PÍSEMKA 1

1) Upravte výraz: $\left[\frac{1}{\sqrt{x}+1} - \frac{2\sqrt{x}}{x-1}\right] \cdot \left(\frac{1}{\sqrt{x}} - 1\right)$

Ř: $\left[\frac{1}{\sqrt{x}+1} - \frac{2\sqrt{x}}{x-1}\right] \cdot \left(\frac{1}{\sqrt{x}} - 1\right) =$

$$= \frac{\sqrt{x}-1-2\sqrt{x}}{x-1} \cdot \frac{1-\sqrt{x}}{\sqrt{x}} = \frac{-1-\sqrt{x}}{x-1} \cdot \frac{1-\sqrt{x}}{\sqrt{x}} =$$

$$= \frac{1+\sqrt{x}}{x-1} \cdot \frac{\sqrt{x}-1}{\sqrt{x}} = \frac{\cancel{\sqrt{x}+1}}{\cancel{(\sqrt{x}+1)}\cancel{(\sqrt{x}-1)}} \cdot \frac{\cancel{\sqrt{x}-1}}{\sqrt{x}} = \underline{\underline{\frac{1}{\sqrt{x}}}}$$

2) Řešte rovnici $9^{x+1} + 27 = 12 \cdot 3^{x+1}$

$$(3^2)^{x+1} + 27 = 12 \cdot 3^{x+1}$$

$$3^{2x+2} + 27 = 12 \cdot 3^{x+1}$$

subst. $3^x = y$

$$9 \cdot (3^x)^2 + 27 = 12 \cdot 3 \cdot 3^x$$

$$9y^2 + 27 = 36 \cdot y \quad | :9$$

$$y^2 - 4y + 3 = 0$$

$$y_{1,2} = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} = \begin{cases} 3 \\ 1 \end{cases}$$

$3^x = 3 \Rightarrow x = 1$

$3^x = 1 \Rightarrow x = 0$

$\underline{\underline{x \in \{0; 1\}}}$

3) Derivujte funkci $y = \sin x \cdot \operatorname{tg} x$

$$y' = \cos x \cdot \operatorname{tg} x + \sin x \cdot \frac{1}{\cos^2 x} = \cos \cdot \frac{\sin x}{\cos x} + \frac{\sin x}{\cos^2 x} =$$

$$= \frac{\sin x \cos^2 x + \sin x}{\cos^2 x} = \frac{\sin x (1 + \cos^2 x)}{\cos^2 x} = \frac{\sin x (1 + 1 - \sin^2 x)}{\cos^2 x} =$$

$$= \frac{\sin x (2 - \sin^2 x)}{\cos^2 x}$$

4) Vypočítejte integrál: $\int\left(\frac{x^3}{4}-\frac{4}{x^3}\right)dx$

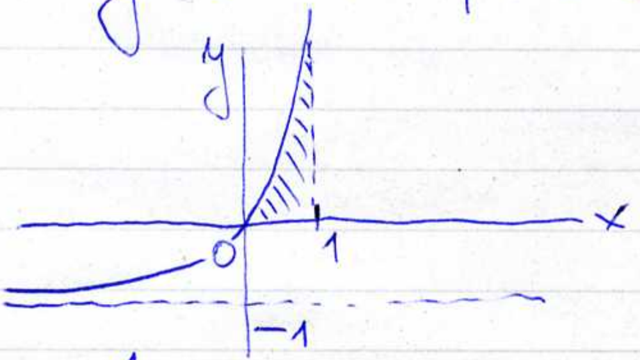
$$\int\left(\frac{x^3}{4}-\frac{4}{x^3}\right)dx = \int\frac{x^3}{4}dx - \int\frac{4}{x^3}dx =$$

$$= \frac{1}{4}\int x^3dx - 4\int x^{-3}dx = \frac{1}{4}\cdot\frac{x^4}{4} - 4\cdot\frac{+x^{-2}}{-2} =$$

$$= \frac{x^4}{16} + \frac{2}{x^2} + C$$

5) Nakreslete rovinný obrazec, který omezuje osa x a graf funkce $y = f(x)$, přičemž $x \in \langle a, b \rangle$. Potom vypočítejte jeho obsah.

$$y = e^x - 1 \quad , x \in \langle 0, 1 \rangle$$



$$S = \int_0^1 (e^x - 1)\,dx = \left[e^x - x\right]_0^1 = (e-1) - (1-0) =$$

$$= e - 2 \doteq 0{,}72$$

PÍSEMKA 2

1) Řešte v $\mathbb{R}$ rovnici: $\sqrt{x+2} - 2\sqrt{x+7} = -4$

$$\sqrt{x+2} - 2\sqrt{x+7} = -4 \quad \Big|^2$$
$$x+2 - 4\sqrt{(x+2)(x+7)} + 4(x+7) = 16$$
$$5x + 30 - 4\sqrt{x^2+9x+14} = 16$$
$$5x + 14 = 4\sqrt{x^2+9x+14} \quad \Big|^2$$
$$25x^2 + 140x + 196 = 16(x^2+9x+14)$$
$$25x^2 + 140x + 196 = 16x^2 + 144x + 224$$
$$9x^2 - 4x - 28 = 0$$
$$x_{1,2} = \frac{4 \pm \sqrt{16+1008}}{18} = \frac{4 \pm \sqrt{1024}}{18} = \frac{4 \pm 32}{18} = \begin{cases} 2 \\ -\frac{14}{9} \end{cases}$$

zkouška: oba kořeny vyhovují $\quad x \in \{-\frac{14}{9}, 2\}$

2) Řešte rovnici: $\dfrac{\log_3 x}{1+\log_3 2} = 2$

$$\log_3 x = 2 + 2\log_3 2$$
$$\log_3 x = \log_3 9 + \log_3 4$$
$$\log_3 x = \log_3 36$$
$$x = 36$$

3) Derivujte funkci: $y = \sqrt{\cos 2x} + x^3$

$$y' = \frac{1}{2}\cdot(\cos 2x)^{-\frac{1}{2}}\cdot(-\sin 2x)\cdot 2 + 3x^2 =$$

$$= \frac{1}{2\sqrt{\cos 2x}}\cdot(-2)\sin 2x + 3x^2 = 3x^2 - \frac{\sin 2x}{\sqrt{\cos 2x}};$$

4) Vypočtěte integrál: $\int \frac{5}{1-3x}\,dx$

substituce $1-3x = t$, pak $-3dx = dt$, tj. $dx = -\frac{dt}{3}$;

$$\int \frac{5}{1-3x}\,dx = 5\int\frac{dx}{1-3x} = 5\int\frac{-\frac{dt}{3}}{t} = 5\int\frac{-dt}{3t} =$$

$$= -\frac{5}{3}\int\frac{dt}{t} = -\frac{5}{3}\cdot\ln t = -\frac{5}{3}\ln(1-3x) + C$$

5) Určete obsah rovinného obrazce, ohraničeného grafem funkce $y = \sin x + \cos x$ a osou x pro $x \in \langle 0, \frac{\pi}{2}\rangle$.

$$\int_0^{\frac{\pi}{2}} (\sin x + \cos x)\,dx = \left[-\cos x + \sin x\right]_0^{\frac{\pi}{2}} =$$

$$= (0+1) - (-1+0) = \underline{\underline{2}}$$