Příklad 1: Vypočtěte následující limity:

2. $\lim\limits_{x\to 0} \frac{1-\cos x}{x\cdot \sin x} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 0} \frac{\sin x}{\sin x + x\cdot \cos x} = \left[\frac{0}{0}\right]$

$\overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 0} \frac{\cos x}{\cos x + \cos x - x\cdot \sin x} = \frac{1}{1+1-0} = \underline{\underline{\frac{1}{2}}}$

3. $\lim\limits_{x\to 1} \frac{\ln x}{\cos\left(\frac{\pi}{2}x\right)} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 1} \frac{\frac{1}{x}}{-\sin\left(\frac{\pi}{2}\cdot x\right)\cdot \frac{\pi}{2}} = \frac{1}{-\frac{\pi}{2}} = \underline{\underline{-\frac{2}{\pi}}}$

Příklad 1: Vypočtěte následující limity:

6. $\lim\limits_{x\to 0^+} x^3 \cdot \ln \frac{1}{x} = [0\cdot\infty] = \lim\limits_{x\to 0^+} \frac{\ln\frac{1}{x}}{\frac{1}{x^3}} = \left[\frac{\infty}{\infty}\right]$

$\left[\frac{1}{0^+}\right]$ $\quad x^{-3}$

$\overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 0^+} \frac{\frac{1}{\frac{1}{x}}\cdot(-1)\cdot x^{-2}}{-3\cdot x^{-4}} = \lim\limits_{x\to 0^+} \frac{\cancel{x}^{1}\cdot\frac{\pm 1}{x^{\cancel{2}\,1}}}{\frac{\pm 3}{x^4}} =$

$= \lim\limits_{x\to 0^+} \frac{1}{\cancel{x}_{1}}\cdot\frac{x^{\cancel{4}\,3}}{3} = \lim\limits_{x\to 0^+} \frac{x^3}{3} = \underline{\underline{0}}$

9. $\lim_{x\to 0^+} x\cdot\ln x = [0\cdot(-\infty)] = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} = \left[\frac{-\infty}{\infty}\right]$ x^{-1}

$$\overset{\text{L'Hosp.}}{=} \lim_{x\to 0^+} \frac{\frac{1}{\cancel{x}}}{-\frac{1}{x^{\cancel{2}}}} = \lim_{x\to 0^+} -x = \underline{\underline{0}}$$

Příklad 1: Vypočtěte následující limity:

11. $\lim_{x\to 0} (\cos 3x)^{\frac{1}{x^2}} = [1^\infty] = \lim_{x\to 0} e^{\ln(\cos 3x)^{\frac{1}{x^2}}}$

$$= e^{\lim_{x\to 0} \ln(\cos 3x)^{\frac{1}{x^2}}} = e^{\lim_{x\to 0} \frac{1}{x^2}\cdot\ln(\cos 3x)} \overset{(*)}{=} \boxed{e^{-\frac{9}{2}}}$$

$$\lim_{x\to 0} \frac{1}{x^2}\cdot\ln(\cos 3x) = [\infty\cdot 0] = \lim_{x\to 0} \frac{\ln(\cos 3x)}{x^2} = \left[\frac{0}{0}\right]$$

$$\overset{\text{L'Hosp.}}{=} \lim_{x\to 0} \frac{\frac{1}{\cos 3x}\cdot(-\sin 3x)\cdot 3}{2x} = \lim_{x\to 0} \frac{-3\cdot \mathrm{tg}\, 3x}{2x} = \left[\frac{0}{0}\right]$$

$$\overset{\text{L'Hosp.}}{=} \lim_{x\to 0} \frac{-3\cdot\frac{1}{\cos^2 3x}\cdot 3}{2} = \frac{-3\cdot 1\cdot 3}{2} = -\frac{9}{2}$$

12. $\lim\limits_{x\to 1^+} x^{\frac{1}{1-x}} = [1^{-\infty}] = \lim\limits_{x\to 1^+} e^{\ln x^{\frac{1}{1-x}}} =$

$\left[\frac{1}{0^-}\right]$

$= e^{\lim\limits_{x\to 1^+} \ln x^{\frac{1}{1-x}}} = e^{\lim\limits_{x\to 1^+} \frac{1}{1-x}\cdot \ln x} \overset{(*)}{=} e^{-1} = \frac{1}{e}$

$\lim\limits_{x\to 1^+} \frac{1}{1-x}\cdot \ln x = [-\infty \cdot 0] = \lim\limits_{x\to 1^+} \frac{\ln x}{1-x} = \left[\frac{0}{0}\right]$

$\overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 1^+} \frac{\frac{1}{x}}{-1} = \lim\limits_{x\to 1^+} -\frac{1}{x} = \boxed{-1}$

Příklad 1: Vypočtěte následující limity:

16. $\lim\limits_{x\to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right) = [\infty - \infty]$

$= \lim\limits_{x\to 1^+} \frac{x\cdot \ln x - (x-1)}{(x-1)\cdot \ln x} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 1^+} \frac{\ln x + x\cdot \frac{1}{x} - 1}{\ln x + (x-1)\cdot \frac{1}{x}}$

1

$= \lim\limits_{x\to 1^+} \frac{\ln x}{\ln x + \frac{x-1}{x}} = \left[\frac{0}{0+0}\right] \overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 1^+} \frac{\frac{1}{x}}{\frac{1}{x} + \frac{1}{x^2}}$

$1 - \frac{1}{x}$

$1 - x^{-1}$

$= \lim\limits_{x\to 1^+} \frac{\frac{1}{\cancel{x}}\,1}{\frac{x+1}{x^{\cancel{2}}}} = \lim\limits_{x\to 1^+} \frac{x}{x+1} = \frac{1}{2}$

17. $\lim\limits_{x\to 0}\left(\frac{1}{x\cdot\sin x}-\frac{1}{x^2}\right) = [\infty-\infty] = \lim\limits_{x\to 0}\frac{x-\sin x}{x^2\cdot\sin x}$

$$= \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim_{x\to 0}\frac{1-\cos x}{2x\cdot\sin x + x^2\cdot\cos x} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=}$$

$$= \lim_{x\to 0}\frac{\sin x}{2\cdot\sin x + \underbrace{2x\cdot\cos x + 2x\cdot\cos x}_{4x\cdot\cos x} - x^2\cdot\sin x} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=}$$

$$= \lim_{x\to 0}\frac{\overset{1}{\cos x}}{\underset{2}{2\cdot\cos x} + \underset{4}{4\cdot\cos x} - \underset{0}{4x\cdot\sin x} - \underset{0}{2x\cdot\sin x} - \underset{0}{x^2\cdot\cos x}} = \frac{1}{6}$$