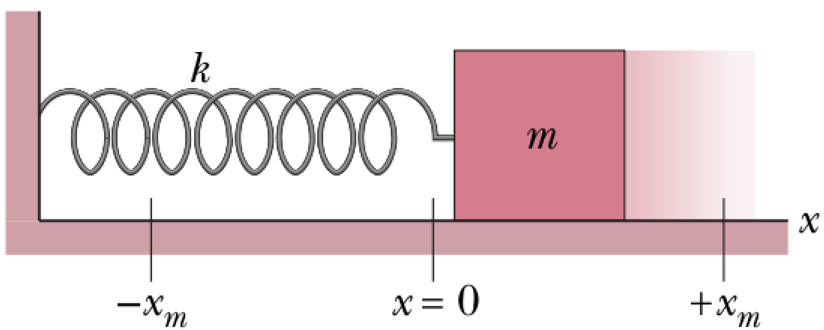
Example:

In the figure, the block has a kinetic energy of 3 J and the spring has an elastic potential energy of 2 J when the block is at x = +2.0 cm.

(a) What is the kinetic energy when the block is at x = 0?

(b) What is the elastic potential energy when the block is at x = -2.0 cm?

(c) What is the elastic potential energy when the block is at $x = -x_m$?



Example: kinetic and potential energy of simple harmonic oscillator

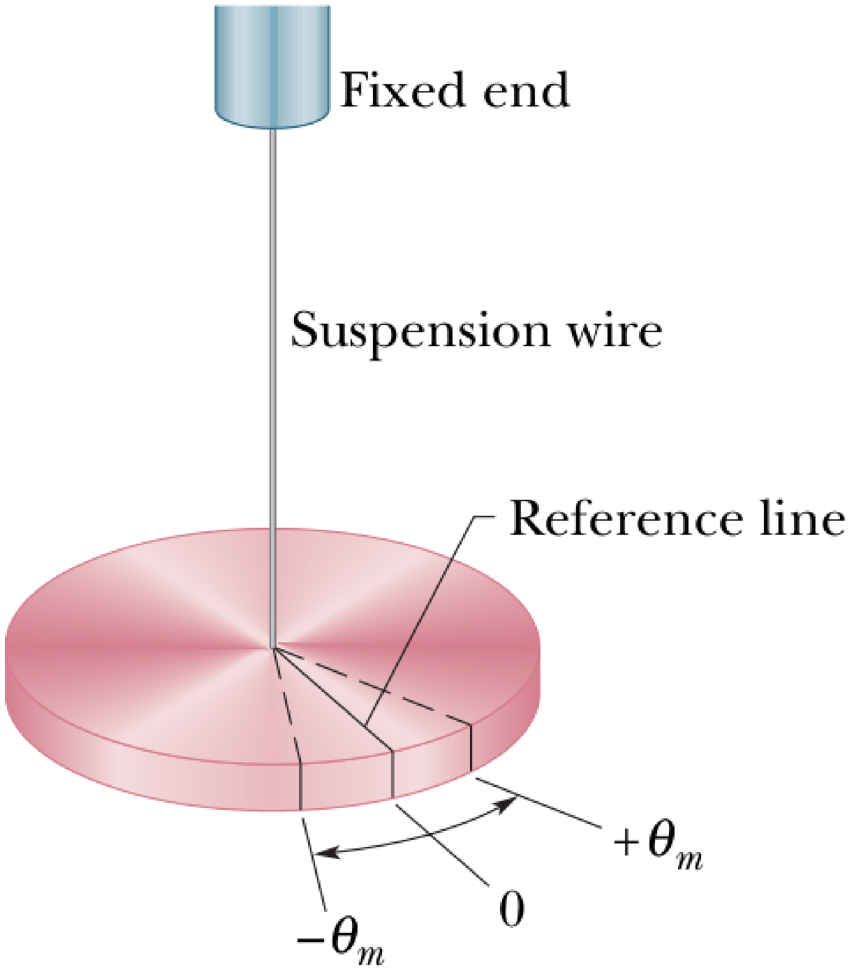
The block has mass m = 2.7 kg and oscillates at frequency f = 10.0 Hz and with amplitude x_m = 20.0 cm.

