1. $a_n = \frac{-3n}{n+2} \cdot \cos\left(\frac{n\pi}{2}\right)$

$a_1 = \frac{-3\cdot 1}{1+2} \cdot \cos\frac{\pi}{2} = \overset{1}{\cancel{3}} \cdot 0 = 0 \longrightarrow a_5 = \frac{-3\cdot 5}{5+2}\cdot \cos\frac{5\pi}{2} = \frac{-15}{7}\cdot\cos\frac{\pi}{2} = -\frac{15}{7}\cdot 0 = 0 \longrightarrow \dots$

$a_2 = \frac{-3\cdot 2}{2+2} \cdot \cos\frac{2\pi}{2} = -\frac{6}{4}\cdot(-1) = \frac{3}{2} \rightarrow a_6 = \frac{-3\cdot 6}{6+2} \cos\frac{6\pi}{2} = -\frac{18}{8}\cdot\cos\frac{2\pi}{2} = -\frac{18}{8}\cdot(-1) = \frac{18}{8} \rightarrow \dots$

$a_3 = \frac{-3\cdot 3}{3+2} \cdot \cos\frac{3\pi}{2} = -\frac{9}{6}\cdot 0 = 0 \rightarrow a_7 = \frac{-3\cdot 7}{7+2} \cos\frac{7\pi}{2} = \frac{-21}{9}\cdot\cos\frac{3\pi}{2} = -\frac{21}{9}\cdot 0 = 0 \rightarrow \dots$

$a_4 = \frac{-3\cdot 4}{4+2} \cdot \cos\frac{4\pi}{2} = -2\cdot 1 = -2 \rightarrow a_8 = \frac{-3\cdot 8}{8+2} \cos\frac{8\pi}{2} = \frac{-24}{10}\cdot\cos\frac{4\pi}{2} = -\frac{24}{10}\cdot 1 = -\frac{24}{10} \rightarrow \dots$

Rozdělme na 3 podposloupnosti:

- $n = 4k+1, n = 4k+3, k \in \mathbb{N}\cup\{0\} \rightarrow n = 2k+1$ $\Big\}$ $\lim\limits_{n\to\infty} \frac{-3n}{n+2}\cdot\cos\left(\frac{n\pi}{2}\right) = \lim\limits_{k\to\infty} \frac{-3\cdot(2k+1)}{2k+1+2} \cdot \lim\limits_{k\to\infty}\left[\cos\frac{(2k+1)\pi}{2}\right] =$
  $= \lim\limits_{k\to\infty} \frac{-6k-3}{2k+3}\cdot \lim\limits_{k\to\infty}\left[\cos\left(\frac{2k\pi}{2} + \frac{\pi}{2}\right)\right] = -3\cdot 0 = 0 \Rightarrow \boxed{0 \in H(a_n)}$
- $n = 4k+2, k\in\mathbb{N}\cup\{0\}$: $\lim\limits_{n\to\infty} \frac{-3n}{n+2}\cdot\cos\left(\frac{n\pi}{2}\right) = \lim\limits_{k\to\infty} \frac{-3(4k+2)}{4k+2+2} \cdot \lim\limits_{k\to\infty}\left[\cos\frac{(4k+2)\cdot\pi}{2}\right] =$
  $= \lim\limits_{k\to\infty} \frac{-12k-6}{4k+4}\cdot \lim\limits_{k\to\infty}\left[\cos\left(\frac{4k\pi}{2} + \pi\right)\right] = -3\cdot(-1) = 3 \Rightarrow \boxed{3 \in H(a_n)}$
- $n = 4k+4, k\in\mathbb{N}\cup\{0\}$: $\lim\limits_{n\to\infty} \frac{-3n}{n+2}\cdot\cos\left(\frac{n\pi}{2}\right) = \lim\limits_{k\to\infty} \frac{-3\cdot(4k+4)}{4k+4+2} \cdot \lim\limits_{k\to\infty}\left[\cos\frac{(4k+4)\pi}{2}\right] =$
  $= \lim\limits_{k\to\infty} \frac{-12k-12}{4k+6}\cdot \lim\limits_{k\to\infty}\left[\cos\left(\frac{4k\pi}{2} + 2\pi\right)\right] = -3\cdot 1 = -3 \Rightarrow \boxed{-3 \in H(a_n)}$

$H(a_n) = \{-3, 0, 3\} \Rightarrow \liminf a_n = -3$
$\limsup a_n = 3$

2. $f(x) = \cos(x) + 3x, \ T[0, ?]$

$f(0) = \cos(0) + 3\cdot 0 = 1 \Rightarrow T[0, 1]$

$f'(x) = -\sin x + 3 \Rightarrow f'(0) = -\sin(0) + 3 = 3 \Rightarrow k = 3$

tečna t: $y = kx + q \Rightarrow y = 3x + q$
Dosazením $T[0,1]$: $1 = 3\cdot 0 + q \Rightarrow \boxed{t: y = 3x + 1}$

normála n: $y = -\frac{1}{k}\cdot x + b \Rightarrow y = -\frac{1}{3}x + b$
Dosazením $T[0,1]$: $1 = -\frac{1}{3}\cdot 0 + b \Rightarrow \boxed{n: y = -\frac{1}{3}x + 1}$

