KARTÉZSKÝ SOUČIN

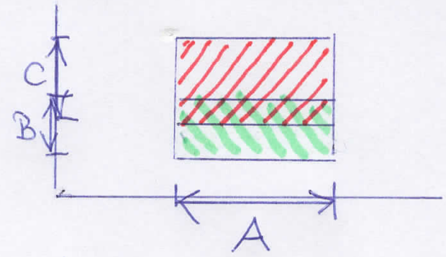
$$A \times B = \{[x,y] : x \in A \wedge y \in B\}$$

$A\times B \sim B\times A$ ekvivalentní

$A\times B = B\times A$, jestliže nastane jeden z těchto případů:

$A=\emptyset$ nebo $B=\emptyset$ nebo $A=B$

Pokaždé: $A\times(B\cup C) = (A\times B)\cup(A\times C)$



„$\subset$" Nechť $[x,y]\in A\times(B\cup C) \Rightarrow x\in A \wedge y\in B\cup C \Rightarrow$

$\Rightarrow x \in A \wedge (y\in B \vee y\in C) \Rightarrow [x\in A\wedge y\in B]\vee[x\in A\wedge y\in C] \Rightarrow$

$\Rightarrow [x,y]\in A\times B \vee [x,y]\in A\times C \Rightarrow [x,y]\in(A\times B)\cup(A\times C)$

„$\supset$" analogicky

Binární relace v množině M je kterákoliv podmnožina $M\times M$.

Má-li množina M n prvků, je počet relací v M celkem 2^{n^2}.

$R' = (M\times M) - R$ relace doplňková

$R^{-1} = \{[a,b]\in M\times M : [b,a]\in R\}$ relace inverzní

Složené relace: Nechť R, S jsou dvě relace v množině M.

Pak $R\circ S = \{[a,b]\in M\times M : (\exists c\in M)\ [a,c]\in R \wedge [c,b]\in S\}$

se nazývá složená relace.

Příklad: $M = \{a,b,c\}$

$R = \{[a,b],[b,b],[c,b],[c,a]\}$

$S = \{[a,a],[b,a],[b,c],[a,c]\}$

$R\circ S = \{[a,a],[a,c],[b,a],[b,c],[c,a],[c,c],\cancel{[c,a]},\cancel{[c,c]}\}$

obecně $R \circ S \neq S \circ R$

$M = \{-2,-1,0,1,2\}$

$R = \{[x,y] \in M^2;\ y = |x|\} = \{[-2,2],[-1,1],[0,0],[1,1],[2,2]\}$

$S = \{[x,y] \in M^2;\ y = -x\} = \{[-2,2],[-1,1],[0,0],[1,-1],[2,-2]\}$

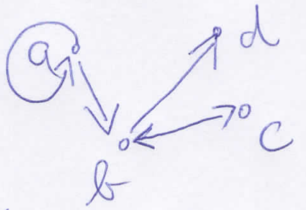
$R \circ S = \{[x,y] \in M^2;\ y = -|x|\} = \{[-2,-2],[-1,-1],[0,0],[1,-1],[2,-2]\}$

$S \circ R = \{[x,y] \in M^2;\ y = |-x|\} = \{[-2,2],[-1,1],[0,0],[1,1],[2,2]\}$

vlastnosti relací

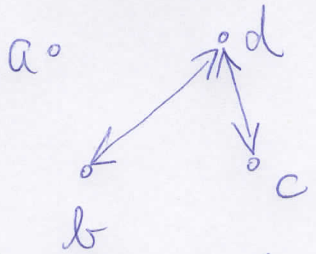
R, AR, S, AS, T, SO

① $M = \{a,b,c,d\}$



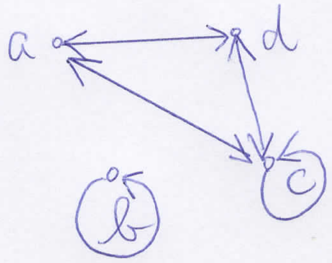
$\cancel{R}, \cancel{AR}, \cancel{S}, \cancel{AS}, \cancel{T}, \cancel{SO}$