(a) What is the total mechanical energy E of the spring – block system?

(b) What is the block's speed as it passes through the equilibrium point?

Angular harmonic oscillator

Torsion pendulum



Restoring torque

$$\tau = -\kappa\,\theta$$

torsion constant dependent on the length, diameter, and material of the wire

Period of oscillations:

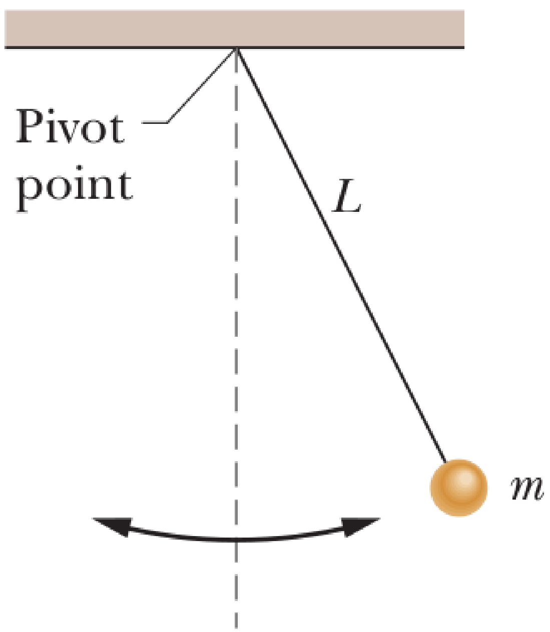
$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

moment of inertia

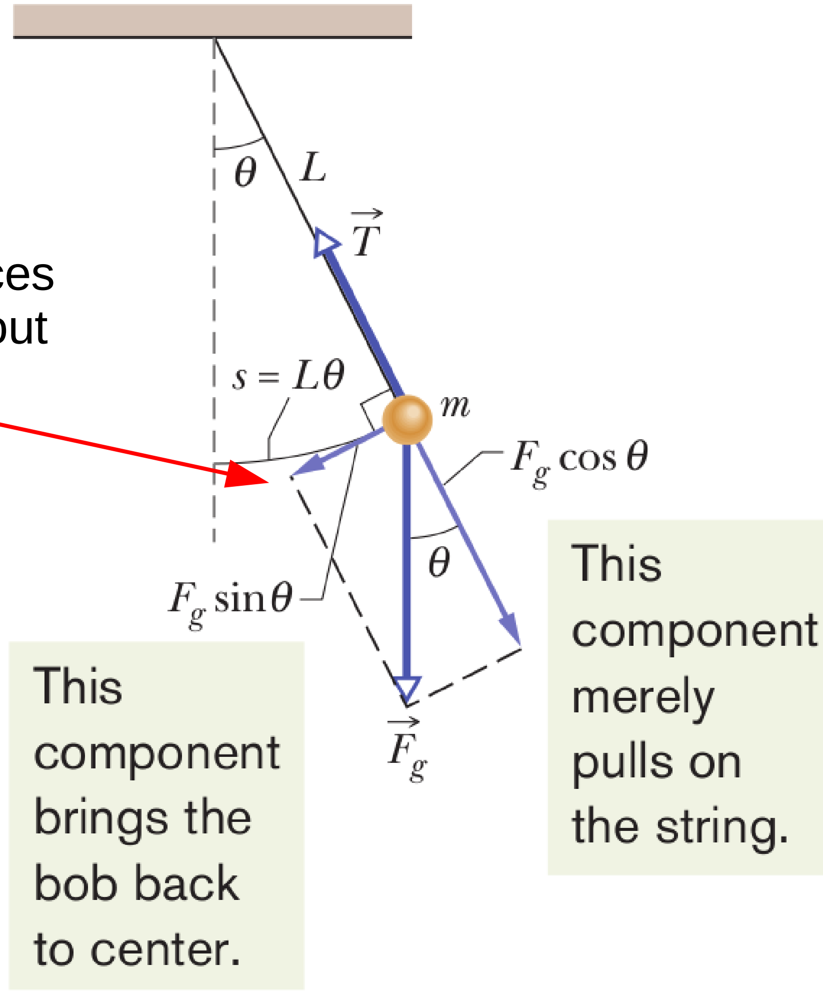
Simple pendulum

Simple pendulum:

