Physics, Foundation Programme – Problem Solving Exercises 2

Algebraic Experessions

1. Write the exponential expression in the simplest form so that all exponents are positive:

   a) $2x^{-4}$ a: $\dfrac{2}{x^4}\,, x \neq 0$

   b) $\dfrac{5}{z^{-5}}$ a: $5z^5\,, z \neq 0$

   c) $(x^3y^2)(xy^5)$ a: x^4y^7

   d) $\dfrac{6a^4}{8a^8}$ a: $\dfrac{3}{4a^4}, a \neq 0$

   e) $(-6x^4y)(7x^{-3}y^{-5})$ a: $-\dfrac{42x}{y^4}, x \neq 0, y \neq 0$

   f) $\dfrac{(-v)^3 \cdot (-v)^5}{v^2} \cdot \dfrac{v^5}{(-v)^2}$ a: $v^9, v \neq 0$

   g) $\dfrac{x^2y^3z^4}{m^5} \cdot \dfrac{m^4z}{2x^3y^2} \div \dfrac{z^4}{4m^2x}$ a: $2yzm, x \neq 0, y \neq 0, z \neq 0, m \neq 0$

   h) $\dfrac{a^6 \cdot (a^3)^2}{(a^3)^4}$ a: $1, a \neq 0$

   i) $\left[16a^4 \cdot \left(\dfrac{1}{a^2}\right)^3 \cdot \left(\dfrac{a}{2}\right)^4\right]^3$ a: $a^6, a \neq 0$

   j) $\dfrac{\left[3(z^2)^3 \cdot y^2\right]^2}{(3zy)^4}$ a: $\dfrac{z^8}{9}, z \neq 0, y \neq 0$

   k) $\dfrac{\left(2u^3v^2\right)^4}{12} \div \dfrac{\left(2u^2v^5 \cdot 3u^4v^2\right)^2}{\left(3u^2v^4\right)^3}$ a: $u^6v^6, u \neq 0, v \neq 0$

2. Evaluate each expression. Express the result in scientific notation. Round using the multiplication/division/addition/subraction rules for significant digits.

   a) $(3.4 \cdot 10^{-4})(2.0 \cdot 10^{-5})$ A: $(6.8 \cdot 10^{-9})$

b) $(6.7 \cdot 10^3)(4.0 \cdot 10^9)$ a: $2.68 \cdot 10^{13} \approx 2.7 \cdot 10^{13}$

c) $(1.5 \cdot 10^5)(2.3 \cdot 10^{-3})$ a: $(3.45 \cdot 10^2) \approx (3.5 \cdot 10^2)$

d) $(4.9 \cdot 10^9) \div (7.0 \cdot 10^4)$ a: $7.0 \cdot 10^4$

e) $(1.11 \cdot 10^{-9}) \div (3.7 \cdot 10^{-5})$ a: $3.0 \cdot 10^{-5}$

f) $(3.29 \cdot 10^{12}) + (4.35 \cdot 10^{12})$ a: $7.64 \cdot 10^{12}$

g) $(5.98 \cdot 10^{-5}) - (3.2 \cdot 10^{-6})$ a: $5.66 \cdot 10^{-5}$

h) $(8.5 \cdot 10^6) - (7.3 \cdot 10^5)$ a: $7.8 \cdot 10^6$

3. For each polynomial determine its a) standard form, b) degree, c) coefficients, d) leading coefficient, e) number of terms

a) $x^3 - 1$ a: a) $x^3 - 1$; b) 3; c) 1, 0, 0, -1; d) 1; e) 2

b) $-3x^2 - 11 - 12x^4$ a: a) $-12x^4 - 3x^2 - 11$;b) 4; c) -12,0,-3,0,-11; d) -12; e) 3

c) $3x^2 - 5x^3 + 7x - 1$ a: a) $-5x^3 + 3x^2 + 7x - 1$; b) 3; c) -5,3,7,-1; d) -5; e) 4

4. Perform indicated operations and simplify if possible by combining like terms. Write the result in standard form.

a) $(5x^4 - 3x^2 + 9) + (3x^3 - 2x^2 - 7x + 3)$ A: $5x^4 + 3x^3 - 5x^2 - 7x + 12$

