$$f(x) = \frac{(x-1)^3}{3x^2} \qquad \text{průběh}$$

$D_f = \mathbb{R} \setminus \{0\}$ $x=0$: bod nespojitosti

$x=0$: $\frac{1}{0}$ – nekonečné jednostr. limity (aspoň jedna z nich).

$$\lim_{x\to 0+} \frac{(x-1)^3}{3x^2} = -\infty$$

$x \to 0,\ x > 0$ $(x-1)^3 < 0$ $3x^2 > 0$ $f(x) < 0$

$\lim_{x\to 0} \frac{1}{x^2} = +\infty$

$$\lim_{x\to 0-} \frac{(x-1)^3}{3x^2} = -\infty \qquad 3x^2 > 0$$

$$\Rightarrow \lim_{x\to 0} f(x) = -\infty$$

nespojitost 2. typu

$x=0$ je asymptotou bez směrnice

Nul. bod: $x=1$.

Monot., lokální extrémy:

$$f(x) = \frac{1}{3}\cdot\frac{(x-1)^3}{x^2}$$

$$f'(x) = \frac{1}{3}\cdot\frac{3(x-1)^2\cdot x^2 - (x-1)^3\cdot 2x}{(x^2)^2} = \frac{(x-1)^2 x}{3}\cdot\frac{3\cdot x - 2(x-1)}{x^4} =$$

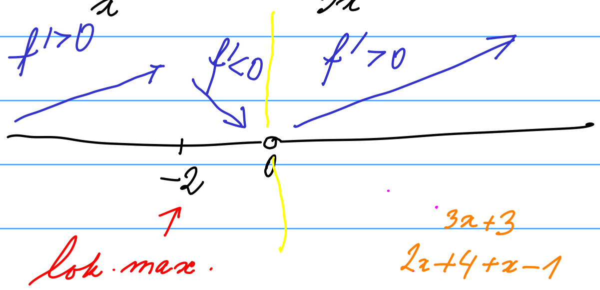
$$= \frac{(x-1)^2}{3}\cdot\frac{3x-2x+2}{x^3} = \frac{(x-1)^2}{3x^3}(x+2) = \boxed{\frac{\boxed{(x-1)^2}(x+2)}{3x^3}}$$

$(x-1)^2 \geq 0$

→ znaménko $f'(x)$

$(x^2)' = 2x$

$((x-1)^2)' = 2(x-1)\cdot\underbrace{(x-1)'}_{1} = 2(x-1)$



$3x+3$

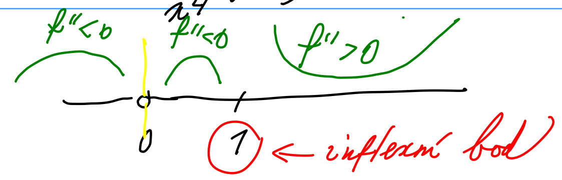
$2x+4+x-1$

$$f''(x) = \frac{1}{3}\left(\frac{(x-1)^2(x+2)}{x^3}\right)' = \frac{1}{3}\cdot\frac{\left(2(x-1)(x+2)+(x-1)^2\right)x^3 - (x-1)^2(x+2)\,3x^2}{x^6} =$$

$$= \frac{(x-1)x^3(2x+4+x-1) - (x-1)^2(x+2)\,3x^2}{3x^6} = \frac{(x-1)x^2\left[x(3x+3) - 3(x-1)(x+2)\right]}{3x^{4}} =$$

$3x+1$ $-2x+6$

$$= \frac{x-1}{3x^4}\left[\cancel{3x^2}+\cancel{3x}-\cancel{3x^2}+\cancel{3x}-\cancel{6x}+6\right] = \frac{2}{x^4}\cdot(x-1)$$



$f(x)=\frac{1}{3}\cdot\frac{(x-1)^3}{x^2}$ $\qquad x\to\pm\infty$?