- consists of a particle of mass *m* (called the *bob* of the pendulum) suspended from one end of an unstretchable, massless string of length *L* that is fixed at the other end



This component produces a **restoring torque** about a pendulum pivot



Two acting forces:

- gravitational force
- tension from the string

Simple pendulum

Restoring torque:

$$\tau = -L(F_g \sin\theta)$$

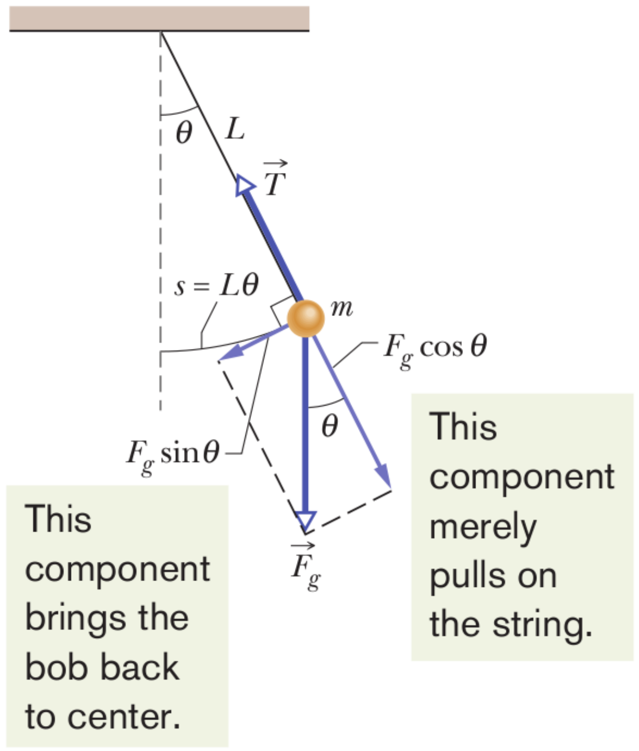
$$I\alpha = -L(mg\sin\theta)$$

I…moment of inertia, α...angular acceleration

We assume the angle θ is small: then $\sin\theta \approx \theta$

$$\alpha = -\frac{mgL}{I}\theta$$

It is similar to the equation: $a_x(t) = -\omega^2 x(t)$



Simple pendulum swinging through only **small angles** is approximately simple harmonic oscillator

All mass of the simple pendulum is concentrated in the bob:

$$I = mL^2$$

Period of swinging:

$$T = 2\pi\sqrt{\frac{I}{mgL}}$$

Period of the simple pendulum swinging is not dependent on the mass of the bob

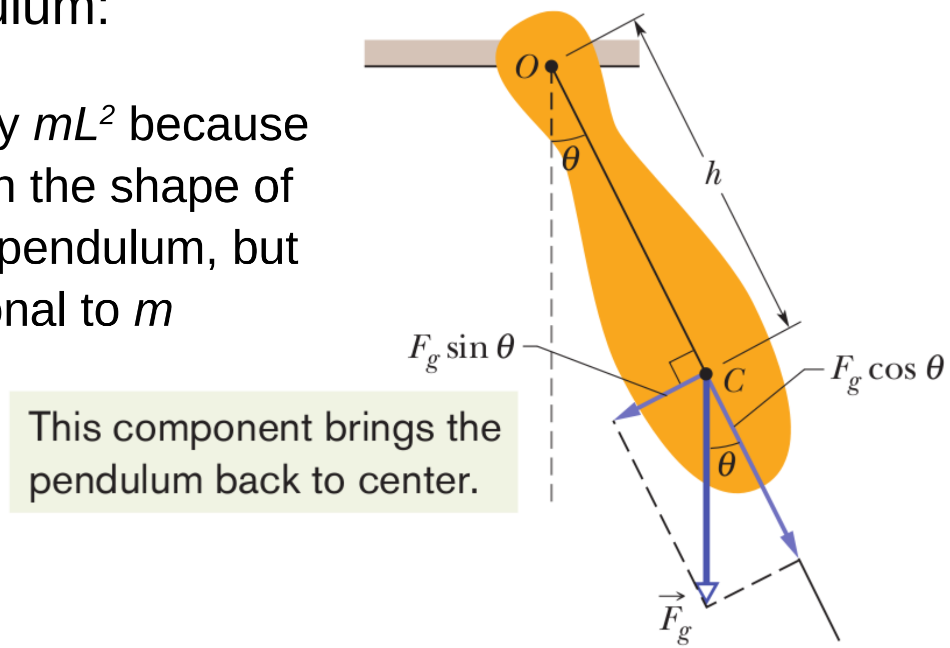
$$T = 2\pi\sqrt{\frac{L}{g}}$$

Physical pendulum

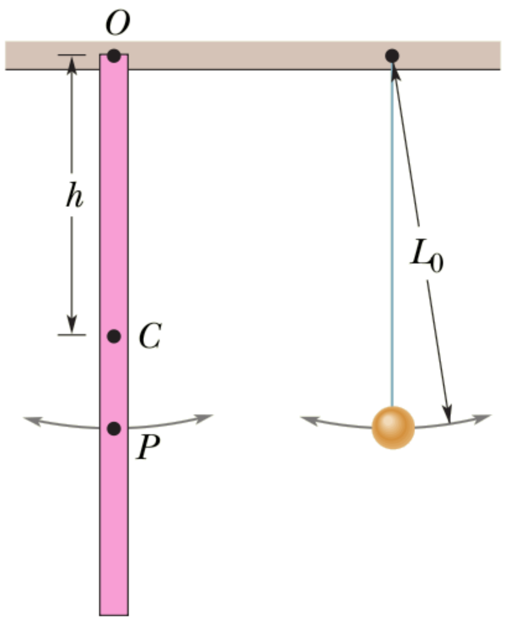
Period of swinging of the physical pendulum:

$$T = 2\pi\sqrt{\frac{I}{m\,g\,L}}$$

I is not simply mL^2 because it depends on the shape of the physical pendulum, but it is proportional to *m*



The physical pendulum does not swing if it pivots at its center of mass



The physical pendulum that oscillates about a given point *O* with period *T* is a simple pendulum of length L_0 with the same period *T* – the point at distance L_0 is called the **center of oscillation**

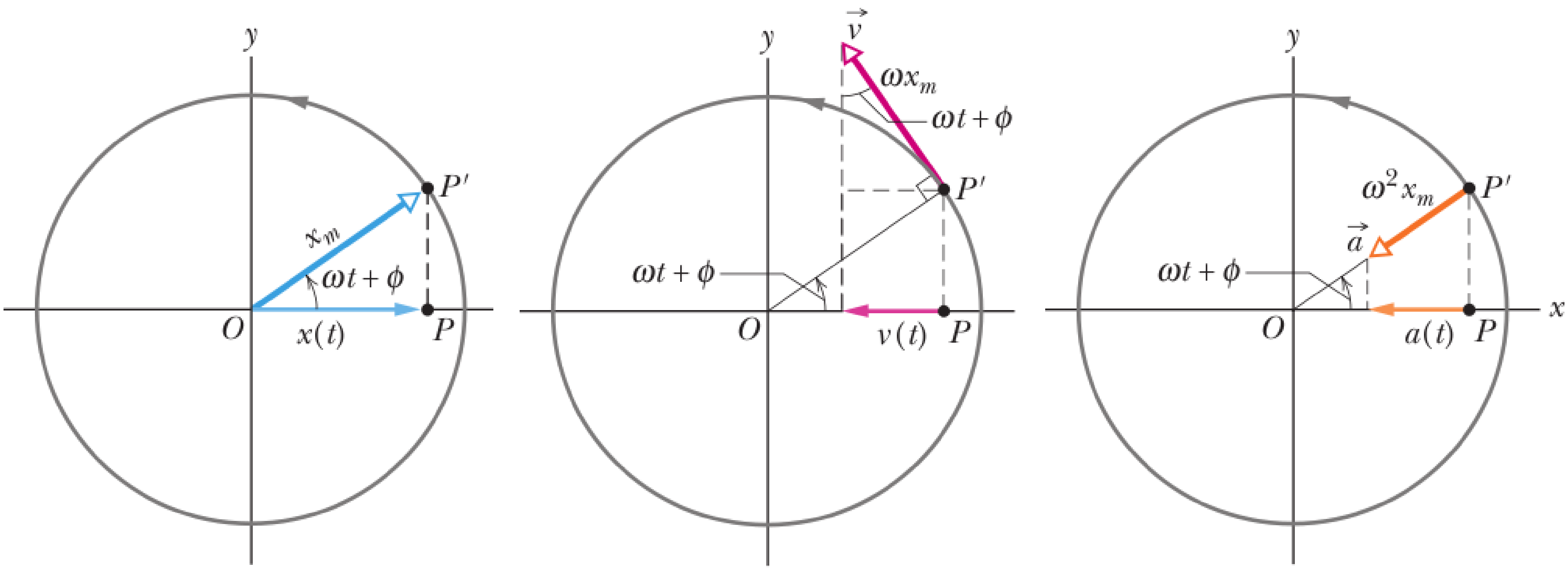
Example:

Three physical pendulums, of masses m_0, $2m_0$, and $3m_0$, have the same shape and size and are suspended at the same point. Rank the masses according to the periods of the pendulums, greatest first.

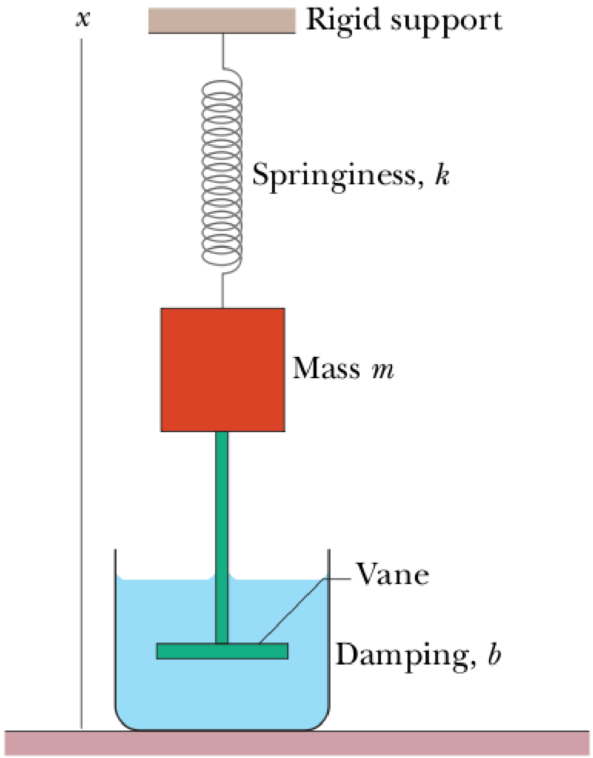
Simple harmonic motion and uniform circular motion

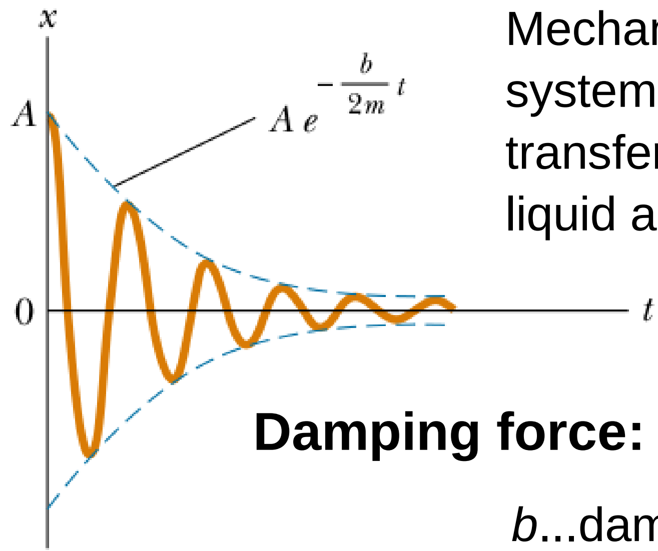
Simple harmonic motion is the projection of uniform circular motion on a diameter of the circle in which the circular motion occurs.

