Geometrie 2

Tabule ze cvičení

Obsah

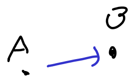
Poslední aktualizace 14. ledna 2021

CVIČENÍ (1) co je a co není AFINNÍ PROSTOR?

obecný postřeh: $V = \vec{a}$

- zejména musíme vidět **ZAMĚŘENÍ** = vekt. prostor
- teprve pak můžeme kontrolovat ostatní požadavky ...

(a) $\{A, B\} \subset a$

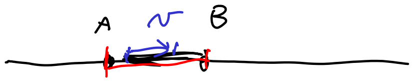


$v = \vec{AB}$ $\rightsquigarrow$ "V" $= \{\vec{AB}\}$

aby byla množina $\{A, B\}$ af. (pod)prostor,
musí být $\underbrace{\{\vec{AB}\} \text{ vekt. (pod)prostor}}$

... což NENÍ!

(b)



$a = \mathbb{R}$

$\mathcal{B}$ = interval $(A, B) \subset \mathbb{R}$

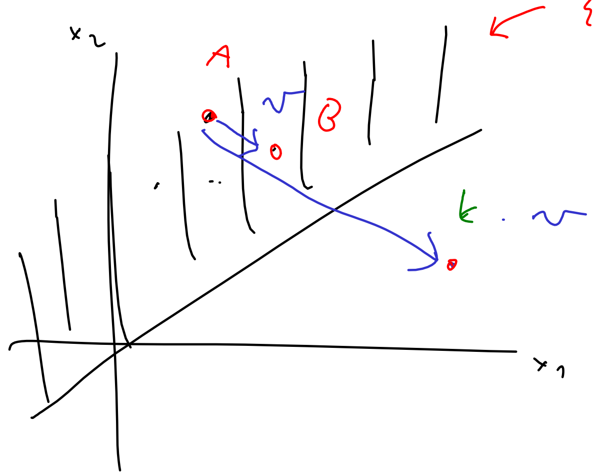
aby byla $\mathcal{B}$ af. (pod)pr.,
musí být "$\vec{\mathcal{B}}$" $= \{\vec{CD} \mid C, D \in \mathcal{B}\}$
$\underbrace{\text{vekt. (pod)pr.}}$

... což NENÍ!

PŘIPOMENUTÍ:

U ... vekt. podpr. ve V
tj. $u, v \in U \Rightarrow u + v \in U$
$u \in U,\ k \in \mathbb{R} \Rightarrow k \cdot u \in U$

(c)



$\{(x_1, x_2) \mid x_2 \geq x_1\} \subset \mathbb{R}^2$

... stejný problém $\Rightarrow$ NENÍ

(d) $\{(x_1, x_2, x_3) \mid \boxed{x_3 = 1 + x_2 + x_3}\} \subset \underline{\mathbb{R}^3}$

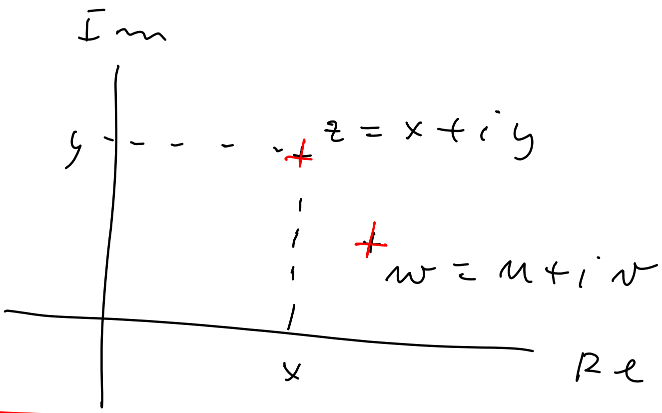
(soustava) lin. rovnic !

$\Rightarrow$ ANO (dim 2)

3 − 1

(e) $\mathbb{C}$ chápáno nad $\underline{\underline{\mathbb{R}}}$

a =



$\mathbb{C} \cong \mathbb{R}^2$

+ ... + ← po složkách

sčítání kompl. čísel atd.

(*)

$z + w = (x + u) + i(y + v)$

... odpovídá ✓

$\underset{\mathbb{C}}{\overset{\shortparallel}{A}} \times \underset{\mathbb{C}}{\overset{\shortparallel}{A}} \to \underset{\mathbb{R}^2}{\overset{\shortparallel}{V}}$ ... obyč. rozdíl

$$\underbrace{(z, w)}_{} \longmapsto w - z$$

(dim = 2)

ANO, splňuje vše, co má
(díky ztotožnění ⊛) !

(f) soust. lin. rovnic nad

$$\mathbb{R} \supset \mathbb{Q} \supset \mathbb{Z}$$

těleso (pole)

jenom "lepší" okruh

- nad TĚLESEM jistě ANO (typický příklad)
- nad OKRUHEM ANO pouze v případě, že soust. má $\boxed{1}$ řešení !!
  (triviální případ dim 0)
- nyní x_3 ... lib.
  ⇓
  soustava má
  - 0 řešení
  - $\boxed{\infty}$ řešení

  ... NENÍ AF. PROSTOR !

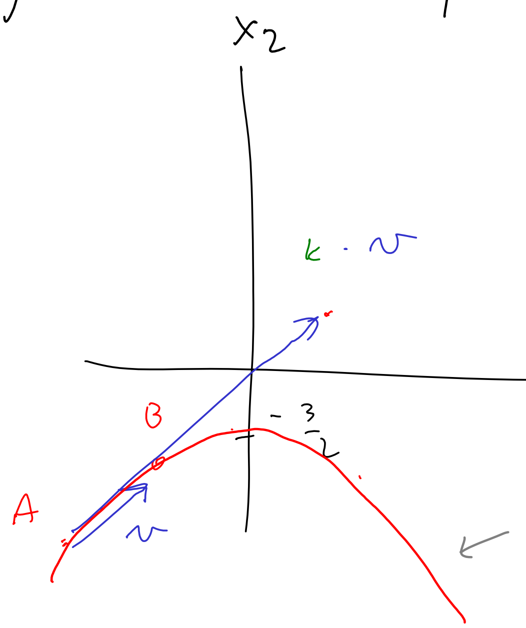
(konkrétní řešení ... $x_1 = -1 - 10\ell$, $x_2 = -1 + 5\ell$, $x_3 = k$, $x_4 = -1 + 4\ell$ | $k, \ell \in \mathbb{Z}$)

nad OKRUHEM více práce
... viz DIOFANTICKÉ ROVNICE

(g) (*) $\boxed{x_1^2 + 2x_2 = -3}$ pro x_1, x_2 ...

