$$f(x) = (x+1)e^{x+1}$$

$$\lim_{x\to+\infty} f(x) = +\infty$$

$$\lim_{x\to-\infty}(x+1)e^{x+1} = \lim_{x\to-\infty}\frac{x+1}{e^{-(x+1)}} \overset{\frac{\infty}{\infty}}{=} \lim_{\substack{x\to-\infty\\ \text{L'H}}}\frac{1}{-e^{-(x+1)}} = \lim_{x\to-\infty}\left(-e^{x+1}\right) = 0$$

$$f'(x) = e^{x+1} + (x+1)e^{x+1} = e^{x+1}(1+x+1) = (x+2)e^{x+1}$$

Stac. body (kde je $f'(x)=0$): $x=-2$

$$f'(x) = (x+2)\underbrace{e^{x+1}}_{>0}$$

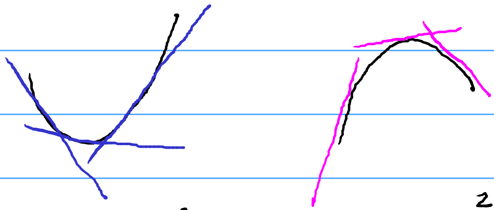
$f'<0$ ↘, $f'>0$ ↗, $-2$: lok. minimum

$$f''(x) = e^{x+1} + (x+2)e^{x+1} = (x+2+1)e^{x+1} = (x+3)e^{x+1}$$

Stac. bod: $f''(-2) = (-2+3)e^{-1} > 0$ (lok. min.)

$f''<0$, $f''>0$, $-3$

**Konvexní – konkávní**



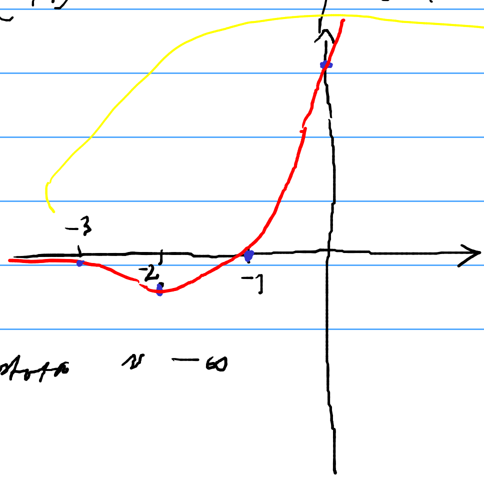
$y=x^2$ $\quad f''(x)>0$

$y=-x^2$ $\quad f''(x)<0$

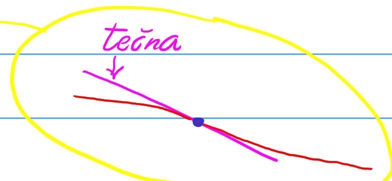
$$f(-2) = (-2+1)e^{-2+1} = -e^{-1} = -\frac{1}{e}$$

$$f(-3) = \dots$$

$f(x) = (x+1)e^{x+1}$ $\quad f>0: x>-1$, $f<0$, $x<-1$









$x=-3$ inflexní bod (změna konvexní – konkávní)

$y=0$ asymptota v $-\infty$

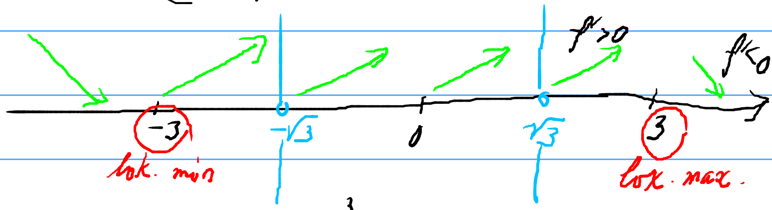
$$f(x) = \frac{x^3}{3-x^2} \qquad \mathcal{D}(f) = \mathbb{R} \setminus \{\pm\sqrt{3}\} \qquad \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{3x^2(3-x^2) - x^3(-2x)}{(3-x^2)^2} = \frac{9x^2 - 3x^4 + 2x^4}{(3-x^2)^2} = \frac{9x^2 - x^4}{(3-x^2)^2} = x^2 \cdot \frac{9-x^2}{(3-x^2)^2}$$

Stac. body: $f'(x) = 0$, $x^2 \cdot \frac{9-x^2}{(3-x^2)^2} = 0$, $x^2(3-x)(3+x) = 0$

$x = 0$, $x = \pm 3$

$$f'(x) = -x^2 \frac{(x-3)(x+3)}{(3-x^2)^2}$$

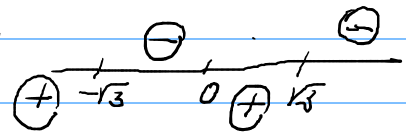



$0$

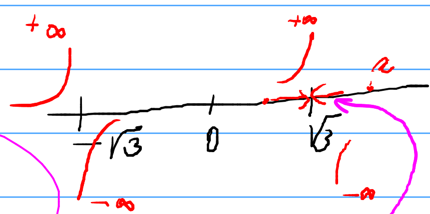
$x \to \sqrt{3}$ nebo $-\sqrt{3}$ ?

