$f(x) = \ln(2x^2+4)$, $\quad f(\frac{1}{2}) \doteq ?$ pomocí Taylorova polynomu st. 2 v okolí bodu $x_0 = 0$

$$f(x) \doteq \underbrace{f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2}_{\text{Taylorův polynom st. 2}} \quad \text{pro } x \text{ blízko } x_0$$

$x_0 = 0$

$$f(x_0) = f(0) = \ln(0+4) = \ln 4$$

$$f'(x) = \left(\ln(2x^2+4)\right)' = \frac{1}{2x^2+4}\cdot(2x^2+4)' = \frac{4x}{2x^2+4} = \frac{2x}{x^2+2}$$

$$f'(x_0) = f'(0) = 0$$

$$f''(x) = 2\left(\frac{x}{x^2+2}\right)' = 2\,\frac{1\cdot(x^2+2) - x(x^2+2)'}{(x^2+2)^2} = 2\cdot\frac{x^2+2-x\cdot 2x}{(x^2+2)^2} = 2\,\frac{2-x^2}{(x^2+2)^2}$$

$$f''(x_0) = f''(0) = 2\cdot\frac{2-0}{(0+2)^2} = 2\cdot\frac{2}{4} = 1.$$

Pak bude $\quad f(x) \doteq f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2$

$$\ln(2x^2+4) \doteq \ln 4 + \frac{0}{1!}(x-0) + \frac{1}{2!}(x-0)^2 =$$

$$= \ln 4 + \frac{x^2}{2}$$

$x = \frac{1}{2}$; $\quad f(\frac{1}{2}) = \ln(\underbrace{2\cdot\frac{1}{4}+4}_{9/2}) \doteq \ln 4 + \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 = \ln 4 + \frac{1}{8} \doteq 1{,}6363$

(vypočítáme-li přibližně $\ln 4$)