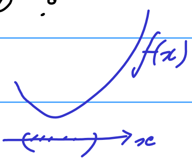
$f(x) = x + e^{-x^2}$, konvexní/konkávní ?



konvexní: $f''(x) > 0$

konkávní: $f''(x) < 0$

$$f'(x) = (x + e^{-x^2})' = 1 + (e^{-x^2})' = 1 + e^{-x^2}\cdot(-x^2)' =$$
$$= \underbrace{1}_{\text{konst.}} - 2xe^{-x^2}$$

$(f(g(x)))' = f'(g(x))\cdot g'(x)$

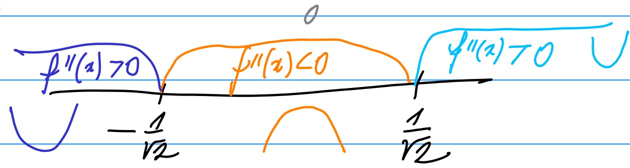
$(x^\alpha)' = \alpha x^{\alpha-1}$

$(f\cdot g)' = f'g + fg'$

$$f''(x) = 0 - 2\cdot(xe^{-x^2})' = -2(1\cdot e^{-x^2} + xe^{-x^2}\cdot(-2x))$$
$$= -2e^{-x^2}(1 - 2x^2) = 2(2x^2-1)e^{-x^2}$$

$$f''(x) = 0 \Leftrightarrow 2(2x^2-1)\underbrace{e^{-x^2}}_{\neq 0} = 0 \Leftrightarrow 2x^2 - 1 = 0 \Leftrightarrow x^2 = \frac{1}{2} \Leftrightarrow x = \pm\frac{1}{\sqrt{2}}$$

$$f''(x) = 2\cdot 2(x^2 - 1/2)e^{-x^2} = \underbrace{4}_{>0}\left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)\underbrace{e^{-x^2}}_{>0}$$



Funkce je konvexní na $(-\infty, -\frac{1}{\sqrt{2}}) \cup (\frac{1}{\sqrt{2}}, \infty)$,
konkávní na $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$.

$\pm\frac{1}{\sqrt{2}}$ jsou inflexní body