$h(x) = f(g(x))$ **Derivace složené funkce**

$$h'(x) = f'(g(x)) \cdot g'(x)$$

$$f(x) = \sin(\sin(2x))$$

$$f'(x) = \cos(\sin(2x)) \cdot (\sin 2x)' = \cos(\sin 2x) \cdot \cos 2x \cdot (2x)'$$
$$= 2\cos(\sin 2x) \cdot \cos 2x$$

**Rovnice tečny:** $y = f(x)$ $[x_0; f(x_0)]$

$$y - f(x_0) = f'(x_0) \cdot (x - x_0)$$

$y = kx + b$ $k = f'(x_0)$ směrnice tečny

$y = f'(x_0)x + b$ $b = ?$

$x = x_0,\ y = f(x_0)$:

$$f(x_0) = f'(x_0)x_0 + b$$
$$b = f(x_0) - f'(x_0) \cdot x_0$$

$$y = f'(x_0)x + f(x_0) - f'(x_0) \cdot x_0$$

$x_0 = \frac{\pi}{2}$ $f(x) = \sin(\sin(2x))$

$$f'(x) = 2\cos(\sin 2x) \cdot \cos 2x$$

$$f(x_0) = \sin(\sin \pi) = \sin 0 = 0$$
$$f'(x_0) = 2\cos(\sin \pi)\cos \pi = -2$$

Rovnice tečny v bodě $[\pi/2, 0]$ je

$$y = -2\left(x - \frac{\pi}{2}\right) = -2x + \pi$$

**Rovnice normály** $k_t = f'(x_0) \longrightarrow k_n = -\frac{1}{f'(x_0)}$

$$k_t k_n = -1$$

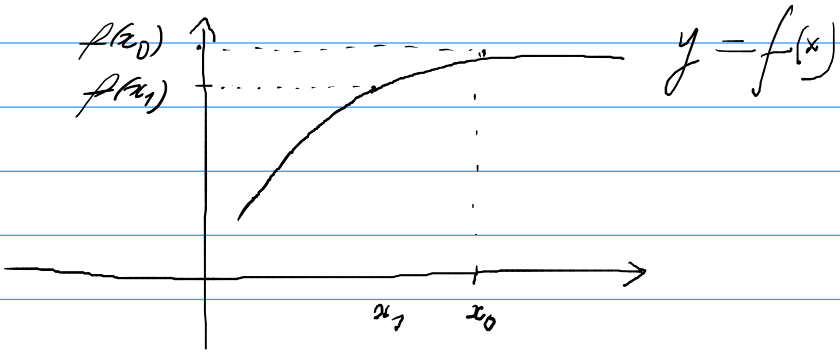
$$y = \frac{1}{2}\left(x - \frac{\pi}{2}\right) = \frac{1}{2}x - \frac{\pi}{4}$$

# Diferenciál funkce

$y=f(x)$ má derivaci

$$df(x)=f'(x)\cdot dx$$

tečna v $[x_0; f(x_0)]$.




$\sqrt{2} \doteq ?$

$f(x_1)\doteq ?$ lze použít diferenciál; myšlenka: přibližně nahradit křivku tečnou přímkou v blízkém bodě, kde se to dá snadněji vypočítat.

$(x^{\alpha})' = \alpha x^{\alpha-1}$

$$f(x)-f(x_0)\doteq f'(x_0)\cdot(x-x_0)$$

$x$ blízko $x_0$

$(\sqrt{x})' = (x^{\frac12})' = \frac12 x^{\frac12-1} = \frac12 x^{-\frac12}$

$f'(x)=\frac{1}{2\sqrt{x}}$

$f(x)=\sqrt{x}$ $\quad f(2)=\sqrt{2} \doteq ?$

$$\underbrace{\sqrt{2}}_{f(2)} - \sqrt{x_0} \doteq \frac{1}{2\sqrt{x_0}}(2-x_0)$$

$x_0=1$: $\quad \sqrt{2}-1 \doteq \frac12(2-1) = \frac12$

$\sqrt{2} \doteq 1+\frac12 = 1{,}5$

$= 1{,}4\ldots$

$\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$ lze odvodit pomocí L'H. pravidla

## L'Hôpitalovo pravidlo:

$\lim\limits_{x\to x_0} \frac{f(x)}{g(x)}$ „$\frac{0}{0}$" nebo „$\frac{\infty}{\infty}$"

$\left( \lim\limits_{x\to x_0} f(x) = 0, \ \lim\limits_{x\to x_0} g(x) = 0 \right)$ ...