jistě **NE** pro stand. struktury na $\mathbb{R}^2$

... "$\vec{a}$" není vekt. prostor



tj. graf fce $x_2 = \dfrac{-3 - x_1^2}{2}$

a = řešení (*)

Lze uvažovat jiné přiřazení $a \times a \to V$

v duchu "UMĚLÉHO PŘÍKLADU" (h)

.. prezentace str. 20

CVIČENÍ (3) SOUŘADNICE

(a)

$x_1, x_2, x_3, x_4 \in \mathbb{R}$

$$a = \left\{ \begin{array}{l} x_1 + 2x_2 \qquad 0 = -3 \\ 0 \quad 4x_2 - 5x_4 = 1 \end{array} \right\}$$

EKVIV. SOUSTAVA (lib. lin. kombinace)

$$= \left\{ \begin{array}{l} 2x_1 + 4x_2 = -6 \\ x_1 + 6x_2 - 5x_4 = -2 \end{array} \right\} = \ldots$$

ŘEŠENÍ

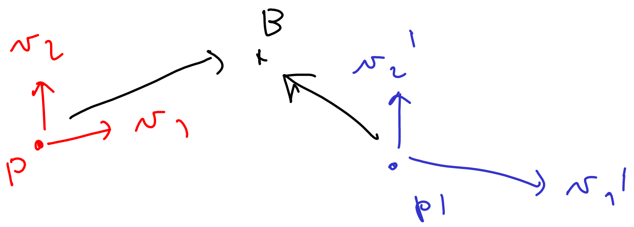
$$= \left\{ \boxed{\begin{array}{l} x_1 = -3 - 2t \\ x_2 = t \\ x_3 = s \\ x_4 = -\frac{1}{5} + \frac{4t}{5} \end{array}} \;\middle|\; t, s \in \mathbb{R} \right\} =$$

dim

$$\boxed{\underbrace{4}_{\text{neznámé}} - \underbrace{2}_{\text{nezáv. rovnice}} = 2}$$

$$= \left\{ \boxed{\begin{array}{l} x_1 = -7/2 - 5/2\,k \\ x_2 = 1/4 + 5/4\,k \\ x_3 = l \\ x_4 = k \end{array}} \;\middle|\; k, l \in \mathbb{R} \right\} = \ldots$$

$$\left(x_1 = -3 - 2x_2 = -3 - \frac{1 + 5k}{2} = -\frac{7}{2} - \frac{5}{2}k \right)$$



(b)

$$\underline{\underline{a}} \Rightarrow \begin{bmatrix} x_1 = -1 \\ x_2 = -1 \\ x_3 = -1 \\ x_4 = -1 \end{bmatrix}$$ je řešení?

$-1-2=-3$ ✓

$-4+5=1$ ✓

$t, s \overset{?}{\in} \mathbb{R}$ tak, aby

$$\left.\boxed{\begin{aligned} -1 &= -3-2t \;\checkmark \\ -1 &= t \\ -1 &= s \\ -1 &= -1/5+4/5\,t \;\checkmark \end{aligned}}\right\} \quad \underline{\underline{t = s = -1}}$$

$\uparrow$

(soustava $\underline{\underline{\text{lin}}}$. rovnic)

$k, l \overset{?}{\in} \mathbb{R}$ tak, aby

$$\left.\boxed{\begin{aligned} -1 &= -7/2-5/2\,k \;\checkmark \\ -1 &= 1/4+5/4\,k \;\checkmark \\ -1 &= l \\ -1 &= k \end{aligned}}\right\} \quad \underline{\underline{l = k = -1}}$$

$\uparrow$

(soustava $\underline{\underline{\text{lin}}}$. rovnic)

(c) OBECNÝ PŘECHOD

$$\boxed{\begin{pmatrix} -3-2t \\ t \\ s \\ -\frac{1}{5}+\frac{4}{5}t \end{pmatrix}} \overset{!}{=} \boxed{\begin{pmatrix} -7/2-5/2\,k \\ 1/4+5/4\,k \\ l \\ k \end{pmatrix}}$$

$$\boxed{\begin{aligned} t &= 1/4 + 5/4\,k \\ s &= l \end{aligned}}$$

[kontrola na zbylých ...] ✓

$$\Updownarrow$$

$$\boxed{\begin{aligned} l &= s \\ k &= -1/5 + \frac{4}{5}t \end{aligned}}$$

[kontrola ...] ✓

pomocí matic:

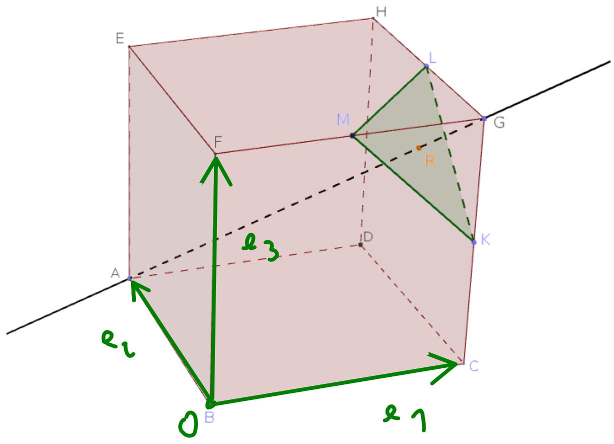
$$\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 5/4 \end{pmatrix} \cdot \begin{pmatrix} l \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 1/4 \end{pmatrix}$$

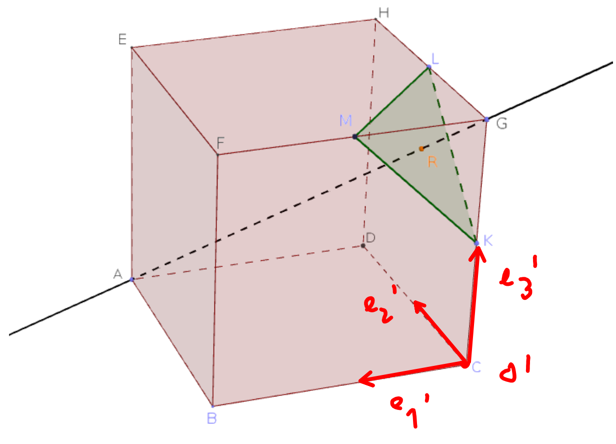
$$\Updownarrow$$

$$\underbrace{\begin{pmatrix} l \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{4}{5} \end{pmatrix} \cdot \begin{pmatrix} s \\ t \end{pmatrix} + \begin{pmatrix} 0 \\ -1/5 \end{pmatrix}}$$

inverzní matice ...

