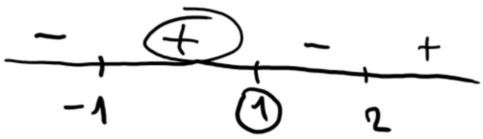
(g) $\lim_{x\to\infty} \frac{\sqrt{x}-6x}{3x+1} = \lim_{x\to\infty} \frac{-6x}{3x} = \underline{\underline{-2}}$

(h) $\lim_{x\to\infty} \left(\sqrt{x-2}-\sqrt{x}\right) \cdot \frac{\sqrt{x-2}+\sqrt{x}}{\sqrt{x-2}+\sqrt{x}} = \lim_{x\to\infty} \frac{\cancel{x}-2-\cancel{x}}{\underbrace{\sqrt{x-2}+\sqrt{x}}_{\infty}} = 0$

(i) $\lim_{x\to 1} \frac{x+1}{x^2-3x+2} = \lim_{x\to 1} \frac{x+1}{(x-2)(x-1)}$



$\lim_{x\to 1^-} f(x) = \infty$

$\lim_{x\to 1^+} f(x) = -\infty$

$\neq \quad \Rightarrow \lim_{x\to 1} f(x)$ neexistuje

$\lim_{x\to 2^-} f(x) = -\infty$

$\lim_{x\to 2^+} f(x) = \infty$

$\neq \quad \Rightarrow \lim_{x\to 2} f(x)$ neexistuje

2 $f(x)=\frac{x+\sqrt{x}+1}{\sqrt{x}}$

Pravidlo pro podíl:

$$f'(x)=\frac{(x+\sqrt{x}+1)'\cdot\sqrt{x}-(\sqrt{x})'\cdot(x+\sqrt{x}+1)}{(\sqrt{x})^2}$$

$$\boxed{(x^n)'=n\cdot x^{n-1}}$$

$$=\frac{\left(1+\frac{1}{2}\cdot x^{-\frac{1}{2}}+0\right)\cdot\sqrt{x}-\frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot(x+\sqrt{x}+1)}{x}$$

$$=\frac{\sqrt{x}+\frac{1}{\cancel{2\sqrt{x}}}\cdot\cancel{\sqrt{x}}-\frac{1}{2\sqrt{x}}\cdot(x+\sqrt{x}+1)}{x}$$

$$=\frac{\sqrt{x}+\cancel{\frac{1}{2}}-\frac{x}{2\sqrt{x}}-\cancel{\frac{1}{2}}-\frac{1}{2\sqrt{x}}}{x}=\frac{\sqrt{x}-\frac{\sqrt{x}}{2}-\frac{\sqrt{x}}{2x}}{x}$$

2 $f(x)=\frac{x+\sqrt{x}+1}{\sqrt{x}}=\frac{x}{\sqrt{x}}+\frac{\sqrt{x}}{\sqrt{x}}+\frac{1}{\sqrt{x}}=x^{\frac{1}{2}}+1+x^{-\frac{1}{2}}$

$$f'(x)=\frac{1}{2}\cdot x^{-\frac{1}{2}}+0-\frac{1}{2}\cdot x^{-\frac{3}{2}}=\dots$$

3 $f(x)=x^2\cdot\ln x$

$$f'(x)=(x^2)'\cdot\ln x+x^2\cdot(\ln x)'$$

$$=2x\cdot\ln x+x^2\cdot\frac{1}{x}$$

$$=\underline{2x\cdot\ln x+x}=x\cdot(2\ln x+1)$$

5 $f(x)=\frac{1+\sin x}{\cos x}$

$$f'(x)=\frac{(1+\sin x)'\cdot\cos x-(\cos x)'\cdot(1+\sin x)}{\cos^2 x}$$

$$=\frac{\cos^2 x+\sin x\cdot(1+\sin x)}{\cos^2 x}=\frac{\overbrace{\cos^2 x+\sin^2 x}^{=1}+\sin x}{\cos^2 x}$$

$$=\frac{1+\sin x}{\cos^2 x}=\frac{1+\sin x}{\underbrace{1-\sin^2 x}_{a^2-b^2}}=\frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1+\sin x)}}$$

