$f(x,y) = 2x^3 + y^3 - 6x - 12y + 3$

lok. extrémy ?

Stacion. body – nutné podmínky

$f'_x(x,y) = 0 \;,\; f'_y(x,y) = 0 \;,\; (x,y) \to ?$

$f'_x(x,y) = 6x^2 - 6 \;,\quad f'_y(x,y) = 3y^2 - 12$

$$\begin{cases} 6x^2 - 6 = 0 \\ 3y^2 - 12 = 0 \end{cases} \Leftrightarrow \begin{cases} x^2 = 1 & x = 1,\; x = -1 \\ y^2 = 4 & y = 2,\; y = -2 \end{cases}$$

Stac. body: $(1;2);\ (1;-2);\ (-1;2);\ (-1;-2)$

Hessova matice: $H_f(x,y) = \begin{pmatrix} f''_{xx}(x,y) & f''_{xy}(x,y) \\ f''_{yx}(x,y) & f''_{yy}(x,y) \end{pmatrix}$

$f''_{xy}(x,y)$ (spojitost parc. der.) – z teorie $f''_{xy} = f''_{yx}$

$f''_{xx} = 12x$

$f''_{yy} = 6y$

$f''_{xy} = f''_{yx} = 0$

$H_f(x,y) = \begin{pmatrix} 12x & 0 \\ 0 & 6y \end{pmatrix}$ $\quad \begin{pmatrix} A & B \\ B & C \end{pmatrix}$

$\det H_f(x,y) = AC - B^2$

- $\det H_f > 0$
  - $A > 0$ l. min
  - $A < 0$ l. max
- $\det H_f < 0$ bez extrému
- ~~$\det H_f = 0$~~ neřešíme

$\cancel{f(x,y)} = g(x)$

$H_f(x,y) = \begin{pmatrix} f''_{xx} & 0 \\ 0 & 0 \end{pmatrix}$

$f''_{xx} = g''(x)$ $\quad g''(x_0) > 0 \to$ lok. min. ; $\quad$ model $g(x) = x^2$ $\quad g''(x) = 2 > 0$

$g''(x_0) < 0 \to$ lok. max.; $\quad$ model: $g(x) = -x^2,\; g''(x) = -2 < 0$

$H_f(x,y) = \begin{pmatrix} 12x & 0 \\ 0 & 6y \end{pmatrix}$

Hodnoty H. m. v S.B.:

Bod $(1;2)$: $\quad x = 1,\quad y = 2,\quad H_f(1,2) = \begin{pmatrix} 12 & 0 \\ 0 & 12 \end{pmatrix}$

$\det H_f(1,2) = 144 > 0$, v bodě $(1,2)$ je lok. min.

Hodnota minima je $f(1,2) =$ (dosadit)

Bod $(1,-2)$: $\quad x = 1,\quad y = -2 \quad H_f(1,-2) = \begin{pmatrix} 12 & 0 \\ 0 & -12 \end{pmatrix} = -144 < 0$ $\quad$ extrém není

$$H_f(x,y) = \begin{pmatrix} 12x & 0 \\ 0 & 6y \end{pmatrix}$$

Bod $(-1;2)$: $x=-1$, $y=2$

$$H_f(-1,2) = \begin{pmatrix} -12 & 0 \\ 0 & 12 \end{pmatrix}, \quad \det H_f(-1,2) < 0$$ extrém není

Bod $(-1;-2)$; $x=-1$, $y=-2$

$$H_f(-1,-2) = \begin{pmatrix} -12 & 0 \\ 0 & -12 \end{pmatrix}, \quad \det H_f(-1,-2) > 0$$

$-12 < 0$

$f''_{xx}(-1,-2) = -12 < 0$ lok. max.

Hodnota maxima je $f(-1,-2) = \ldots$

$$f(x,y) = \sqrt{x^2+y^2}$$

$$f'_x(x,y) = \left(\left(x^2+y^2\right)^{\frac{1}{2}}\right)'_x = \frac{1}{2}\left(x^2+y^2\right)^{-\frac{1}{2}} 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f'_y(x,y) = \frac{y}{\sqrt{x^2+y^2}}$$

$f'_x(0,0)$, $f'_y(0,0)$ nejsou defin.

