Note

The 47th proposition . . .: Pythagoras's Theorem.
Gnomon: "the part of a parallelogram which remains after a similar parallelogram is taken away from one of its corners." (*OED*).

Pythagoras's Theorem

Edmund Scarburgh, 1705

For generations of schoolchildren, geometry meant Euclid, and it meant in particular a handful of key theorems, of which we present one of the most famous here: Pythagoras's Theorem. Euclid had first been translated into English in 1570 (there is a section of the preface to that translation in Chapter 10); many other translations followed, often differing little in their choice of words, diagrams, or notation but showing more variation in the annotations and supplements they provided. Scarburgh, as we see here, attempts to give some hint of how the theorem might have been first discovered: compare Joseph Fenn's imagined "first analysts" in Chapter 3.

Edmund Scarburgh, *The English Euclide, being The First Six Elements of Geometry, Translated out of the Greek, with Annotations and useful Supplements* (Oxford, 1705), pp. 108–109.

In a Right-angled Triangle, the Square of the side subtending the Right angle is equal to the Squares of the sides containing the Right angle

Let the Right-angled Triangle be *ABC*, having the Right angle *BAC* (see Figure 4.6). I say that the square of *BC* is equal to the squares of *BA*, *AC*.

For on *BC*, let be described the square *BDEC*, and on *AB*, *AC*, the squares *GB*, *HC*, and by *A* let *AL* be drawn parallel to either of the lines *BD*, *CE*, and let be joined *AD*, *FC*.

Now forasmuch as each of the angles *BAC*, *BAG*, is a Right angle, and to the straight line *BA*, and to a point in the same *A*, the two straight lines *AC*, *AG*, not lying the same way, make the consequent angles equal to two Right, therefore *CA* is ⌞parallel⌟ to *AG*. By the same reason also *AB* is ⌞parallel⌟ to *AH*. And because the angle *DBC* is equal to the angle *FBA*, for each is a Right angle, let the angle *ABC* be added in common, therefore

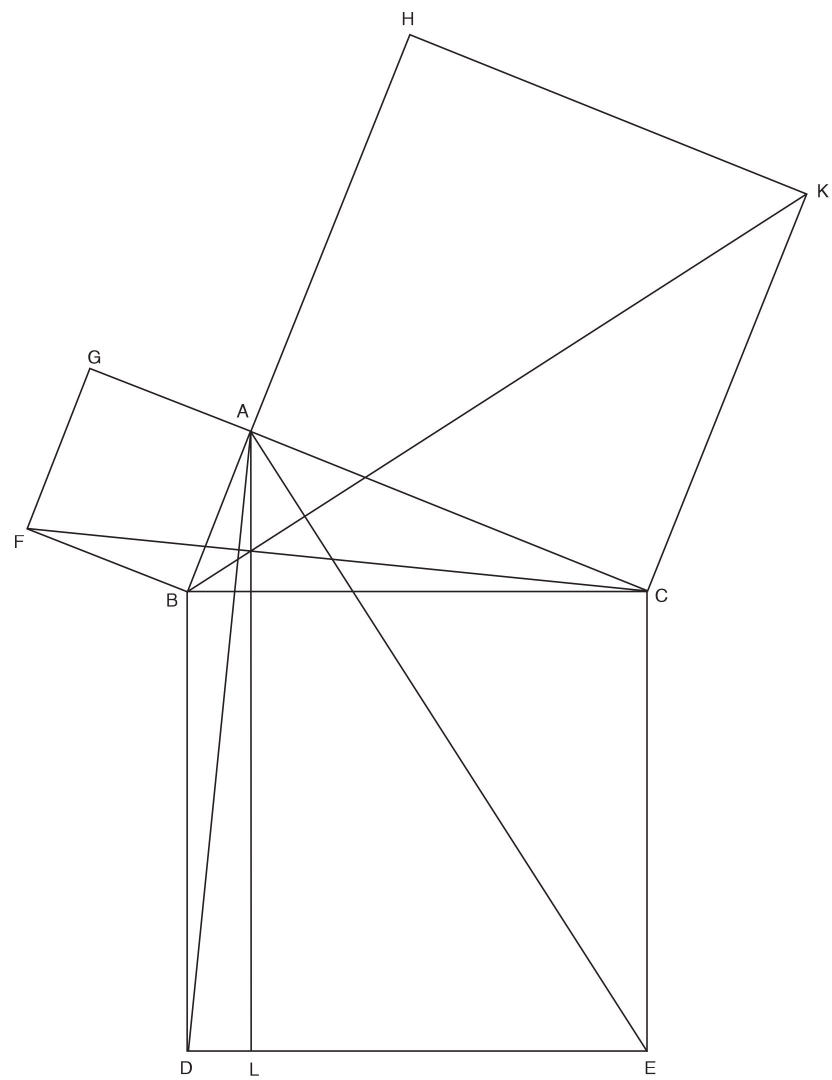

Figure 4.6. The diagram for Pythagoras's Theorem.

the whole angle *DBA* is equal to the whole angle *FBC*. And because the two lines *DB*, *BA*, are equal to the two lines *CB*, *BF*, each to each, and the angle *DBA* is equal to the angle *FBC*, therefore the base *AD* is equal to the base *FC*, and the Triangle *ABD* is equal to the Triangle *FBC*.

