Making Things 'Write:' Supporting Mathematically Promising Students

Tutita M. Casa
University of Connecticut
Samantha R. Schaaf
W. B. Sweeney Elementary School
Erica L. Ambrogio
Redding Elementary School

Dr. Tutita M. Casa is an Associate Professor of elementary mathematics education in the Neag School of Education. She was the Co-PI and co-author on Project M2 and staff for Project M3 that produced researched-based advanced math units

Samantha and Erica are elementary school teachers in Connecticut. They were master's students in the Integrated Bachelor's/Master's Teacher Preparation Program when they collaborated on this work and continue to implement these ideas with their current students.

"I just know…because it's right." Zan's response to our attempt to get him to further explain his thinking in writing (see Figure 1) caught us off guard since he had scored five grade levels above his peers on a standardized test. Although he could readily provide answers, we knew that he was missing a critical aspect of mathematics: communicating his reasoning (Common Core State Standards Initiative [CCSSI], 2010; National Council of Teachers of Mathematics [NCTM], 2000). In fact, NCTM's *Providing Opportunities for Students with Exceptional Mathematical Promise* Position Statement (2016) noted, "Students with exceptional mathematical promise include those who…are particularly good at explaining complex concepts to others or demonstrate in other ways that they understand mathematical material deeply" (p. 1).

Figure 1. Zan found his numerical answer in two minutes, but then spent 30 minutes putting nothing else on the page, even when prompted

This experience and similar ones with the other excelling students in our second- and third-grade classes made us consider how we could help them advance their abilities to communicate their mathematical reasoning. It so happens that recent work being done in elementary mathematical writing emphasizes writing for the purpose of reasoning and communicating mathematically (Casa et al., 2016). Writing therefore was a way we felt we could provide services for students, like Zan, who were underachieving in this area.

Planning to Teach Argumentative Writing

We decided specifically to target argumentation, one of the four types of mathematical writing identified by the Elementary Mathematical Writing Task Force (Casa et al., 2016) because of its direct connection to the Common Core's mathematical practices. Specifically, Mathematical Practice 3 states, "Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is" (CCSSI, 2016, p. 7). The core elements of an argument include a claim and evidence to support that position. An example of a written mathematical argument is when a student writes the claim, "I disagree that there are enough goodie bags," and backs it by stating, "There are three blue ones and two reds. I know two and two is four, plus one is five. There are six kids, so there's not enough."

Following NCTM's position that "each and every student has mathematical promise" (2016, p. 1), we were committed to providing all our students with the opportunity to develop their abilities to communicate their reasoning through mathematical arguments. We also hoped their writing would more accurately show us whether there were gaps in their understanding to push their thinking further

and, at the same time, provide them with the opportunity to showcase a unique way of thinking about any given topic. We therefore sought to differentiate our supports as suggested by NAGC's Pre-K-Grade 12 Gifted Programming Standards (2019) Learning and Development and Curriculum & Instruction strands. To do this, we identified excelling, average, and struggling students based on their scores on mathematics district assessments and unit tests. We further tweaked these three groups based on our observations of their performance and participation in discussions during class.

This work with our second and third graders took place across two months. Prior to responding to each of six writing tasks, students discussed the prompt. They became accustomed to solving the problem first, then deciding whom they agreed with and why. We specifically prompted our students to write mathematical arguments by asking them to choose one of two solutions we provided from fictional students and defend their argument by explaining their choice, as recommended by Firmender, Casa, and Colonnese (in press). It took students about 30 minutes to complete each task, which included time to talk and write.

Lessons Learned from Our Students

Despite our designation of the three student groups, we attended to the needs of our excelling students with an aim to have the strategies that helped them serve other groups. Nonetheless, students are more successful in learning when they are taught in ways that meet the needs of their individual readiness levels. We decided to differentiate our instruction and identify strategies that could help us best serve our students. We therefore had all our students work on the same task with different levels of support. Additionally, we designed a student-friendly rubric (Figure 2) that outlined expectations. We used the rubric to assess our students' work and provide them with feedback. They also used it to evaluate their own writing and that of their peers.