Podezřelý bod: $(0,0)$

$f(x,y) = \sqrt{x^2+y^2} \geq 0$, přičemž $\sqrt{x^2+y^2} = 0 \Leftrightarrow (x,y) = (0,0)$

$f(x,y) > f(0,0)$ pro každé $(x,y) \neq (0,0)$

↳ v bodě $(0,0)$ bude min.

$$f(x,y) = x^3 + y^3$$

polynomy, proto derivace $\exists$ všude

$f'_x(x,y) = 3x^2$, $f'_y(x,y) = 3y^2$

Stac. body: $\begin{cases} 3x^2 = 0 \\ 3y^2 = 0 \end{cases} \Leftrightarrow (x,y) = (0,0)$

$f''_{xx} = 6x$, $f''_{yy} = 6y$, $f''_{xy} = f''_{yx} = 0$

Hessova matice: $H_f(x,y) = \begin{pmatrix} 6x & 0 \\ 0 & 6y \end{pmatrix}$

Jediný podezřelý bod $(0,0)$; v tomto bodě bude

$H_f(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, pak $\det H_f(0,0) = 0$.

↳ postačující podmínka odpověď neposkytuje

Jak se funkce chová v okolí bodu $(0,0)$?

$f(x,y) = x^3 + y^3$

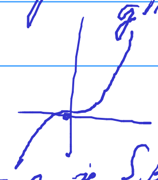
$g(x) = x^3$, $x = 0$

$\forall$ okolí bodu $x=0$ $\exists x_1$: $f(x_1) > f(0) = 0$

$\exists x_2$: $f(x_2) < f(0) = 0$

$g(x) = x^3$

$g''(0) = 0$



$x=0$ je S.B. (inflexe)

$f(x,y) = x^3 + y^3$, podezř. bodem je pouze $(0,0)$, kde $f(0,0)=0$.

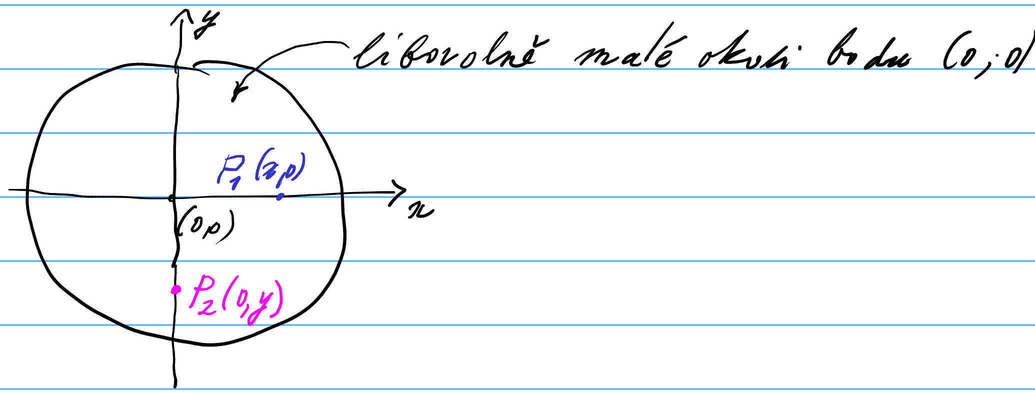
V každém okolí bodu $(0,0)$ lze najít

body P_1 a P_2, $P_2 \neq P_1$: $f(P_1) > 0$, $f(P_2) < 0$

↳ dle definice lok. extr. zde není.