(4) SOUŘADNICE





volba 1

počátek = B

$e_1 = \vec{BC}$, $e_2 = \vec{BA}$, $e_3 = \vec{BF}$

souřadnice

$B = [0,0,0]$

$C = [1,0,0]$

⋮

$A = [0,1,0]$ ....

$G = [1,0,1]$ ✓ ...

⋮

$K = [1,0,1/2]$ ...

$L = [1,1/2,1]$ ...

$M = [1/2,0,1]$ ✓ ...

volba 2

počátek$'$ = C

$e_1' = \frac{1}{2}\vec{CB}$, $e_2' = \frac{1}{2}\vec{CG}$, $e_3' = \frac{1}{2}\vec{CD}$

souřadnice

$A' = C + 2e_1' + 2e_2' + 0\cdot e_3'$

$A' = [2,2,0]$

$G' = [0,0,2]$

$K' = [0,0,1]$

$L' = [0,1,2]$

$M' = [1,0,2]$ ✓

obecný přechod ?

$X = [x_1, x_2, x_3] \rightsquigarrow [x_1', x_2', x_3']$

... určeno vztahem mezi souř. repéry.

(*)

$$\boxed{\begin{aligned} B &= C + 2e_1' + 0e_2' + 0e_3' \\ \hline e_1 &= -2e_1' \\ e_2 &= \qquad +2e_2' \\ e_3 &= \qquad\qquad +2e_3' \end{aligned}}$$

$[2, 0, 0]$

$(-2, 0, 0)$

$(0, 2, 0)$

$(0, 0, 2)$

obecný bod ...

$X = [x_1, x_2, x_3] \rightsquigarrow [x_1', x_2', x_3']$

$\Updownarrow \qquad\qquad \Updownarrow$

$X = B + x_1 e_1 + x_2 e_2 + x_3 e_3 \rightsquigarrow C + x_1' e_1' + x_2' e_2' + x_3' e_3'$

... po dosazení (*) a úpravě:

$$X = (C + 2e_1' + 0e_2' + 0e_3') + {}$$
$$+ x_1(-2e_1') + x_2(+2e_2') + x_3(+2e_3')$$
$$= C + \underbrace{(2-2x_1)}_{x_1'} e_1' + \underbrace{(0+2x_2)}_{x_2'} e_2' + \underbrace{(0+2x_3)}_{x_3'} e_3'$$

$$\boxed{\begin{aligned} x_1' &= 2 - 2x_1 \\ x_2' &= 0 + 2x_2 \\ x_3' &= 0 + 2x_3 \end{aligned}}$$

kontrola pro K:

$0 \overset{?}{=} 2 - 2 \cdot 1$ ✓

$0 = 0 + 2 \cdot 0$ ✓

$1 = 0 + 2 \cdot 1/2$ ✓

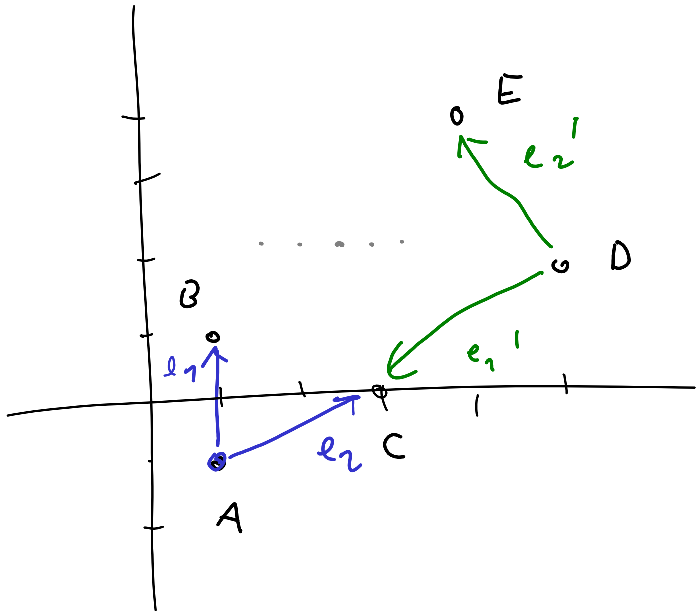
TOTÉŽ pomocí matic:

$$\boxed{\begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}}$$

... lze skládat přímo z (*)

---

(5)



Nic moc NOVÉHO

(6) ZOBRAZENÍ

(b) stejnolehlosti = PODOBNOSTI
$\implies$ AFINNÍ

(c) symetrie krychle = SHODNOSTI
$\implies$ AFINNÍ

... vzpomínáme z konstrukční geom.
(kolineárnost + poměry ~~xx~~ + rovnoběžnost)

$f: \underbrace{\mathbb{R}}_{a} \to \underbrace{\mathbb{R}}_{a'}$

(a) $k = 0 \;..\; f(x) = 1$ ANO

$k = 1 \;..\; f(x) = x + 1$ ANO

$k = 2 \;..\; f(x) = x^{2} + 1$ NE

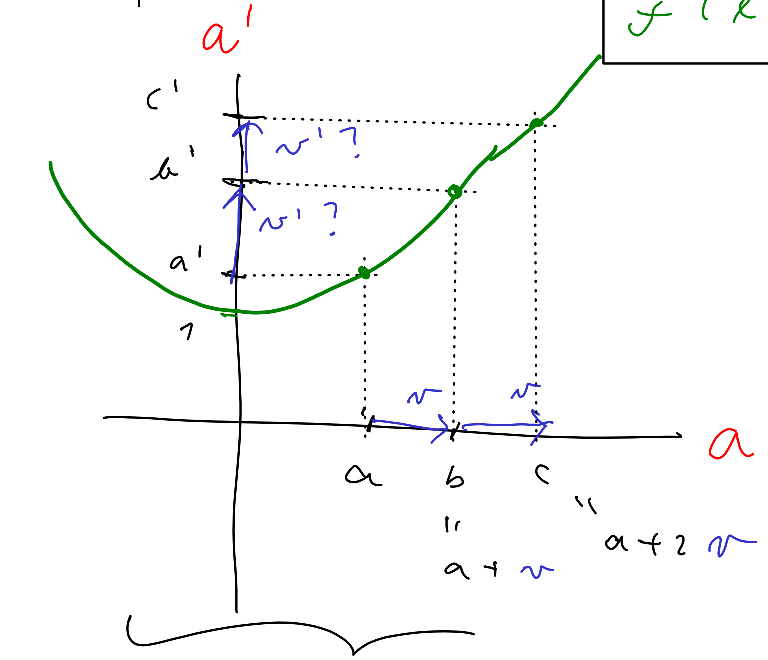
$a = \mathbb{R}^1$ ... stand. str. = obyč. rozdíl

OBECNÝ POSTŘEH:

- zejména musíme vidět indukované zobrazení mezi **VEKTORY**
- teprve potom můžeme kontrolovat zbytek ...