② $M = \{a,b,c,d\}$



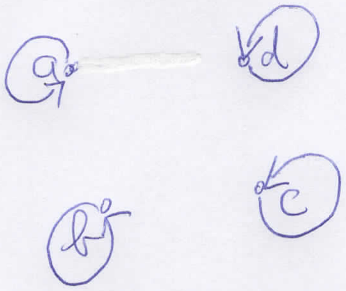
$\cancel{R}, \underline{AR}, \underline{S}, \cancel{AS}, \cancel{T}, \cancel{SO}$

③ $M = \{a,b,c,d\}$



$\cancel{R}, \cancel{AR}, \underline{S}, \cancel{AS}, \cancel{T}, \cancel{SO}$

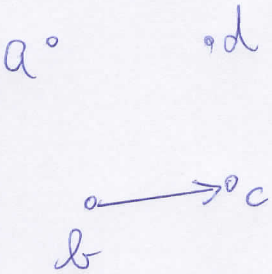
④ $M = \{a,b,c,d\}$



ekvivalence

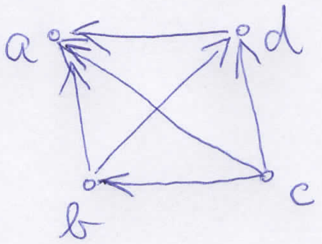
$\underline{R}, \cancel{AR}, \underline{S}, \underline{AS}, \underline{T}, \cancel{SO}$

⑤ $M = \{a,b,c,d\}$



$\cancel{R}, \underline{AR}, \cancel{S}, \underline{AS}, \underline{T}, \cancel{SO}$

⑥ $M = \{a,b,c,d\}$



$c < b < d < a$

$\cancel{R}, \underline{AR}, \cancel{S}, \underline{AS}, \underline{T}, \underline{SO}$

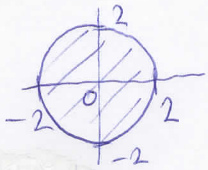
ostré lineární uspořádání

Určete vlastnosti relací:

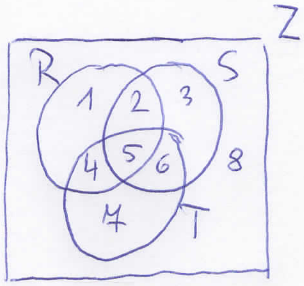
a) $R=\{[x,y]\in\mathbb{N}\times\mathbb{N};\ 2\,|\,(x+y)\}$ $\quad$ $\underline{R}$, $\underline{S}$, $\underline{T}$, $\cancel{AR}$, $\cancel{AS}$, $\cancel{SO}$

b) $S=\{[x,y]\in\mathbb{R}\times\mathbb{R};\ x=2y\}$ $\quad$ $\cancel{R}$, $\cancel{AR}$, $\cancel{S}$, $\underline{AS}$, $\cancel{T}$, $\cancel{SO}$