Např. $(x,0) \to (0,0)$, $x>0$; $(0,y) \to (0,0)$, $y<0$;

hodnoty f budou: $f(x,0) = x^3 > 0$ pro $x>0$; $f(0,y) = y^3 < 0$ pro $y<0$



$$f(x,y) = e^{x+y}\cdot(x^2-2y^2) \quad \to \text{l. extr.}$$

S. body: $\quad f'_x=0,\ f'_y=0 \quad$ (parc. derivace $\exists$ na R^2)

$$f'_x(x,y) = \left(e^{x+y}\right)'_x\cdot(x^2-2y^2) + e^{x+y}\cdot(x^2-2y^2)'_x =$$
$$= e^{x+y}(x^2-2y^2) + 2x\cdot e^{x+y} = \underbrace{e^{x+y}}_{>0}(x^2-2y^2+2x)$$

$$f'_y(x,y) = e^{x+y}\cdot(x^2-2y^2) + e^{x+y}(-4y) = \underbrace{e^{x+y}}_{>0}(x^2-2y^2-4y)$$

Stac. body: $\begin{cases} x^2+2x-2y^2=0 \\ x^2-2y^2-4y=0 \end{cases}$

$2x+4y=0, \quad 4y=-2x, \quad x=-2y$

$(-2y)^2-2\cdot 2y-2y^2=0$

$4y^2-2y^2-4y=0, \quad 2y^2-4y=0$

$y\cdot(y-2)=0$

$y=0 \Rightarrow x=0$; $\quad y=2 \Rightarrow x=-2\cdot 2=-4$

Stac. body $(0;0)$ a $(-4;2)$

$$f'_x = e^{x+y}(x^2-2y^2+2x)$$
$$f'_y = e^{x+y}(x^2-2y^2-4y)$$

($x=-4,\ y=2$: $e^{-2}(16-2\cdot$

$$f''_{xx} = e^{x+y}(x^2-2y^2+2x)+e^{x+y}(2x+2) = e^{x+y}(x^2-2y^2+4x+2)$$
$$f''_{yy} = e^{x+y}(x^2-2y^2-4y)+e^{x+y}(-4y-4) = e^{x+y}(x^2-2y^2-8y-4)$$
$$f''_{xy} = e^{x+y}(x^2-2y^2+2x)+e^{x+y}(-4y) = e^{x+y}(x^2-2y^2+2x-4y)$$

$H_f(0;0) = \begin{pmatrix} 2 & 0 \\ 0 & -4 \end{pmatrix}$, $\quad \det H_f(0;0) = -8 < 0 \quad$ extr. není

$H_f(-4;2) = \ldots$

lok. max, hodnota maxima je $f(-4;2) = \frac{8}{e^2}$.

$$x_n = (-1)^n \cdot \frac{n-1}{n+1} + \sin^2\left(\frac{3\pi n}{4}\right)$$

$n = 2k$: $\quad x_n = \frac{n-1}{n+1} + \sin^2\left(\frac{3\pi n}{4}\right) = \frac{2k-1}{2k+1} + \sin^2\left(\frac{3\pi}{4} \cdot 2k\right) =$

x_{2k}

$$= \frac{2k-1}{2k+1} + \sin^2\left(\frac{3\pi}{2} \cdot k\right)$$

$\to 1,\ k \to +\infty$ $\qquad$ k sudé? liché

$\to k = 2m$

$$x_{2k} = x_{4m} = \frac{4m-1}{4m+1} + \underbrace{\sin^2(3\pi \cdot m)}_{0} = \frac{4m-1}{4m+1} \to 1$$

$m \to \infty$

$\to k = 2m+1$

$$x_{2k} = x_{2(2m+1)} = x_{4m+2} = \underbrace{\frac{2(2m+1)-1}{2(2m+1)+1}}_{\downarrow\ 1} + \underbrace{\sin^2\left(\frac{3\pi}{2} \cdot (2m+1)\right)}_{1} \to$$

$\to 1 + 1 = 2$ pro $m \to \infty$

$n = 2k+1$:

$$x_n = (-1)^n \cdot \frac{n-1}{n+1} + \sin^2\left(\frac{3\pi n}{4}\right)$$

$$x_{2k+1} = \underbrace{-\frac{2k}{2k+2}}_{\downarrow\ -1} + \sin^2\left(\frac{3\pi}{4} \cdot (2k+1)\right) =$$

$$= -\frac{2k}{2k+2} + \underbrace{\sin^2\left(\frac{3\pi}{2}k + \frac{3\pi}{4}\right)}_{\left(\pm\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}} \longrightarrow -1 + \frac{1}{2} = -\frac{1}{2},\ k \to +\infty$$

Hrom. body jsou $1;\ 2;\ -\frac{1}{2}$