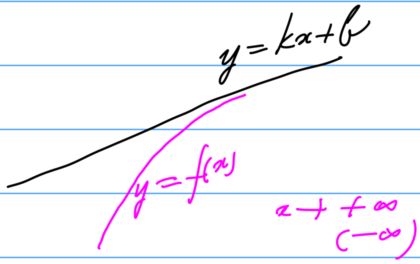
$$\lim_{x\to+\infty} f(x) = +\infty \qquad \left[\ \frac{(x-1)^3}{x^2}\right.$$

$(x-1)^3 \sim x^3$, $x\to+\infty$

$\frac{x^3}{x^2}=x\to+\infty$

$$\lim_{x\to-\infty} f(x) = -\infty$$

Asymptota se směrnicí: přímka $y=kx+b$



$\underbrace{f(x)-(kx+b)}$ $\to 0$ pro $x\to+\infty$ $(-\infty)$

$f(x)-kx-b=u(x)$, $u(x)\to 0$ pro $x\to+\infty$ $(-\infty)$

$$\frac{f(x)}{x}-k-\underbrace{\frac{b}{x}}_{\to 0}=\underbrace{\frac{u(x)}{x}}_{\to 0}$$

$$k=\lim_{\substack{x\to+\infty\\(-\infty)}}\frac{f(x)}{x}$$

$\exists \lim_{\substack{x\to\infty\\(-\infty)}}\frac{f(x)}{x}=?$

$f(x)-kx-b=u(x)$, $u(x)\to 0$

$f(x)-kx\to b$

↓ nižší ř.

$$\frac{f(x)}{x}=\frac{1}{3}\frac{(x-1)^3}{x^2\cdot x}=\frac{1}{3}\frac{(x-1)^3}{x^3}=\frac{1}{3}\cdot\frac{x^3+\dots}{x^3}\to\frac{1}{3},\quad x\to+\infty\ (-\infty)$$

$\boxed{k=\frac{1}{3}}$

$$f(x)-kx=\frac{1}{3}\frac{(x-1)^3}{x^2}-\frac{x}{3}=\frac{1}{3}\cdot\left[\frac{(x-1)^3-x^3}{x^2}\right]$$

$$=\frac{1}{3x^2}(\cancel{x}-1-\cancel{x})\left((x-1)^2+x(x-1)+x^2\right)\to -1,\quad x\to+\infty,\ -\infty$$

$x^2-2x+1+x^2-x+x^2$

$$-\frac{3x^2-3x+1}{3x^2}\to -1$$

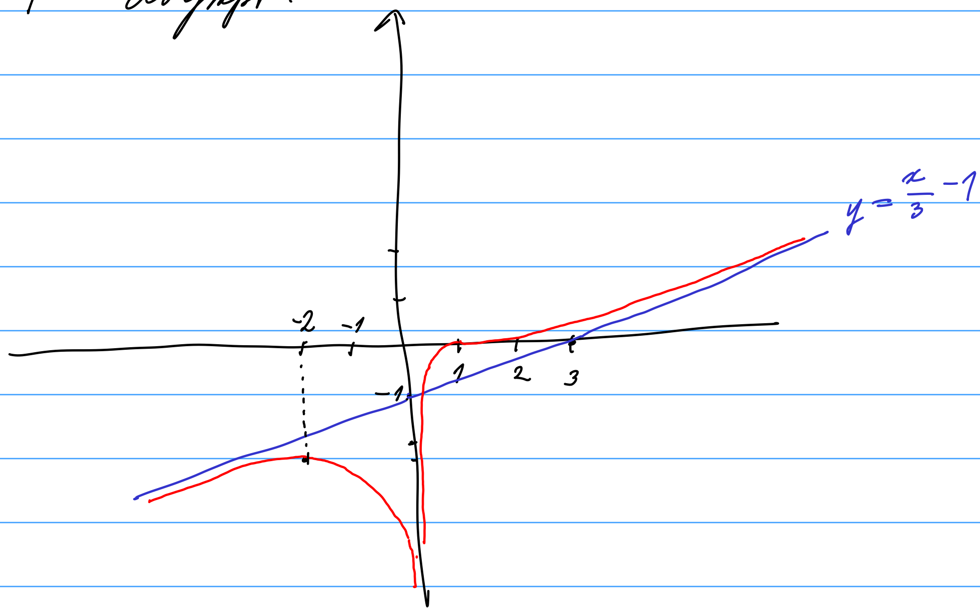
$b=-1.$

$y=\frac{1}{3}x-1$ — asymptota pro $x\to\pm\infty$

$f(x) = \frac{(x-1)^3}{3x^2}$ $f(1) = 0$ (infl. bod)
stac. bod: $f'(1) = 0$
$f''(1) = 0.$