Např.

$\boxed{f(x) = x^2 + 1}$



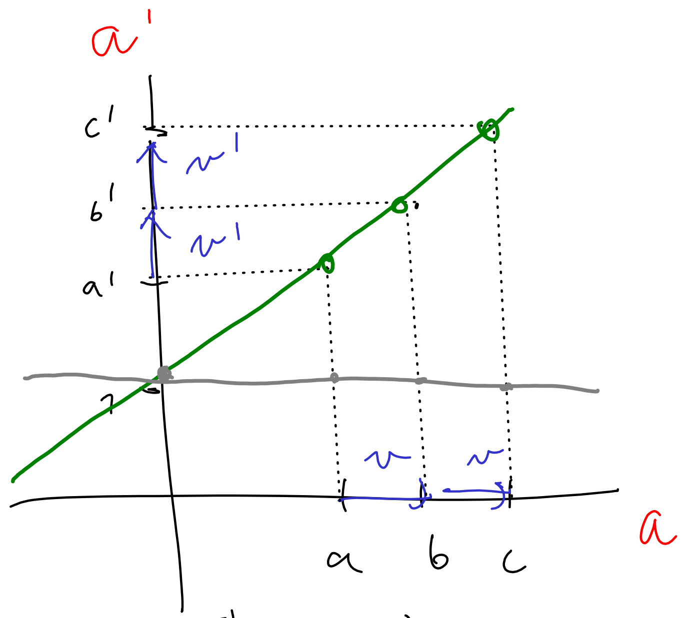
$\overrightarrow{ab} = \overrightarrow{bc}$

$\overrightarrow{a'b'} \neq \overrightarrow{b'c'}$!

nevymyslitelné zobr. $\vec{f}: V \to V$

$\Rightarrow$ NENÍ AFINNÍ!

Pro $\boxed{f(x) = kx + l}$ ... lib. $k, l \in \mathbb{R}$



$\leftarrow$ spec. $f(x) = 0 \cdot x + 1$

$\vec{f}(x) = 0$

$\overrightarrow{ab} = \overrightarrow{bc}$ ... v

$\overrightarrow{a'b'} \overset{!}{=} \overrightarrow{b'c'}$ ... $v' = k v$

Tedy pro $\boxed{f(x) = k x + l}$ $\quad f: a \to a$

1) indukt. lin. zobrazení je

$$\boxed{\vec{f}(x) = k x} \qquad \vec{f}: \vec{a} \to \vec{a}$$

2) a skutečně platí

- $\vec{f}(\overrightarrow{ab}) = \vec{f}(b - a) = k(b - a) = kb - ka$
- $\overrightarrow{f(a)\,f(b)} = (kb + \cancel{l}) - (ka + \cancel{l}) = kb - ka$

$f(a) = (ka + l)$, $f(b) = (kb + l)$

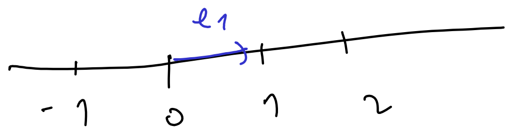
pro lib. $a, b \in \mathbb{R}^a$

LINEÁRNÍ FUNKCE $f: \mathbb{R} \to \mathbb{R}$

JE AFINNÍ ZOBR.

(7)

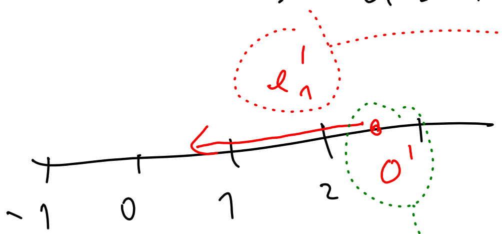
(a) O = počátek, "$e_1 = \overrightarrow{O1}$"



souř. vyjádření = původní předpis

$$\boxed{f(x) = kx + \ell}$$

(jiná souř. soustava $\rightsquigarrow$ jiný předpis)

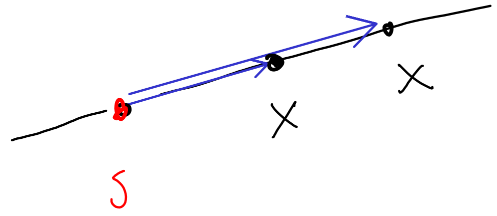


$$\boxed{f(x) = \square\, x + \square}$$

(b) stejnolehlost

… určená středem S
a koeficientem $k \in \mathbb{R}$

$(k \doteq +1{,}6)$



(pevný bod)

$$\boxed{\overrightarrow{SX'} = k \cdot \overrightarrow{SX}} \qquad (*)$$

Pro obecné vyjádření obrazu X' stačí přepsat definující rovnost $(*)$:

$$\overrightarrow{SX'} = k \cdot \overrightarrow{SX}$$

$$X' - S = k\,(X - S)$$

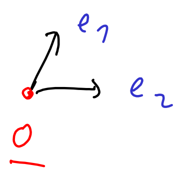
$$X' = k(X - S) + S \overset{!}{=} kX - kS + S$$

Nezávislé na **dimenzi** a volbě **báze**

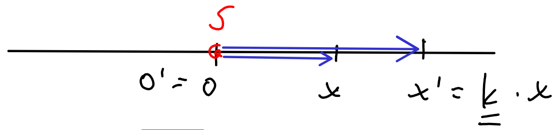
$$(**) \qquad \boxed{X' = \boxed{k}\,X + \boxed{-kS + S}}$$

lin. část
$(k \cdot \mathrm{id})$

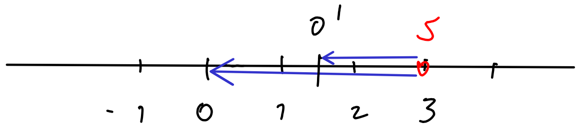
obraz počátku



Např. dim 1



- počátek = S $\rightsquigarrow$ $f(x) = \boxed{k}\, x + \boxed{0}$



- počátek $\neq$ S ...