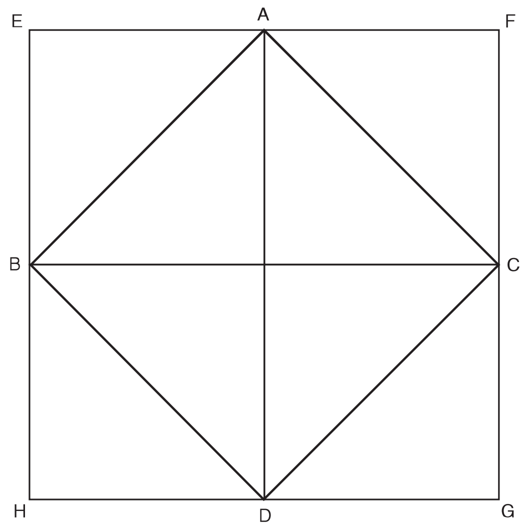

Figure 4.7. How Pythagoras's Theorem might have been first conceived. (Scarburgh, p. 109.)

Now the Parallelogram *BL* is double of the Triangle *ABD*, for they have the same base *BD*, and are in the same parallels *DB*, *AL* (Proposition 41). Also the square *GB* is double of the Triangle *FBC*, for they have the same base *FB*, and are in the same parallels *FB*, *GC*. Now the doubles of equals are equal to one another; therefore the Parallelogram *BL* is equal to the square *GB*.

In like manner, *AE*, *BK* being joined, may be proved that the Parallelogram *CL* is equal to the square *HC*; therefore the whole square *BDEC* is equal to the two squares *GB*, *HC*. And the square *BDEC* is described on *BC*, and *GB*, *HC* on *BA*, *AC*; wherefore the square of the side *BC* is equal to the squares of the sides *BA*, *AC*.

Therefore in Right-angled Triangles, the square of the side subtending the Right angle is equal to the squares of the sides containing the Right angle. Which was to be demonstrated.

This Proposition, among Geometricians most famous, is said to have been found out by *Pythagoras*, and the Invention publicly celebrated with a Sacrifice to the Muses. Yet the hint from whence the discovery of this Truth might first arise, seems to be very obvious.

For in this Figure (4.7) the square *EFGH* is apparently double of the square *ABDC*. But *EFGH* is described on *EF*, which is equal to

BC, the side subtending the Right angle *BAC* of the ⌊isosceles⌋ Triangle *ABC*; and the square *ABDC* is described on either of the sides *AB*, *AC*, containing the Right angle *BAC*, of the same ⌊isosceles⌋ Triangle *ABC*. It is therefore hereupon very reasonable to conceive that the same property might likewise belong to Scalene Right-angled Triangles, and give the occasion of a farther enquiry into this matter.

Thus Geometricians often happen to discover a Truth before they have framed a legitimate demonstration of it, and find out their Propositions one way (which they usually conceal) but prove them in another. We have an Example of this kind in the Remains of *Archimedes*, who shows how first he found the Quadrature of a Parabola° Mechanically, as he calls it, and afterwards gives a Geometrical demonstration.

Note

Quadrature of a parabola: the area under a parabola.

Trigonometrical Definitions

Edward Wells, 1714

It is very difficult to find approachable introductions to trigonometry from this period because—as this extract shows—the terms "sine," "cosine," and so on, were used for *lengths* in a particular construction involving a circle: not, as now, for *ratios* of lengths in a right-angled triangle. This way of using the terms remained in use until the second half of the nineteenth century, and it is not immediately obvious that—as is in fact the case—it is numerically equivalent to the modern way.

Edward Wells's training was in divinity and his profession that of an Anglican clergyman, but he had an abiding interest in education and put together textbooks ("more voluminous than distinguished," according to a recent biographer) on subjects from mathematics to geography, history, and religion. His trigonometry is, for its period, unusually lucid.

Edward Wells (1667–1727), *The Young Gentleman's Trigonometry, Mechanicks, and Opticks. Containing such Elements of the said Arts or Sciences as are most Useful and Easy to be known.* (London, 1714), pp. 1–5.