$\exists \lim\limits_{x\to x_0} \frac{f'(x)}{g'(x)} = L \Rightarrow \exists \lim\limits_{x\to x_0} \frac{f(x)}{g(x)} = L.$

$\frac{f(x)}{g(x)} = \frac{\sin x}{x}$

$\lim\limits_{x\to 0} f(x) = \sin 0 = 0$

$\lim\limits_{x\to 0} g(x) = 0$

„$\frac{0}{0}$" v okolí bodu $x = 0$

$\frac{f'(x)}{g'(x)} = \frac{(\sin x)'}{x'} = \frac{\cos x}{1} = \cos x$

$x\to 0: \ \cos x \to \cos 0 = 1$

$\hookrightarrow \exists \lim\limits_{x\to 0} \frac{f'(x)}{g'(x)} = 1 \xRightarrow{\text{L'Hôpital (Hospital)}} \exists \lim\limits_{x\to 0} \frac{f(x)}{g(x)} = \lim\limits_{x\to 0} \frac{\sin x}{x} = 1$

$$\lim_{x\to+\infty} \frac{-3x^5+6x^4-2x+4}{81x^5-x-1} = -\frac{3}{81} \qquad \frac{\infty}{\infty}$$

$(x^\alpha)' = \alpha x^{\alpha-1}$ $\quad (x^5)' = 5x^4$ $\quad (x^4)' = 4x^3 \dots$

$$\lim_{x\to+\infty}\frac{-3x^5+6x^4-2x+4}{81x^5-x-1} = \lim_{x\to+\infty}\frac{-15x^4+24x^3-2}{5\cdot 81x^4-1} = \dots = -\frac{3}{81}$$

$$\lim_{x\to+\infty}\frac{\overset{\to\infty}{x}}{\underset{\to\infty}{e^x}} = 0 \qquad \frac{\infty}{\infty}$$


L'H.: $\frac{(x)'}{(e^x)'} = \frac{1}{e^x} = e^{-x} \to 0$ pro $x\to+\infty$.

Pro $x\to+\infty$ $e^{-x}\to 0$

$m\in\mathbb{N}$

$$\lim_{x\to+\infty}\frac{x^m}{e^x} = ?$$

$\frac{x^2}{e^x}$ $\frac{\infty}{\infty}$ : L'H. $\frac{(x^2)'}{(e^x)'} = \frac{2x}{e^x} = 2\cdot\frac{x}{e^x} \to 0$ $\quad x\to+\infty$

$\lim_{x\to+\infty}\frac{x^2}{e^x} = 0$. Proto bude

— " — atd. $\quad \lim_{x\to+\infty}\frac{x^m}{e^x} = 0$.

$(a^x)' = ?$

$(e^x)' = e^x$

$(a^x)' = \left(\left(e^{\ln a}\right)^x\right)' = \left(e^{x\ln a}\right)' = \underbrace{e^{x\ln a}}_{a^x} \cdot (x\ln a)' = a^x \cdot \ln a$

## Srovnání růstu funkcí v nekonečnu.

$+\infty \qquad \ln x \to x^n \to e^x$

$$\lim_{x\to 0+} x^{\frac{3}{4+\ln x}} = \cancel{?}\ e^3$$

$\boxed{0^0 = ?}$

$$x^{\frac{3}{4+\ln x}} = e^{\ln\left(x^{\frac{3}{4+\ln x}}\right)} = e^{\frac{3}{4+\ln x}\ln x} = e^{\frac{3\ln x}{4+\ln x}}$$

$1 \cdot 3 = 3$

$x \to 0+$

$\frac{\infty}{\infty}$

$$\lim_{x\to 0+} \frac{\ln x}{4+\ln x} \overset{L'H}{=} \lim_{x\to 0+} \frac{\frac{1}{x}}{\frac{1}{x}} = 1$$

$$\lim_{n\to+\infty} \sqrt[n]{n} = 1.$$

$$n^{\frac{1}{n}} \qquad \infty^{0}$$

$$n^{\frac{1}{n}} = e^{\ln n^{\frac{1}{n}}} = e^{\frac{1}{n}\ln n} = e^{\frac{\ln n}{n}} \to e^{0} = 1, \quad n\to\infty$$

$$\lim_{n\to+\infty} \frac{\ln n}{n} = 0$$ (viz 2 hw)