$$\left.\begin{array}{l} S \doteq 3 \\ k \doteq 1/2 \end{array}\right\} \; O' \doteq \frac{3}{2}$$

$$\rightsquigarrow \; x' \doteq \boxed{\tfrac{1}{2}}\, x + \boxed{\tfrac{3}{2}}$$

obecně pro $\left.\begin{array}{l} S = s \\ \text{koef} = \underline{k} \end{array}\right\} \; O' = (1-k)s$

$$\rightsquigarrow \; x' = \boxed{k}\, x + \boxed{(1-k)s}$$

... souhlasí s ob. vyjádřením (**) ✓

... viz též př. (7a)

$$\boxed{f(x) = kx + l}$$

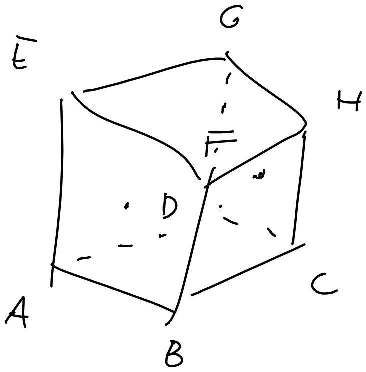
$\rightsquigarrow$ STEJNOLEHLOST

s koef $= k$

a středem $= \dfrac{l}{1-k}$

$f(x) = x \iff x = \dfrac{l}{1-k}$ ✓

(c) symetrie krychle



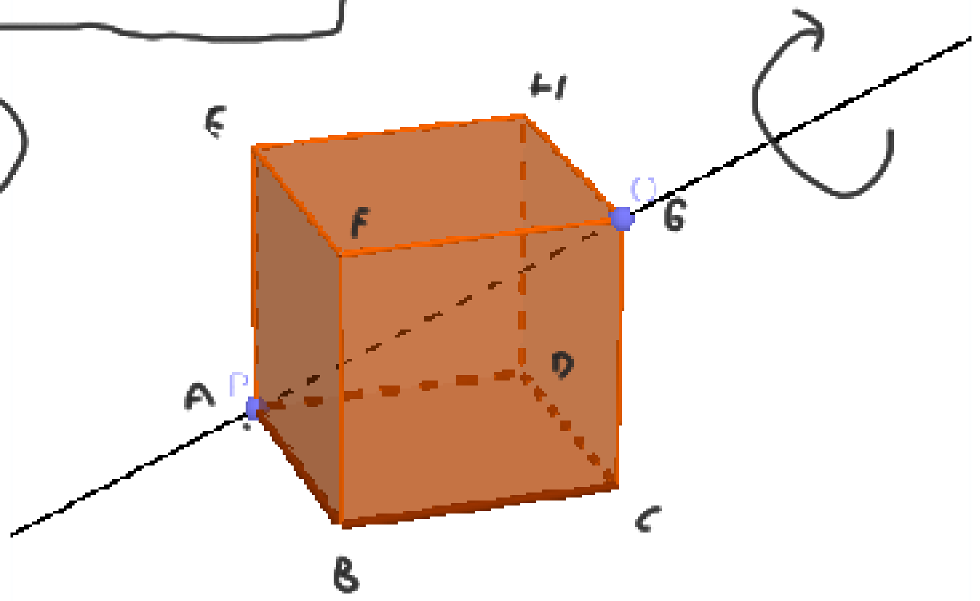
- střed. sym.
- osové sym
- obecnější rotace
  (úhly 90°, 120°)
- sym. podle rovin

Pro některé volby z př. (4)
a některé symetrie chceme:

$$\overset{?}{\rightsquigarrow} \quad \underbrace{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}}_{X'} = \underbrace{\boxed{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}}}_{\vec{f}} \cdot \underbrace{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}}_{X} + \underbrace{\boxed{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}}}_{O'}$$

cvič. (7)

(C)

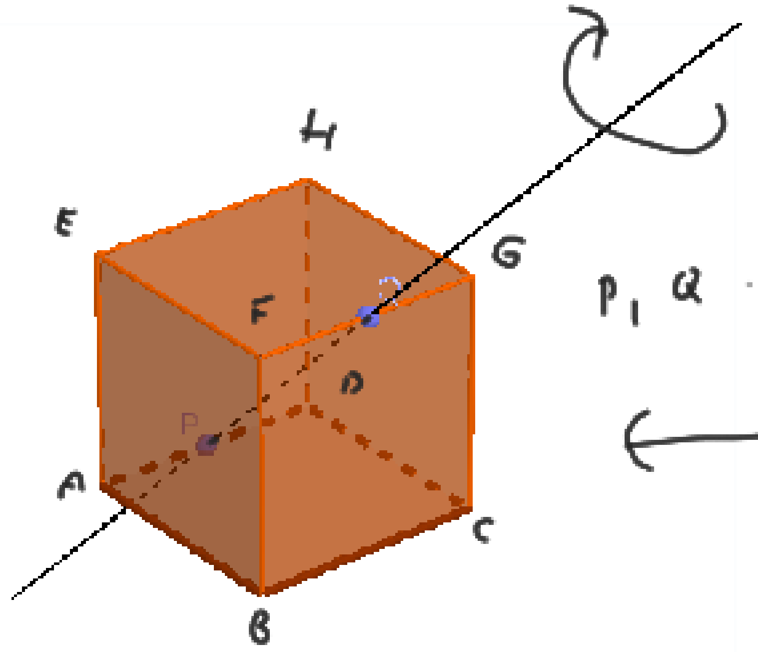


možné úhly: ~~180°~~ ±120°

možné permutace vrcholů:

A	B	C	D	E	F	G	H
A	E	F	B	D	H	G	C

(B)

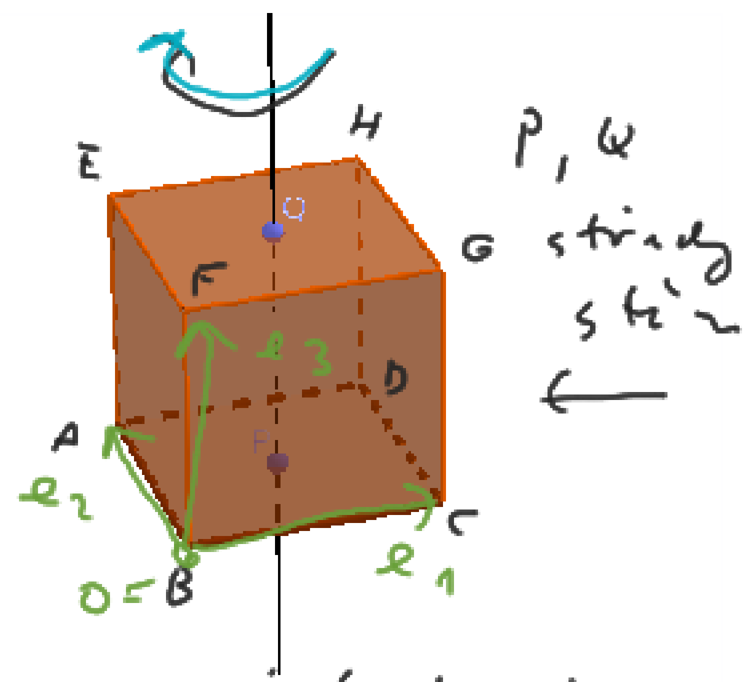


P, Q ... středy stran

možné úhly: 180°

A	B	C	D	E	F	G	H
D	H	E	A	C	G	F	B

(A)



P, Q středy stěn

možné úhly: 180° 90°

A	B	C	D	E	F	G	H
A	A	B	C	H	E	F	G

OBECNĚ: e_1', e_2', e_3' obrazy bázových vektorů!