$$=\underline{\frac{1}{1-\sin x}}$$

2 $f(x) = e^{x^2-2x+1}$

$(e^x)' = e^x$ vnější funkce

x^2-2x+1 vnitřní funkce

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$

$$\left[e^{x^2-2x+1}\right]' = e^{x^2-2x+1} \cdot (x^2-2x+1)'$$

$$= e^{x^2-2x+1} \cdot (2x-2)$$

3 $f(x) = \ln^3(x^2-1)$

x^3 $\boxed{\ln(x^2-1)}$

$$f'(x) = 3 \cdot \ln^2(x^2-1) \cdot \frac{1}{x^2-1} \cdot 2x$$

$$= \frac{6x \cdot \ln^2(x^2-1)}{x^2-1}$$

8 $f(x) = \operatorname{arctg}\frac{1+x}{1-x}$

$$f'(x) = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \left(\frac{1+x}{1-x}\right)'$$

$$= \frac{1}{1+\frac{(1+x)^2}{(1-x)^2}} \cdot \frac{1\cancel{-x}+1\cancel{+x}}{(1-x)^2}$$

$$= \frac{1}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}} \cdot \frac{2}{(1-x)^2}$$

$$= \frac{\cancel{(1-x)^2}}{1\cancel{-2x}+x^2+1\cancel{+2x}+x^2} \cdot \frac{2}{\cancel{(1-x)^2}} = \frac{2}{2+2x^2} = \frac{1}{1+x^2}$$

1 $f(x) = x^x = e^{\ln x^x} = e^{x \cdot \ln x}$

$$f'(x) = e^{x \cdot \ln x} \cdot (x \cdot \ln x)'$$

$$= e^{x \cdot \ln x} \cdot \left(\ln x + x \cdot \frac{1}{x}\right)$$

$$= e^{x \cdot \ln x} \cdot (\ln x + 1)$$

$$= \underline{x^x \cdot (\ln x + 1)}$$

Příklad 4: Napište rovnici tečny a normály grafu dané funkce v bodě $T = [x_0, y_0]$.

1 $f(x) = \frac{3x-1}{2x+3}$, $T = [2, ?]$

$$f(2) = \frac{3 \cdot 2 - 1}{2 \cdot 2 + 3} = \frac{5}{7} \rightarrow T\left[2, \frac{5}{7}\right]$$

$$f'(x) = \frac{3 \cdot (2x+3) - 2 \cdot (3x-1)}{(2x+3)^2} = \frac{\cancel{6x}+9-\cancel{6x}+2}{(2x+3)^2} = \frac{11}{(2x+3)^2}$$

$$f'(2) = \frac{11}{(2 \cdot 2+3)^2} = \frac{11}{49}$$

směrnice tečny

tečna: $y = kx + q$

$$y = \frac{11}{49} \cdot x + q$$

Dosadím T: $\frac{5}{7} = \frac{11}{49} \cdot 2 + q$

$$\frac{5}{7} - \frac{22}{49} = q$$

$$\frac{35-22}{49} = q \Leftrightarrow q = \frac{13}{49}$$

tečna: $\boxed{y = \frac{11}{49} x + \frac{13}{49}}$

normála: je-li k směrnice tečny, tak normála má směrnici $-\frac{1}{k}$.

$$y = ax + b, \text{ kde } a = -\frac{49}{11}$$

$$y = -\frac{49}{11} x + b$$

Dosadím T: $\frac{5}{7} = -\frac{49}{11} \cdot 2 + b$

$$\frac{5}{7} + \frac{98}{11} = b$$

$$\frac{55+686}{77} = b \Leftrightarrow b = \frac{741}{77}$$

normála: $\boxed{y = -\frac{49}{11} x + \frac{741}{77}}$

3 $f(x) = \frac{8}{x^2+4}$, $T = [2, ?]$

$f(2) = 1 \Rightarrow T[2,1]$

$f'(x) = \left[8 \cdot (x^2+4)^{-1}\right]' = 8 \cdot (-1) \cdot (x^2+4)^{-2} \cdot 2x$

$= \frac{-16x}{(x^2+4)^2}$

$f'(2) = \frac{-32}{64} = -\frac{1}{2}$

tečna: $y = -\frac{1}{2}x + q$

Dosadím T: $1 = -\frac{1}{2} \cdot 2 + q$

$q = 2 \Rightarrow \boxed{y = -\frac{1}{2}x + 2}$ tečna

normála: $y = 2x + b$

Dosadím T: $1 = 2 \cdot 2 + b$

$-3 = b \Rightarrow \boxed{y = 2x - 3}$ normála