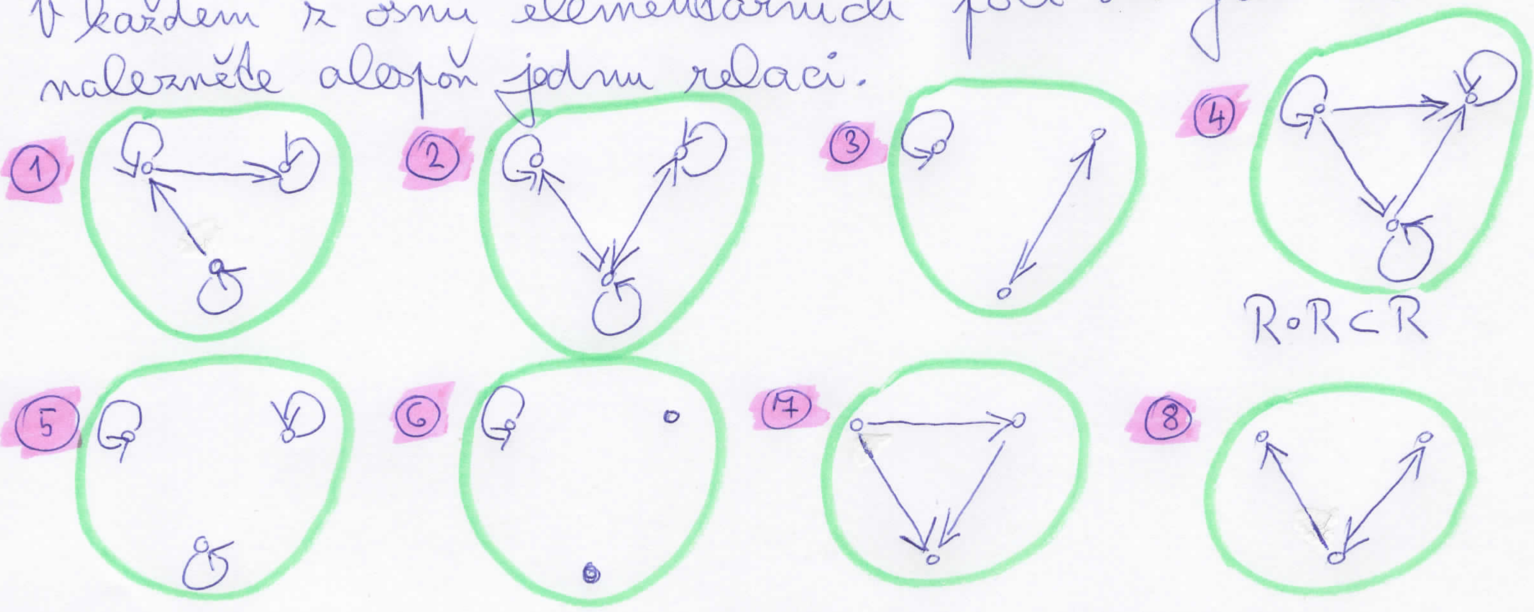
c) $T=\{[x,y]\in\mathbb{R}\times\mathbb{R};\ x^2+y^2\leqq 4\}$ $\quad$ $\cancel{R}$, $\cancel{AR}$, $\underline{S}$, $\cancel{AS}$, $\cancel{T}$, $\cancel{SO}$



Př: $M=\{a,b,c\}$,



Z je množina všech relací v množině M,
R je množina všech reflexivních relací v M,
S je množina všech symetrických relací,
T je množina všech tranzitivních relací.
V každém z osmi elementárních polí diagramu nalezněte alespoň jednu relaci.



$R\circ R\subset R$

příklady z geometrie:

shodnost $\quad$ R, S, T

rovnoběžnost $\quad$ S, T (reflexivnost ?)

kolmost $\quad$ AR, S, $\cancel{T}$

Ekvivalence a rozklady

$M = \{a, b, c, d\}$

$T_1 = \{\{a\}, \{b, c, d\}\}$

$E_1 = \{[a,a], [b,b], [c,c], [d,d], [b,c], [c,b], [b,d], [d,b], [c,d], [d,c]\}$

$T_2 = \{\{a, b\}, \{c, d\}\}$

$E_2 = \{[a,a], [b,b], [c,c], [d,d], [a,b], [b,a], [c,d], [d,c]\}$

$T_3 = \{\{a\}, \{b\}, \{c\}, \{d\}\}$

$E_3 = \{[a,a], [b,b], [c,c], [d,d]\}$

$T_4 = \{\{a, b, c, d\}\}$

$E_4 = M \times M$

Uspořádání

- uspořádání AS, T
  - částečné AS, T, ~~SO~~
    - ostré AS, T, SO, AR
    - neostré AS, T, SO, R
    - AS, T, SO, ~~R~~, ~~AR~~
  - lineární AS, T, SO
    - ostré AS, T, SO, AR
    - neostré AS, T, SO, R
    - AS, T, SO, ~~R~~, ~~AR~~

ostré lineární uspořádání

AR, AS, T, SO

$M = \{a, b, c, d\}$

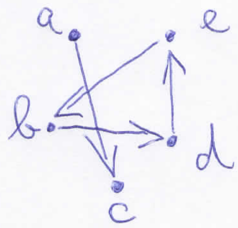
$U = \{[b,a], [b,d], [b,c], [a,d], [a,c], [d,c]\}$

$b < a < d < c$

zapisujeme $[M] = [\{b, a, d, c\}]$

Př. Doplňte uzlové grafy, aby představovaly ostré lin. uspořádání.

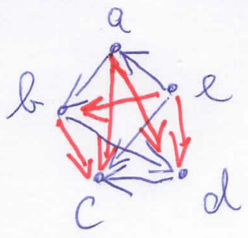
a)



nelze

$\not\vdash$

b)



$e < a < b < d < c$