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \boxed{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \boxed{\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}}$$

X' $\vec{f}$ X O' ... obraz počátku...!

$e_1 = \vec{BC} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ Analyticky vzhledem k soustavě ZELENÉ

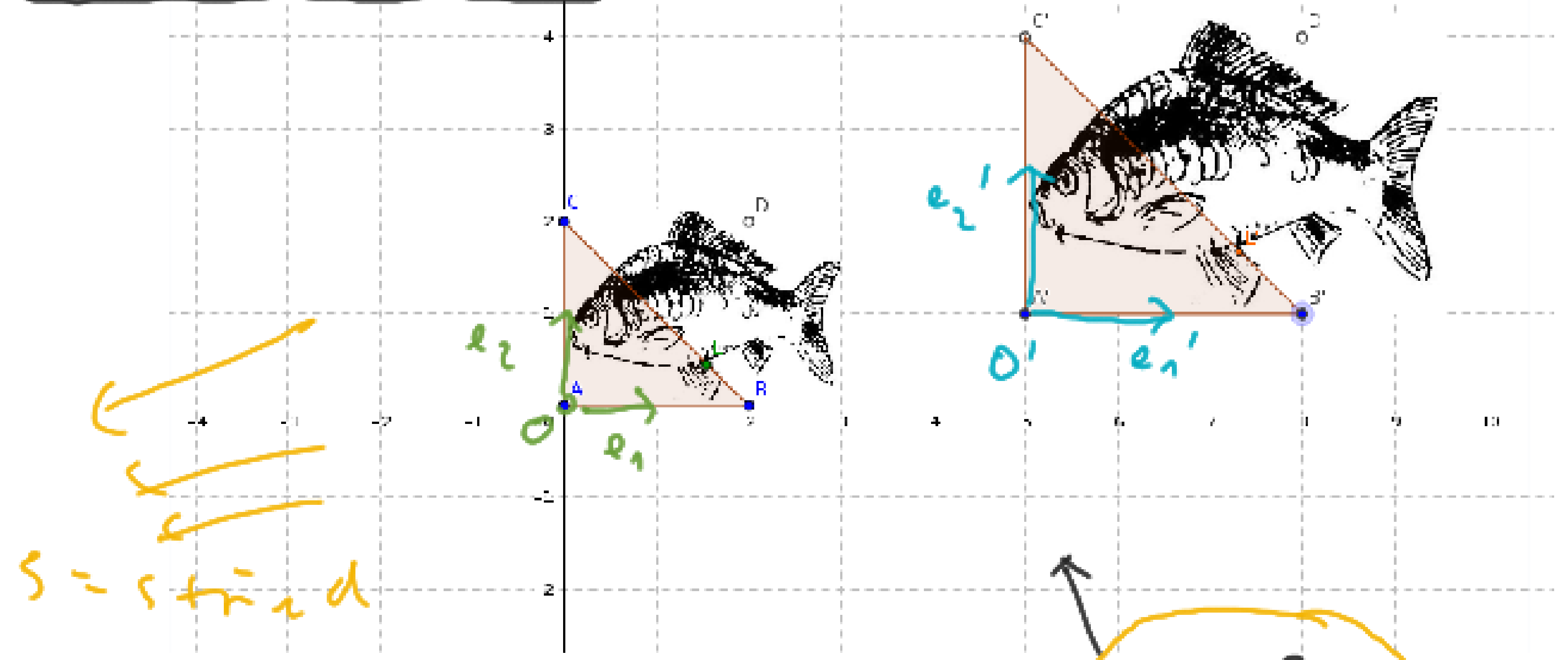
$e_1' = \vec{B'C'} = \vec{AB} = -e_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