$f(-2) = \frac{(-2-1)^3}{3\cdot 4} = \frac{-3\cdot 9}{3\cdot 4} = -\frac{9}{4} = -2{,}25$

$y = \frac{x}{3} - 1$ asympt.



je sudá, $f(x) = f(-x)\ \forall x$

$f(x) = 1 - x^{\frac{2}{5}} + x^{2}$ $f(0) = 1$

$D_f = \mathbb{R}$

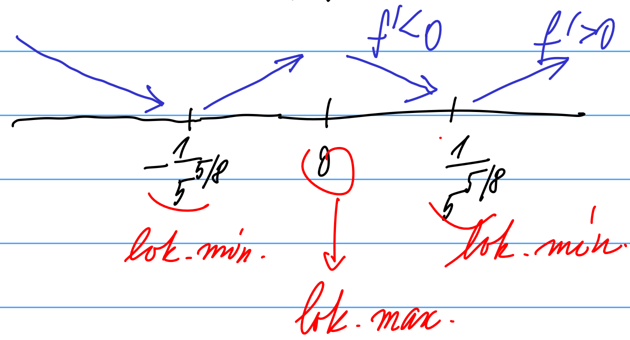
$f'(x) = -\frac{2}{5}x^{\frac{2}{5}-1} + 2x = -\frac{2}{5}x^{-\frac{3}{5}} + 2x$ $\nexists f'(0)$ - nevlastní hodnota.

Stac. bod: $f'(x) = 0$ (nebo $f'(x)\ \nexists$)

$f'(x) = 0$: $\frac{2}{5}x^{-\frac{3}{5}} = 2x$, $\frac{1}{5}x^{-\frac{3}{5}} = x$, $\frac{1}{5} = x^{1+\frac{3}{5}}$

$\frac{1}{5} = x^{\frac{8}{5}}$, $x = \pm\left(\frac{1}{5}\right)^{5/8} = \pm\frac{1}{5^{\frac{5}{8}}}$

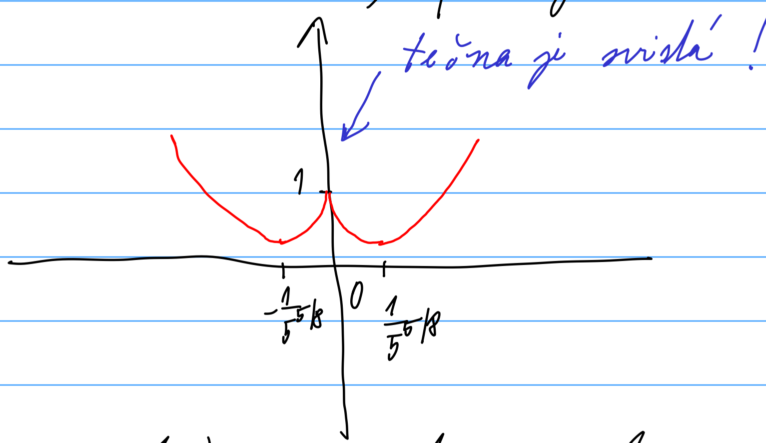
$$f'(x) = 2\left(x - \frac{1}{5}x^{-\frac{3}{5}}\right) = \frac{2}{x^{\frac{3}{5}}}\left(x^{1+\frac{3}{5}} - \frac{1}{5}\right) = \frac{2}{x^{\frac{3}{5}}}\left(x^{\frac{8}{5}} - \frac{1}{5}\right)$$



$$f'(x) = -\frac{2}{5}x^{-\frac{3}{5}} + 2x$$

$$f''(x) = -\frac{2}{5}\cdot\left(-\frac{3}{5}\right)x^{-\frac{3}{5}-1} + 2 = \frac{6}{25}x^{-\frac{8}{5}} + 2 = \frac{6}{25}\cdot\underbrace{\frac{1}{(x^{1/5})^8}}_{>0} + \underbrace{2}_{>0}$$