3. $f(x) = \ln x - x^2$

$D(f) = \mathbb{R}^+ = (0, \infty)$

$f'(x) = \frac{1}{x} - 2x = \frac{1-2x^2}{x} \Rightarrow f'(x) = 0 \Leftrightarrow 1 - 2x^2 = 0 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \pm\frac{\sqrt{2}}{2}$

Nulové body či body, v nichž $f'(x)$ neex.: $x = \frac{\sqrt{2}}{2}$ (s ohledem na $D(f)$)

$f(x)$ je rostoucí pro $x \in (0, \frac{\sqrt{2}}{2})$
je klesající pro $x \in (\frac{\sqrt{2}}{2}, \infty)$

$f(\frac{\sqrt{2}}{2}) = \ln(\frac{\sqrt{2}}{2}) - (\frac{\sqrt{2}}{2})^2 = \ln(\frac{\sqrt{2}}{2}) - \frac{1}{2} \Rightarrow$ lokální maximum v bodě $[\frac{\sqrt{2}}{2}, \ln(\frac{\sqrt{2}}{2}) - \frac{1}{2}]$

4. $f(x) = \dfrac{3x^2+2x}{x-4}$

Asymptota se směrnicí $y = ax+b$

$$\to a = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{\frac{3x^2+2x}{x-4}}{x} = \lim_{x\to\pm\infty} \frac{3x^2+2x}{x(x-4)}$$

$$= \lim_{x\to\pm\infty} \frac{3x^2+2x}{x^2-4x} = 3$$

$$\to b = \lim_{x\to\pm\infty} (f(x) - ax) = \lim_{x\to\pm\infty} \frac{3x^2+2x}{x-4} - 3x$$

$$= \lim_{x\to\pm\infty} \frac{3x^2+2x-3x\cdot(x-4)}{x-4} = \lim_{x\to\pm\infty} \frac{\cancel{3x^2}+2x-\cancel{3x^2}+12x}{x-4} =$$

$$= \lim_{x\to\pm\infty} \frac{14x}{x-4} = 14$$

Pro $x \to \pm\infty$ je asymptotou se směrnicí přímka $\boxed{y = 3x+14}$

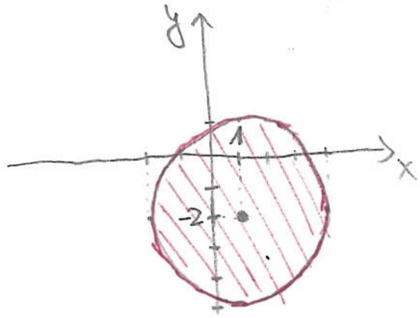
5. $z = 1 + \sqrt{4+2x-4y-x^2-y^2}$

Podmínka pro $D(z)$: $4+2x-4y-x^2-y^2 \geq 0$

$$4 \geq (x^2-2x+1) - 1 + (y^2+4y+4) - 4$$

$$3^2 = 9 \geq (x-1)^2 + (y+2)^2 \to \text{kružnice o poloměru 3 se středem } [1,-2]$$

$D(z) = \{[x,y] \in \mathbb{R}^2 \mid (x-1)^2 + (y+2)^2 \leq 3^2\}$



6. $f(x,y) = -6xy - x^2 - y^3 + 4$

$D(f) = \mathbb{R}^2$

$$\left.\begin{aligned} f'_x &= -6y - 2x \\ f'_y &= -6x - 3y^2 \end{aligned}\right\} \begin{aligned} &-6y - 2x = 0 \Rightarrow 2x = -6y \Rightarrow x = -3y \\ &-6x - 3y^2 = 0 \end{aligned}$$

$$-6(-3y) - 3y^2 = 0 / :3$$

$$+6y - y^2 = 0$$

$$y\cdot(6-y) = 0 \quad \begin{cases} y_1 = 0 \Rightarrow x_1 = -3\cdot 0 = 0 \Rightarrow S_1[0,0] \\ y_2 = 6 \Rightarrow x_2 = -3\cdot 6 = -18 \Rightarrow S_2[-18,6] \end{cases}$$

$$\left.\begin{aligned} f''_{xx} &= -2,\ f''_{xy} = -6 \\ f''_{yx} &= -6,\ f''_{yy} = -6y \end{aligned}\right\} H(x,y) = \begin{vmatrix} -2 & -6 \\ -6 & -6y \end{vmatrix} = (-2)\cdot(-6y) - (-6)^2 = 12y - 36$$

$S_1[0,0]$: $H(0,0) = 12\cdot 0 - 36 < 0 \Rightarrow S_1$ sedlový bod

$S_2[-18,6]$: $H(-18,6) = 12\cdot 6 - 36 > 0 \Rightarrow S_2$ lok. extrém

$f''_{xx}(-18,6) = -2 < 0 \Rightarrow S_1$ lokální maximum