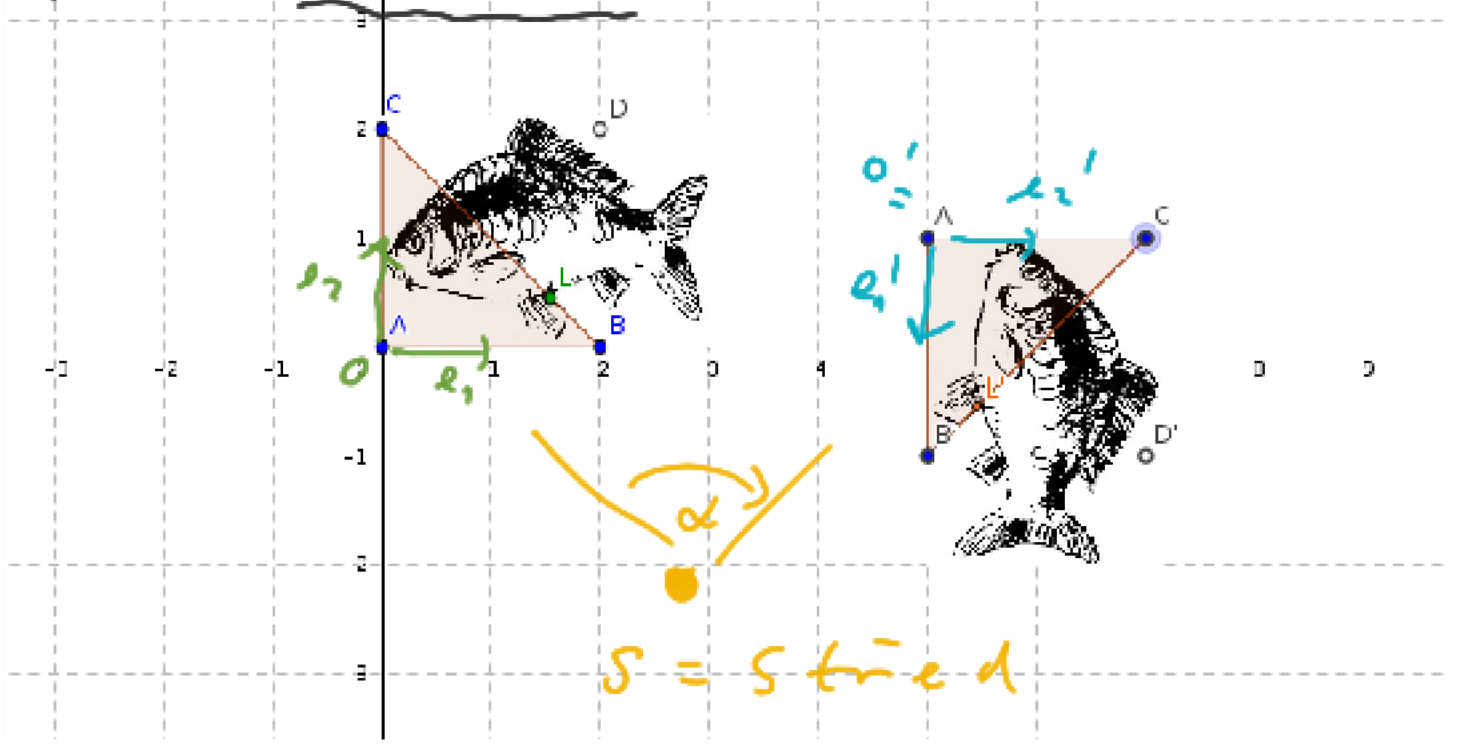
$e_2' \dots = e_1$

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \cdot \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} + \boxed{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}$$

X' $\vec{f}$ $e_3' = \dots = e_3$ X $O' = B' = A$

CVIČ. (7)–(8) AF. ZOBR. V ROVINĚ ZADÁNO TŘEMI BODY:





$k = \frac{3}{2}$

STEJNOLEHLOST

$[0,0] \mapsto [5,1]$
$[2,0] \mapsto [8,1]$
$[0,2] \mapsto [5,4]$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$e_1' = k e_1$ $\quad e_2' = k e_2$ $\quad O'$

OTÁČENÍ ...

$[0,0] \mapsto [5,1]$
$[2,0] \mapsto [5,-1]$
$[0,2] \mapsto [7,1]$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$e_1' = -e_2$ $\quad e_2' = e_1$ $\quad O'$

S = STŘED $\Rightarrow$ $S' \overset{!}{=} S$ $\Leftrightarrow$ $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \boxed{\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \cdot \\ \cdot \end{pmatrix} \overset{!}{=} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}$ SOUSTAVA LIN. ROVNIC

POZN... STEJNOLEHLOST OBECNĚ (minule)

$$\boxed{\vec{SX'} = k \cdot \vec{SX}}$$

$k = \frac{3}{2}$ $\quad S = ?$

$$x' - s = k(x - s)$$

$$x' = k(x-s) + s \overset{!}{=} \underline{\underline{k}}\, x - \underbrace{ks + s}$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$(1-k)S = -\frac{1}{2}S = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\Longleftrightarrow \quad S = \underline{\underline{\begin{bmatrix} -10 \\ -2 \end{bmatrix}}}$$

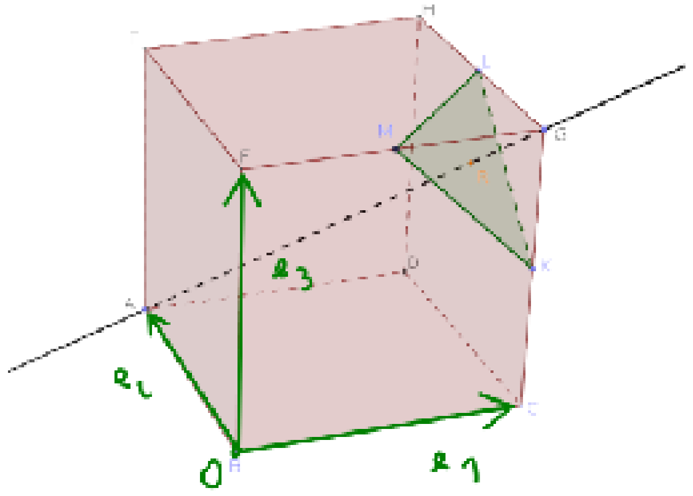
kontrola

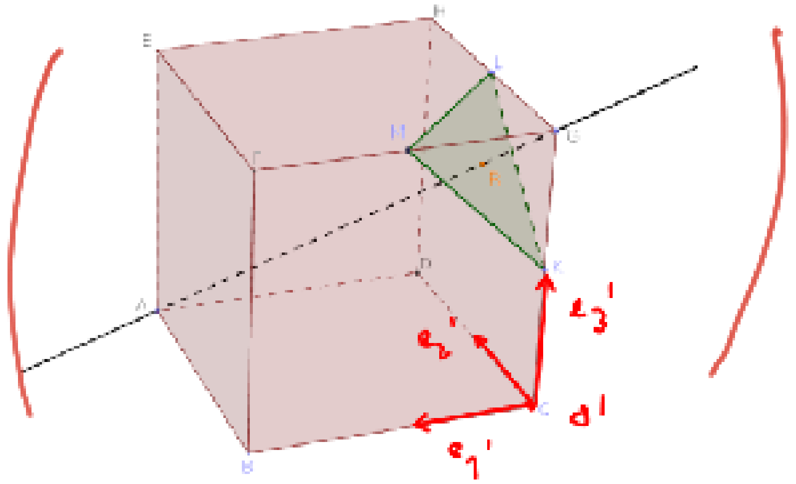
$$S' = \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -10 \\ -2 \end{pmatrix} = S \quad \checkmark$$

cv. (10)

(A)

$A = [0, 1, 0]$
$G = [1, 0, 1]$
$\vdots$
$K = [1, 0, 1/2]$
$L = [1, 1/2, 1]$
$M = [1/2, 0, 1]$





$p = AG$
$\alpha = KLM$

$X = A + t\,\overrightarrow{AG}$

$\overrightarrow{AG} = (1, -1, 1)$

a) parametricky

$$p = \{ A + t\,\overrightarrow{AG} \mid t \in \mathbb{R} \} = \left\{ \begin{array}{l} x_1 = 0 + t \cdot 1 \\ x_2 = 1 + t\,(-1) \\ x_3 = 0 + t \cdot 1 \end{array} \middle| \; t \in \mathbb{R} \right\}$$

$$\alpha = \{ K + r\,\overrightarrow{KL} + s\,\overrightarrow{KM} \mid r, s \in \mathbb{R} \} = \left\{ \begin{array}{l} x_1 = 1 + r \cdot 0 + s \cdot (-1/2) \\ x_2 = 0 + r \cdot 1/2 + s \cdot 0 \\ x_3 = 1/2 + r \cdot 1/2 + s \cdot 1/2 \end{array} \middle| \; \dots \right\}$$