Over time, we found that different groups of students benefited from word banks, sentence starters, manipulatives, and hints. We initially thought that our excelling students would be able to write their ideas clearly enough to build an argument that would demonstrate their reasoning, and therefore did not provide them with any scaffolds. It turned out that they needed more support in attending to precision (CCSSI, 2010), and a word bank allowed them to further their abilities to write strong arguments. This scaffold turned out to be beneficial for our average group. Although they often figured out the correct answers, they initially were lost about what to write when it came time to construct their argument. We therefore also provided them with sentence starters. Lastly, our students who were struggling, not surprisingly, needed help with the mathematics and writing. Thus, in addition to using the word bank and sentence starters, we initially made manipulatives available to them, and then hint cards.

Strategies That Supported Our Students' Argumentative Writing

We recognize that the strategies we implemented across groups might be applicable to other teachers whose students are excelling at different levels with regards to their command of mathematics and ability to communicate their arguments in writing. Therefore, we present the strategies we used in more detail. Our goal was to ensure all our second- and third graders could reach their highest potential in writing mathematical arguments, and we hope these insights support other teachers' work in this area.

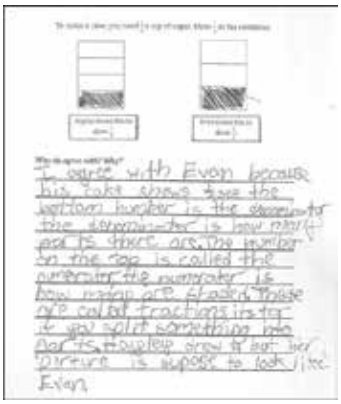
1. Using Word Banks to Improve Accuracy

Our goal for utilizing the word bank was to help our students hone in on the ideas they could address to help readers understand the argument. While many students had command of the mathematics content, utilizing precise mathematical language pushed them to describe more accurately what they knew clearly and efficiently. We provided students terms written on large index cards that had student-friendly definitions on the back. Students engaged with the cards, yet students who knew the definitions already did not need to flip them over. We changed the word banks to correspond to the content, such as including "numerator" and "whole" for a prompt about fractions. Eli's response in Figure 3 shows how he incorporated relevant fraction terms and their definitions to convince others why his claim was correct.

Figure 2. The student-friendly language in this rubric made it possible for students to self- and peer-assess each other's writing.

	Things you need to work on	Things you almost have	Things you do very well
Vocabulary	I didn't find any math vocabulary.	I saw some vocabulary words, but some are missing.	I see a lot of math vocabulary used correctly.
Clarity	I have a hard time understanding the writing.	I can understand what the author is trying to say, but the writing isn't clear.	I completely understand the author's response.
Explanation	I don't understand how the author got their answer.	I am starting to understand how the author got their answer, but I still have questions.	I fully understand how the author got their answer and I have no questions.

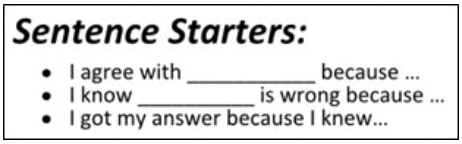
Figure 3. Eli used the fraction related vocabulary words and their definitions to explain to the reader why Evan was right.



2. Using Sentence Starters to Bridge Between Student Solutions and Their Writing

We needed to help some students connect their solutions with their written arguments. We therefore provided them

Figure 4. Students received a copy of these sentences starters with their prompts.



with sentence starters (Figure 4) to help them initially realize that instead of listing the steps they took to solve the problem, the structure of a written argument includes a claim and evidence supporting that claim.

Figure 5 shows how although Stephanie wrote a relatively long piece, she only wrote out her computational steps and did not attend to the misconception introduced in the prompt. In contrast, Lily (Figure 6) adapted the "I knew ____ was wrong because…" sentence starter to structure her argument in a way that provided important details about how she chose which claim to agree with and why the other one provided was invalid.

