Příklad 1: Vypočtěte následující limity:

2. $\lim\limits_{x\to 0} \frac{1-\cos x}{x\cdot\sin x}$ $= \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 0} \frac{0+\sin x}{1\cdot\sin x + x\cdot\cos x}$

$$= \lim_{x\to 0} \frac{\sin x}{\underbrace{\sin x}_{\downarrow\, 0} + x\cdot\cos x} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim_{x\to 0} \frac{\cos x}{\cos x + \cos x - x\cdot\sin x}$$

$$= \frac{1}{1+1-0\cdot 0} = \underline{\underline{\frac{1}{2}}}$$

3. $\lim\limits_{x\to 1} \frac{\ln x}{\cos\left(\frac{\pi}{2}x\right)}$ $= \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim\limits_{x\to 1} \frac{\frac{1}{x}}{-\sin\frac{\pi}{2}x\cdot\frac{\pi}{2}} = \frac{1}{-\frac{\pi}{2}}$

$$= -\frac{2}{\pi}$$

Příklad 1: Vypočtěte následující limity:

6. $\lim\limits_{x\to 0^+} x^3 \cdot \ln \frac{1}{x} = [0 \cdot \infty] = \lim\limits_{x\to 0^+} \dfrac{\ln \frac{1}{x}}{\frac{1}{x^3}} = \left[\frac{\infty}{\infty}\right]$

$\frac{1}{x} \to \infty$, $\quad \frac{1}{x^3} = x^{-3}$

L'Hosp.

$= \lim\limits_{x\to 0^+} \dfrac{\frac{1}{\frac{1}{x}} \cdot (-1) \cdot x^{-2}}{-3 \cdot x^{-4}} = \lim\limits_{x\to 0^+} \dfrac{\cancel{x} \cdot \frac{-1}{x^{\cancel{2}}}}{\frac{-3}{x^4}}$

$= \lim\limits_{x\to 0^+} \dfrac{x^{\cancel{4}\,3}}{3\cancel{x}} = \lim\limits_{x\to 0^+} \dfrac{x^3}{3} = \underline{\underline{0}}$

9. $\lim\limits_{x\to 0^+} x \cdot \ln x = [0 \cdot (-\infty)] = \lim\limits_{x\to 0^+} \dfrac{\ln x}{\frac{1}{x}} = \left[\frac{-\infty}{\infty}\right]$

L'Hosp.

$= \lim\limits_{x\to 0^+} \dfrac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim\limits_{x\to 0^+} \dfrac{-x^{\cancel{2}}}{\cancel{x}\,1} = \underline{\underline{0}}$

Příklad 1: Vypočtěte následující limity:

11. $\lim\limits_{x\to0}(\cos 3x)^{\frac{1}{x^2}} = [1^\infty] = \lim\limits_{x\to 0} e^{\ln(\cos 3x)^{\frac{1}{x^2}}}$

$$= e^{\lim\limits_{x\to0}\ln(\cos 3x)^{\frac{1}{x^2}}} \overset{(*)}{=} \boxed{e^{-\frac{9}{2}}}$$

$$\lim_{x\to0}\ln(\cos 3x)^{\frac{1}{x^2}} = \lim_{x\to0}\frac{1}{x^2}\cdot\ln(\cos 3x) = [\infty\cdot 0]$$

$$= \lim_{x\to0}\frac{\ln(\cos 3x)}{x^2} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim_{x\to0}\frac{\underbrace{\frac{1}{\cos 3x}\cdot(-\sin 3x)}_{-\mathrm{tg}\,3x}\cdot 3}{2x}$$

$$= \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim_{x\to0}\frac{-3\cdot\frac{1}{\cos^2 3x}\cdot 3}{2} = \underline{\underline{\frac{-9}{2}}}\ (*)$$

$-3\cdot\mathrm{tg}\,3x$

12. $\lim\limits_{x\to1^+} x^{\frac{1}{1-x}} = [1^{-\infty}] = \lim\limits_{x\to1^+} e^{\ln x^{\frac{1}{1-x}}}$

$$= e^{\lim\limits_{x\to1^+}\ln x^{\frac{1}{1-x}}} = e^{\lim\limits_{x\to1^+}\frac{1}{1-x}\cdot\ln x} \overset{(*)}{=} e^{-1} = \frac{1}{e}$$

$$\lim_{x\to1^+}\frac{\ln x}{1-x} = \left[\frac{0}{0}\right] \overset{\text{L'Hosp.}}{=} \lim_{x\to1^+}\frac{\frac{1}{x}}{-1} = \underline{\underline{-1}}$$

Příklad 1: Vypočtěte následující limity:

16. $\lim\limits_{x\to 1^+}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right) = [\infty - \infty]$

$$= \lim_{x\to 1^+}\frac{x\cdot\ln x - x + 1}{(x-1)\cdot\ln x} = \left[\frac{0}{0}\right] \overset{L'Hosp.}{=}$$

$$= \lim_{x\to 1^+}\frac{\ln x + \cancel{x\cdot\frac{1}{x}} \cancel{-1}}{\ln x + (x-1)\cdot\frac{1}{x}} = \lim_{x\to 1^+}\frac{\ln x}{\ln x + \underbrace{\frac{x-1}{x}}_{=1-\frac{1}{x}}} = \left[\frac{0}{0}\right]$$

$$\overset{L'Hosp.}{=} \lim_{x\to 1^+}\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{x^2}} = \lim_{x\to 1^+}\frac{\frac{1}{\cancel{x}\,1}}{\frac{x+1}{x^{\cancel{2}}}} = \lim_{x\to 1^+}\frac{x}{x+1} = \frac{1}{2}$$

17. $\lim\limits_{x\to 0}\left(\frac{1}{x\cdot\sin x}-\frac{1}{x^2}\right) = [\infty - \infty]$

$$= \lim_{x\to 0}\frac{x-\sin x}{x^2\cdot\sin x} = \left[\frac{0}{0}\right] \overset{L'Hosp.}{=} \lim_{x\to 0}\frac{1-\cos x}{2x\cdot\sin x + x^2\cdot\cos x} = \left[\frac{0}{0}\right]$$

$$\overset{L'Hosp.}{=} \lim_{x\to 0}\frac{\sin x}{2\cdot\sin x + \underbrace{2x\cdot\cos x + 2x\cdot\cos x}_{4x\cdot\cos x} - x^2\cdot\sin x} = \left[\frac{0}{0}\right]$$

$$\overset{L'Hosp.}{=} \lim_{x\to 0}\frac{\cos x}{\underbrace{2\cos x + 4\cdot\cos x}_{6\cos x} - \underbrace{4x\cdot\sin x}_{0} - \underbrace{2x\cdot\sin x}_{0} - \underbrace{x^2\cdot\cos x}_{0}} = \frac{1}{6}$$