$$f(x) = \frac{x^3}{3-x^2} = -\frac{x^3}{(x-\sqrt{3})(x+\sqrt{3})}$$

znaménko $f(x)$:




$$f(x) = -\frac{x^3}{(x-\sqrt{3})(x+\sqrt{3})}$$




$$\lim_{x \to -\sqrt{3}^+} f(x) = -\infty$$

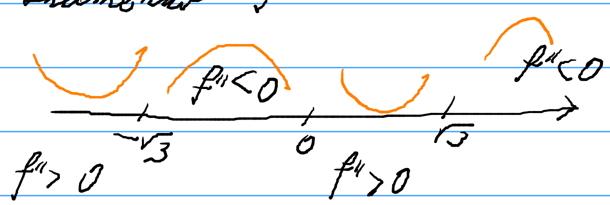
$x = \pm\sqrt{3}$ asymptoty svislé

znaménko $f''$

$$f''(x) = -6x \cdot \frac{x^2+9}{(x^2-3)^3}$$

$>0$

$$= -6x \cdot \frac{x^2+9}{(x-\sqrt{3})^3(x+\sqrt{3})^3}$$




Asymptota $\quad y = kx + b$

$$k = \lim_{\substack{x \to +\infty \\ (-\infty)}} \frac{f(x)}{x}, \qquad b = \lim_{\substack{x \to +\infty \\ (-\infty)}} (f(x) - kx)$$

$$\lim_{\substack{+\infty \\ (-\infty)}} (f(x) - kx - b) = 0 \qquad f(x) - kx - b = g(x), \quad g(x) \to 0 \quad x \to \pm\infty$$

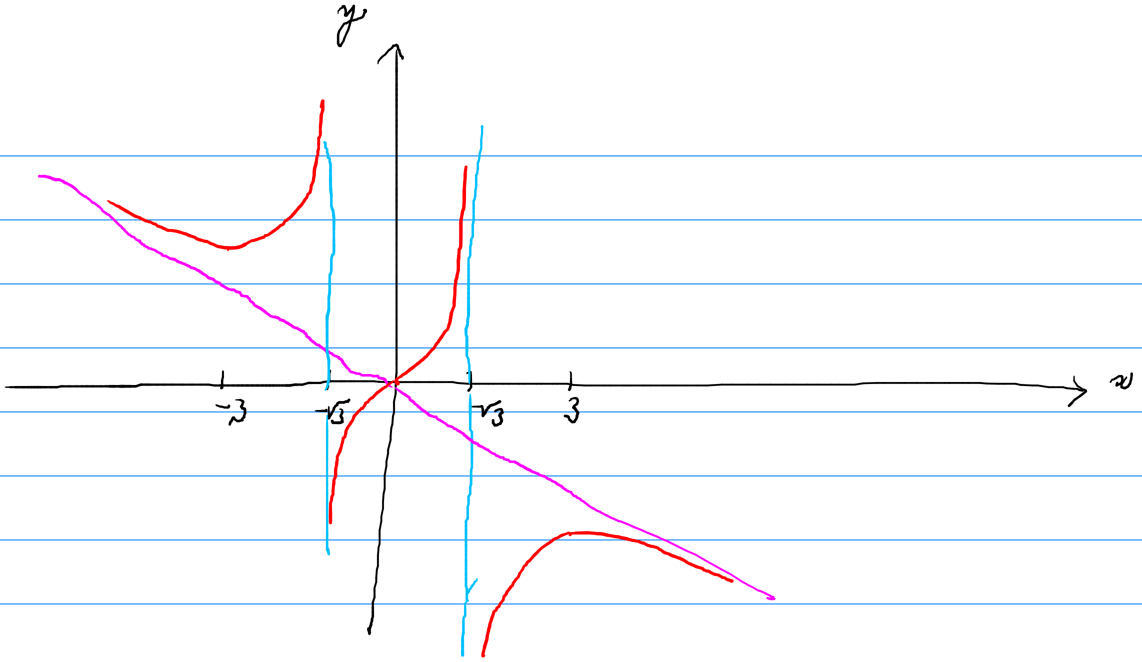
$$\frac{f(x)}{x} - k - \frac{b}{x} = \frac{g(x)}{x}$$

$x \to +\infty$ $(-\infty)$ $\quad \frac{b}{x} \to 0 \quad \frac{g(x)}{x} \to 0$

$$f(x) = \frac{x^3}{3-x^2} \qquad \frac{f(x)}{x} = \frac{x^2}{3-x^2} \longrightarrow -1, \quad x \to +\infty \ (-\infty)$$

$$f(x) - (-1)\cdot x = \frac{x^3}{3-x^2} + x = \frac{x^3 + 3x - x^3}{3-x^2} = \frac{3x}{3-x^2} \to 0, \quad x \to +\infty \ (-\infty)$$

Asymptota $\quad y = -x$


y
x
-3
-√3
√3
3