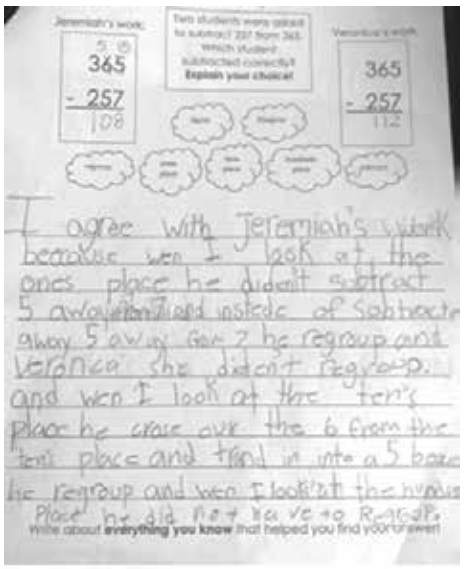
3. Using Manipulatives to Solve the Problem

We recognized that having command of the mathematics was necessary for students to write a compelling argument. Therefore, one way we tried to support our students was to provide them with the option of using manipulatives like base-ten blocks (for regrouping) and square tiles (for area) to help them determine a solution prior to writing their arguments. Although students who chose to use the manipulatives eventually settled on a valid answer,

Figure 5. Stephanie simply listed her steps rather than defending an argument.

they ran out of time to construct their argument. While utilizing manipulatives was a strategy that was beneficial for some of our students, we wished to have our whole group focused on one task at the same time. We therefore considered another alternative that still would give them the foundation they needed to figure out the mathematics and time to write their arguments.

Figure 6. Lily gives important details about how she chose which claim to agree with and why Veronica was wrong.



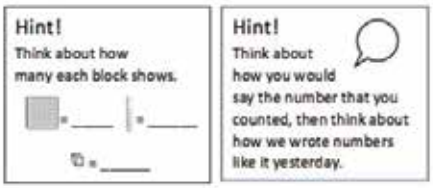
4. Using Hint Cards as a Reference of Content

Eventually, we designed hint cards to give our students a jumpstart on thinking about the argument presented in the prompts. These cards served to remind students about key ideas we had discussed in class to allow them to apply it to the specific problem they were given, such as the one in Figure 7 that we used with a prompt focused on place value. We passed out a hint card with the prompts, which allowed students to choose whether they would like to use it. Students initially resist using it and instead asked us for help. We suggested they check the hint card. Over time, students learned to be more independent about using the cards when they needed it while thinking on their own whenever possible.

Final Thoughts

Although our students had differing

Figure 7. These are hint cards that students were given for a prompt about place value.



needs, our work taught us that all students, including our excelling ones, may need some level of support to start the process of mathematical writing. Focusing on writing mathematical arguments required our students to go beyond the answer and communicate their reasoning. We envisioned all our students as having mathematical promise and providing them with scaffolds to allow them to delve more deeply into the content through writing seemingly empowered them. Even for excelling students, like Zan, just one support may make a drastic difference in their writing. While Zan was stumped about what to record when we began asking our students to write mathematical arguments, in the

Figure 8. While every other student decomposed the shape into two rectangles to find the area, Zan's creative approach showcased a strategy he used that was more sophisticated.

end, he realized he had the voice to convey a solution path that was advanced among his classmates (see Figure 8). Opening the possibility for our students to communicate their unique reasoning through mathematical writing is something we plan to continue to provide all our students. **THP**

References

Casa, T. M., Firmender, J. M., Cahill, J., Car-

detti, F., Choppin, J. M., Cohen, J., Zawodniak, R. (2016). *Types of and purposes for elementary mathematical writing: Task force recommendations*. Retrieved from http://mathwriting.education.uconn.edu

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. Authors: Washington, DC: The National Governors Association Center for Best Practices and Council of Chief State School Officers. Retrieved from www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Firmender, J. M., Casa, T. M., & Colonnese, M. W. (in press). Write on: Reasoning through mathematical writing. To be published in *Teaching Children Mathematics*.

National Association for Gifted Children. (2010). *Pre-K-Grade 12 gifted programming standards*. Retrieved from www.nagc.org/resources-publications/resources/national-standards-gifted-and-talented-education/pre-k-grade-12

National Council of Teachers of Mathematics. (2000). *Executive summary: Principles and standards for school mathematics*. Retrieved from www.nctm.org/Standards-and-Positions/Principles-and-Standards/

National Council of Teachers of Mathematics. (2016). *Providing opportunities for students with exceptional mathematical promise: A position of the national council of teachers of mathematics*. Retrieved from www.nctm.org/Standards-and-Positions/Position-Statements/Providing-Opportunities-for-Students-with-Exceptional-Promise/