General Physics Foundation Course – Problem Solving Exercises 5

Exponential and Logarithmic Functions

1. Evaluate the exponential function for the given x-values.

   a) $g(x) = 3^x; x = 0 \text{ and } x = 4$ A: g(0)=1; g(4)=81

   b) $g(x) = 10^x; x = -2 \text{ and } x = 3$ a: g(-2)=0.01; g(3)=1000

   c) $h(x) = \left(\frac{3}{2}\right)^x; x = 2 \text{ and } x = -3;$ a: h(2)=9/4; h(-3)=8/27

   d) $j(x) = \left(\frac{1}{4}\right)^x; x = -1 \text{ and } x = 5$ a: j(-1)=4; j(5)=1/1024

2. Use the calculator to evaluate the exponential function for the given x-values. Round to the nearest hundredth.

   a) $g(x) = 2^x; x = 3.2$ A: g(3.2)=9.19

   b) $h(x) = 0.5^x; x = \pi$ a: h(pi)=0.11

   c) $g(x) = \mathrm{e}^x; x = 2, x = -2, x = 0 \text{ and } x = 2.2$
   a: g(2)=7.39, g(-2)=-0.14, g(0)=1, g(2.2)=9.03

3. For photo-chromatic eyeglass lenses their transparency changes depending on the intensity of the UV light. The intensity of UV rays is measured using a scale called UV index. On this scale, a value near 0 indicates a low UV intensity and a value near 10 indicates a high UV intensity. For a particular type of the photo-chromatic lenses their transparency P depends on the current UV index x as:
   $P(x) = 0.9^x$.

   a) Find the transparency of the lenses, to the nearest tenth of a percent,when they are exposed to light rays with a UV index of 3.5.

   a: P=0.692 <=> 69.2%

b) What is the UV index of light rays that cause these photo-chromatic lenses to have a transparency of 45%? Round to the nearest tenth.

A: use log(0.45)=x*log(0.9) → x=log(0.45)/log(0.9)=7.6

4. A sample of 40.0 g of tritium, $^{3}_{1}H$, is produced. What will be the mass of the tritium remaining after 61.5 years? ($^{3}_{1}H$ half life is 12.3 years)

a: $40*(1/2)^{5}$..... m=1.25 g

5. The half-life of plutonium $^{336}_{94}$Pu is 2.9 years. How long does it take before 1 g of $^{336}_{94}$Pu out of the original amount of 8 g ?

a: $(1/8)=(1/2)^{\text{n-half-lifes}}$ n=3 => $t=t_{\text{half-life}} * 3$; t=8.7 years

6. Write each equation in its exponential form:
   a) $1 = \log 10$ a: $10^1 = 10$
   b) $2 = \log_8 64$ a: $8^2 = 64$
   c) $4 = \log 10{,}000$ a: $10^4 = 10{,}000$
   d) $-4 = \log_3 \frac{1}{81}$ a: $3^{-4} = \frac{1}{81}$
   e) $\ln x = -3$ a: $e^{-3} = x$
   f) $\ln 1 = 0$ a: $\mathrm{e}^0 = 1$

7. Write each equation in its logarithmic form:
   a) $3^2 = 9$ a: $\log_3 9 = 2$
   b) $4^{-2} = \frac{1}{16}$ a: $\log_4 \frac{1}{16} = -2$
   c) $b^x = y$ a: $\log_b y = x$
   d) $y = \mathrm{e}^x$ a: $\ln y = x$
   e) $10^0 = 1$ a: $\log 1 = 0$
   f) $\mathrm{e}^2 = x + 5$ a: $\ln (x + 5) = 2$

8. Evaluate, do not use calculator:
   a) $\log_4 16$ a: 2

   b) $\log_3 \frac{1}{243}$ a: -5

   c) $\log_b b$ a: 1

   d) $\log \frac{1}{100}$ a: -2

   e) $\log_{10} 10^6$ a: 6

   f) $\log_5 125^2$ a: 2*log_5(125)=6

   g) $\log_3 \sqrt[5]{9}$ a: 1/5*log_3(9)= 2/5

   h) $\log_6 \sqrt[3]{36}$ a: 2/3

9. Write each expression as a single logarithm with a coefficient 1:
   a) $\log(x+5) + 2\log x$ a: $\log(x+5) \cdot x^2$

   b) $\frac{1}{2}\log_8(x+5) + 3\log_8 y$ a: $\log_8 (x+5)^{\frac{1}{2}} y^3$

   c) $2\ln(x+4) - \ln x - \ln(x^2-3)$ a: $\ln \frac{(x+4)^2}{x(x^2-3)}$

10. Use the change-of-base formula to approximate the logarithm accurate to the nearest tenthousandth.
   a) $\log_7 20$ A: 1.5395

   b) $\log_{50} 22$ a: 0.7901

   c) $\log_3 \frac{7}{8}$ a: -0.1215

11. What is decibel level (given by dB(I)=10 log(I/I0)) of a fire-alarm with sound intensity I=I0*10^{14}, a hand-drill with sound intensity I=I0*10^{10}, a typical conversation with sound intensity I=I0*10^{6}?

    A: 140, 100, and 60

12. What multiple of the base intensity is the intensity of a moderate rainfall with the decibel level of 50 and a quiet room with decibel level of 40.

    a: 10^{5}*I0, 10^{4}*I0