$$x(t) = x_m \cos(\omega t + \phi) \qquad v_x(t) = -\omega x_m \sin(\omega t + \phi) \qquad a_x(t) = -\omega^2 x_m \cos(\omega t + \phi)$$



Damped oscillations





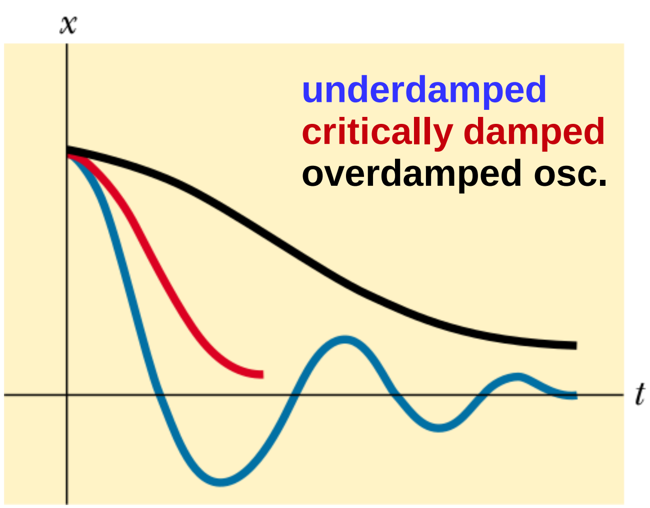
Mechanical energy of the block – spring system decreases: energy is transferred to thermal energy of the liquid and vane

Damping force: $F_d = -bv$

b...damping constant, *v*...velocity

The force on the block from the spring: $F_s = -kx$

$$ma = -bv - kx$$

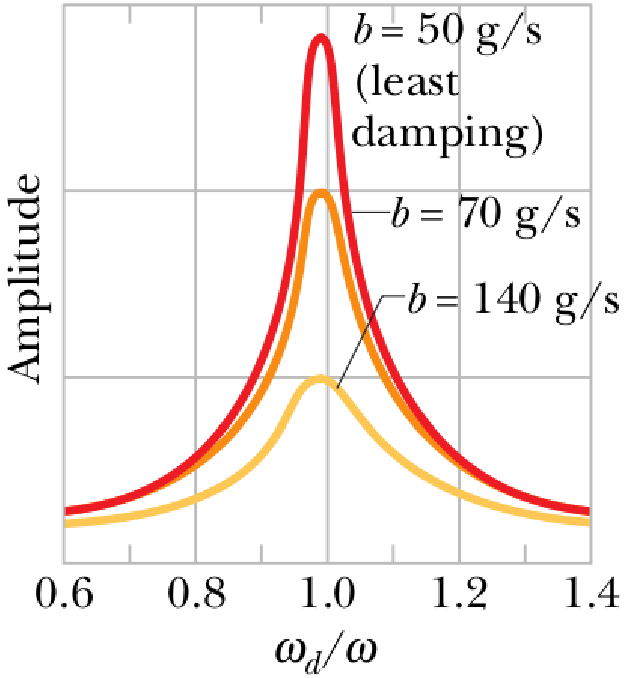


Angular frequency of a damped oscillator:

$$\omega = \sqrt{\omega_0 - \left(\frac{b}{2m}\right)^2}$$

Forced oscillations and resonance

- **free oscillations**
  - *natural* angular frequency ω causing the free oscillation
- **driven (forced) oscillations**
  - angular frequency ω_d of the external driving force causing the driven oscillations



- the system oscillates with angular frequency ω_d
- the velocity amplitude is greatest when $\omega_d = \omega$
- the amplitude x_m of the system is (approximately) greatest under the same condition.

$$\omega_d = \omega$$

...this condition is called **resonance**