$$\lim_{x \to 0} \frac{\cos x - e^{x^2}}{x^2} = ? \quad \left(\frac{0}{0}\right)$$

$e^{x^2} = 1$

$f(x) = e^{x^2}, \; f(0) = 1$

$f'(x) = 2x e^{x^2}$

$f''(x) = 2\left(e^{x^2} + x \cdot e^{x^2} \cdot 2x\right) =$

$= 2e^{x^2}(1 + 2x^2)$

$x_0 = 0 \qquad f(0) = 1, \; f'(0) = 0, \; f''(0) = 2$

$e^{x^2} \doteq 1 + 2 \cdot \frac{1}{2!} x^2 = 1 + x^2$

$\overline{\cos x} \doteq 1 + \frac{-1}{2!} x^2$

$$\frac{\cos x - e^{x^2}}{x^2} = \frac{\cancel{1} - \frac{x^2}{2} - \cancel{1} - x^2 + (\text{čl. vyššího ř.})}{x^2}$$

$$= -\frac{3}{2} + (\text{členy s „}x\text{", „}x^2\text{" atd})$$

Φro $x \to 0 \quad \frac{\cos x - e^{x^2}}{x^2} \to -\frac{3}{2}$

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$

$f(x) = \sin x$

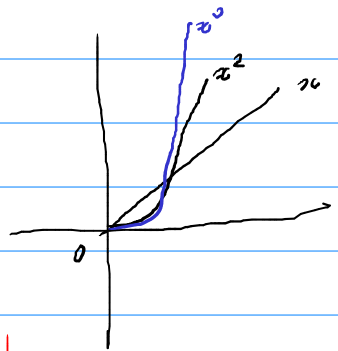
$x_0 = 0$

$\sin x \doteq x - \frac{x^3}{3!}$ pro $x \to 0$

$f'(x) = \cos x, \; f'(0) = \cos 0 = 1$

$f''(x) = -\sin x, \; f''(x_0) = -\sin 0 = 0$

$f'''(x) = -\cos x, \; f'''(0) = -\cos 0 = -1$

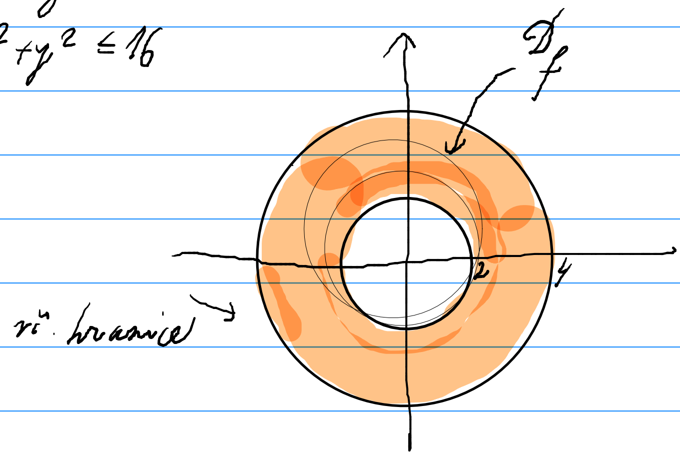


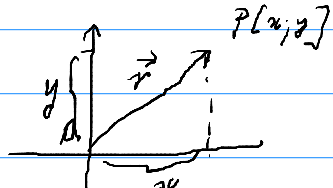
$$f(x,y) = \sqrt{x^2+y^2-4} - 3\cdot\sqrt{16-x^2-y^2}$$

Definiční obor $D_f = ?$

$$D_f = \{(x,y):\ x^2+y^2-4 \geq 0 \wedge 16-x^2-y^2 \geq 0\}$$

$$\begin{cases} x^2+y^2 \geq 4 \\ x^2+y^2 \leq 16 \end{cases}$$



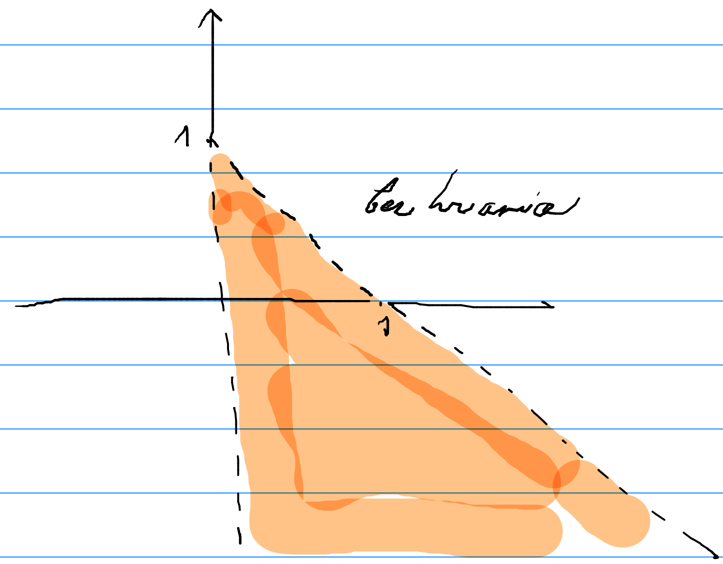


$$|\vec{r}| = \sqrt{x^2+y^2}$$

$$f(x,y)=\frac{\ln(1-x-y)}{(y^2-2y+1)\sqrt{x}}, \qquad D_f = ?$$

$$\begin{cases} y^2-2y+1 \neq 0, \quad x \neq 0 \\ x > 0 \\ 1-x-y > 0 \end{cases}$$

$$\begin{cases} y^2-2y+1 \neq 0 \\ x > 0 \\ x+y < 1 \end{cases}$$



$\frac{1}{x}$ $\qquad\qquad$ $\frac{1}{x^2}$

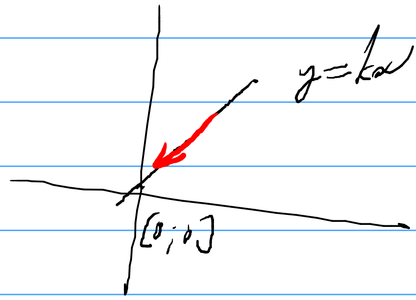
$x \to 0$

" $\frac{1}{0}$ "

Jednoznačnost limity: výsl. nesmí být závislý na cestě

$$f(x,y) = \frac{x+y}{x-y}$$

$$\lim_{(x,y)\to(0,0)} \frac{x+y}{x-y} = ?$$



$y = kx \quad k \in R$

$y = kx$;

$$\frac{x+y}{x-y} = \frac{x+kx}{x-kx} = \frac{x(1+k)}{x(1-k)} = \frac{1+k}{1-k}$$

závisí na směrnici k!

$k=0$: $\frac{x+y}{x-y} = 1$

$k=-1$: $\frac{x+y}{x-y} = 0$

Limita neexistuje.