↳ $f''(x) > 0 \quad \forall x \in \mathbb{R}$, fce je konvexní



$x = \frac{1}{5^{5/8}}$ $\qquad f\left(\frac{1}{5^{5/8}}\right) = 1 - \frac{1}{5^{5/8\cdot 2/5}} + \frac{1}{5^{5/8\cdot 2}} = 1 - \frac{1}{5^{1/4}} + \frac{1}{5^{5/4}} > 0$

$$f(x) = 1 - x^{\frac{2}{5}} + x^{2}$$

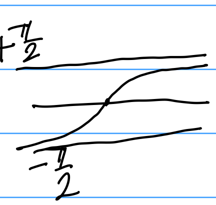
$f(x) = x + \operatorname{arctg} x$ $\qquad D_f = \mathbb{R}$

$f(0) = 0$ $\qquad$ spojitá

$f'(x) = 1 + \frac{1}{1+x^2} > 0$, rostoucí

0 pro $x \to \pm\infty$

$\frac{f(x)}{x} = \frac{x + \operatorname{arctg} x}{x} = 1 + \frac{\operatorname{arctg} x}{x} \to 1, \; x \to \pm\infty$



$k = 1$

$\lim_{x \to \pm\infty} \operatorname{arctg} x = \pm\frac{\pi}{2}$

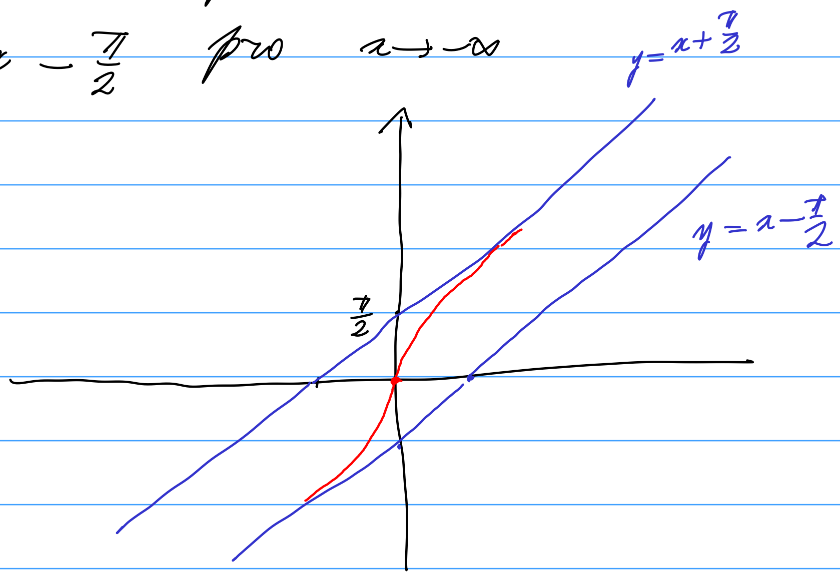
$|\operatorname{arctg} x| \le \frac{\pi}{2}$

$\frac{|\operatorname{arctg} x|}{x} \to 0$ pro $x \to \pm\infty$

$f(x) - kx = \cancel{x} + \operatorname{arctg} x - \cancel{x} = \operatorname{arctg} x$ $\to \frac{\pi}{2}$ pri $x \to +\infty$, $\to -\frac{\pi}{2}$ pri $x \to -\infty$

$y = x + \frac{\pi}{2}$ pro $x \to +\infty$

$y = x - \frac{\pi}{2}$ pro $x \to -\infty$



$f'(x) = 1 + \frac{1}{1+x^2}$, $\quad f''(x) = -\frac{1}{(1+x^2)^2} \cdot 2x = -2 \cdot \frac{x}{(1+x^2)^2}$

$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

f'': $\cup$ | $\cap$ ($f'' < 0$)

0 (inflexní bod).