b) $(u^3 - 3u^2 - 4u + 8) - (u^3 - 2u + 4)$ a: $-3u^2 - 2u + 4$

c) * $(4x-5)(2x^2+7x-8)$ a: $8x^3+18x^2-67x+40$

d) * $(5x-7)(3x^2-8x-5)$ a: $15x^3-61x^2+31x+35$

5. Use the FOIL method to find the indicated product and simplify if possible. Write the result in standard form.

a) $(2x+4)(5x+1)$ A: $10x^2+22x+4$

b) $(b-4)(b+6)$ a: $b^2+2b-24$

c) $(5x-11y)(2x-7y)$ a: $10x^2-57xy+77y^2$

d) $(4z-3)(z-4)$ a: $4z^2-19z+12$

e) $(6w-11x)(2w-3x)$ a: $12w^2-40wx+33x^2$

6. Use special product formulas to perform indicated operation.

a) $(x+5)^2$ A: $x^2+10x+25$

b) $(3x^2-y)^2$ a: $9x^4-6x^2y+y^2$

c) $(5x-y)(5x+y)$ a: $25x^2-y^2$

d) $[(x+5)+y]\,[(x+5)-y]$ a: $x^2+10x+25-y^2$

7. Evaluate the given polynomial for the indicated value of the variable:
   a) $x^2 + 7x - 1$, for $x = 3$ a: 29

   b) $-x^2 - 5x + 4$, for *x=5* a: -46

   c) $3x^3 - 2x^2 - x + 3$, for x=-1 a: -1

8. The number of committees consisting of exactly 3 people that can be formed from a group of n people is given by the polynomial

   $$\frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{1}{3}n.$$

   Find the number of committees consisting of exactly 3 people that can be formed from a group of 6 people.

   Ans.: 20

9. Factor out the greatest common factor (GCF) for each polynomial:
   a) $-15x^2 - 12x$ a: -3x(5x+4)

   b) $(x-4)(2a-b) + (x+4)(2a-b)$ a: $2x(2a-b)$

   c) $6w^3 + 4w^2 - 15w - 10$ (use grouping) a: $(2w^2 - 5)(3w + 2)$

10. Factor each difference of squares:
    a) $x^2 - 9$ a: $(x-3)(x+3)$

    b) $y^4 - 81$ a: $(y^2-9)(y^2+9)=(y-3)(y+3)(y^2+9)$

    c) $(x+5)^2 - 4$ a: (x+7)(x+3)

11. Factor each perfect square trinomial:

a) $x^2 + 10x + 25$ a: $(x + 5)^2$

b) $a^2 - 14a + 49$ a: $(a - 7)^2$

c) $25y^2 + 40y + 16$ a: $(5y + 4)^2$

12. Factor each sum or difference of cubes:

a) $x^3 - 8$ a: $(x - 2)(x^2 + 2x + 4)$

b) $8 - y^6$ a: $(2 - y^2)(4 + 2y^2 + y^4)$

13. Factor each trinomial by the trial and error method:

a) $x^2 + 7x + 12$ a: $(x + 4)(x + 3)$

b) $b^2 + 12b - 28$ a: $(b + 14)(b - 2)$

c) $x^2 + 12x + 35$ a: (x+5)(x+7)

d) $x^2 - 9x + 18$ a: (x-3)(x-6)

14. Simplify each rational expression:

a) $\dfrac{y + 1}{n + ny}$ a: $\dfrac{1}{n},\ n \neq 0,\ y \neq -1$

b) $\frac{a^2-ab}{ab-b^2}$ a: $\frac{a}{b}, b\neq 0, b\neq a$

c) $\frac{9z^2-12z+4}{3z-2}$ a: 3z-2, $z\neq\frac{2}{3}$

d) $\frac{p^2-4}{pq+2q-p-2}$ a: $\frac{p-2}{q-1}, q\neq 1, p\neq -2$

15. Perform the indicated operation(s). State the result in the simplest form:

a) $\frac{p+5}{r}+\frac{2p-7}{r}$ a: $\frac{3p-2}{r}$, $r\neq 0$

b) $\frac{2s+5t}{4t}+\frac{-2s+3t}{4t}$ a: 2, $t\neq 0$

c) $\frac{x}{x-5}+\frac{x}{x+3}$ a: $2x\frac{x-1}{(x-5)(x+3)}, x\neq 5, x\neq -3$

d) $\frac{5y-7}{y+4}-\frac{2y-3}{y+4}$ a: $\frac{3y-4}{y+4}$, $y\neq -4$

e) $\frac{x}{x^2-9}-\frac{3x-1}{x^2+7x+12}$ a: $\frac{-2x^2+14x-3}{(x-3)(x+3)(x+4)}$, $x\neq\pm 3, x\neq -4$

f) $\left[\left(\frac{u}{v}\right)^2 - 1\right] \div \left(\frac{1}{v}\right)^2$ a: $u^2 - v^2, \quad v \neq 0$

g) $\left(1 + \frac{m}{1-m}\right) \div \frac{1+m}{1-m}$ a: $\frac{1}{1+m}, \quad m \neq \pm 1$

h) $\frac{a^2 + ax}{x - x^2} \div \frac{x^2 + ax}{a - ax}$ a: $\frac{a^2}{x^2}, \quad x \neq 0, a \neq -x, x \neq 1$