$\overrightarrow{KL}$ $\qquad$ $\overrightarrow{KM}$

b) rovnicemi

$$p = \left\{ \boxed{\begin{array}{l} 1x_1 + 1x_2 + 0x_3 = 1 \\ 0x_1 + 1 \cdot x_2 + 1 \cdot x_3 = 1 \end{array}} \right\}$$

$$\alpha = \{ (+1)x_1 + (-1)x_2 + 1x_3 = \tfrac{3}{2} \}$$

z h(l)avy (ty, kdo chtějí) eliminovat t)
resp. r, s

pozn ... $(1, -1, 1)$ = "normála α"

CO KDYŽ

$$\alpha = \left\{ \begin{array}{l} x_1 = 1 + r \cdot 1 + s \cdot (-2) \\ x_2 = 0 + r \cdot 3 + s \cdot 1 \\ x_3 = 1/2 + r \cdot 7 + s \cdot 3 \end{array} \;\middle|\; \dots \right\}$$

$$= \left\{ \begin{array}{l} x_1 - 1 = r - 2s \\ x_2 = \boxed{3r} + s \\ x_3 - 1/2 = \boxed{7r} + 3s \end{array} \right\} \begin{array}{l} \cdot(-3) \quad \cdot(-7) \end{array}$$

$$= \left\{ \begin{array}{l} x_1 - 1 = r - 2s \\ -3x_1 + x_2 + 3 = 0 + 7s \\ -7x_1 + x_3 + \frac{13}{2} = 0 + \boxed{17s} \end{array} \right\} \begin{array}{l} \cdot(-17) \\ \cdot 7 \end{array}$$

$$= \left\{ \begin{array}{l} x_1 - 1 = r - 2s \\ -3x_1 + x_2 + 3 = 0 + 7s \\ 2x_1 - 17x_2 + 7x_3 - \frac{11}{2} = 0 \end{array} \right\}$$

toto nás nezajímá

toto je hledaná rovnice

GAUSSOVA ELIMINACE

NIKDY NEZKLAME!

$$\alpha = \left\{ 2x_1 - 17x_2 + 7x_3 = \frac{11}{2} \right\}$$

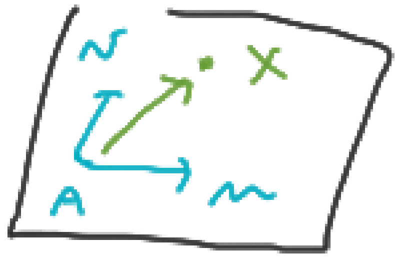
MINULE ... rovnicové vyjádření podpr. (cv. 110)

$m, s \in \mathbb{R}$

$$\alpha = \left\{ \begin{array}{l} x_1 = 1 + m\,0 + s\,(-1/2) \\ x_2 = 0 + m\,1/2 + s\,0 \\ x_3 = 1/2 + m\,1/2 + s\,1/2 \end{array} \middle| \ \dots \right\} = \{\ \dots\ \} = \{ x_1 - x_2 + x_3 = -\tfrac{1}{2} \}$$

$K \qquad \vec{KL} \qquad \vec{KM}$

z hlavy



$$\beta = \left\{ \begin{array}{l} x_1 = 1 + m\,1 + s\,(-2) \\ x_2 = 0 + m\,3 + s\,1 \\ x_3 = 1/2 + m\,7 + s\,3 \end{array} \middle| \ \dots \right\} = \{\ \dots\ \} = \{ 2x_1 - 17x_2 + 7x_3 = \tfrac{11}{2} \}$$

$X \qquad A \qquad m \qquad n$

systematická (GAUSSOVA) eliminace

NAJDĚTE

- jiné řešení ← např. pomocí DETERMINANTŮ

$\vec{AX} \qquad m \qquad n$

$$\begin{vmatrix} x_1 - 1 & 1 & -2 \\ x_2 & 3 & 1 \\ x_3 - \tfrac{1}{2} & 7 & 3 \end{vmatrix} = 2x_1 - 17x_2 + 7x_3 - \frac{11}{2} = 0$$

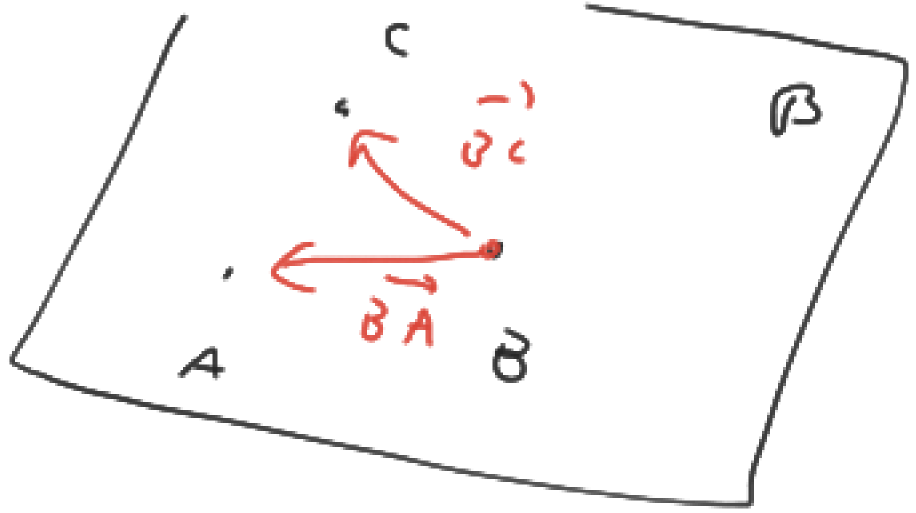
- jiné vyjádření ← "AF. KOMBINACE" bodů

$K \qquad L \qquad M$

$$\alpha = \{ \text{"}X = t_0K + t_1L + t_2M\text{"} \mid \boxed{t_0 + t_1 + t_2 = 1} \} = \left\{ \begin{array}{l} x_1 = t_0\,1 + t_1\ \ + t_2 \\ x_2 = t_0\,0 + t_1\ \ + t_2 \\ x_3 = t_0\,1/2 + t_1\ \ + t_2 \end{array} \middle| \ t_0 + t_1 + t_2 = 1 \right\}$$

TOTÉŽ pro af. podpr. $\mathcal{B}$ určený body | cv. (11)

$A = [1, -1, 0, 2]$ $B = [4, 1, 0, 2]$ $C = [2, -1, 1, 1]$



- Af. kombinace

$$\mathcal{B} = \left\{ \begin{array}{l} x_1 = t_0 + 4t_1 + 2t_2 \\ x_2 = -t_0 + t_1 - t_2 \\ x_3 = t_2 \\ x_4 = 2t_0 + 2t_1 + t_2 \end{array} \middle| \; \underbrace{t_0 + t_1 + t_2 = 1}_{t_1 = 1 - t_0 - t_2} \right\}$$

- Parametricky

$$\mathcal{B} = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \underbrace{\begin{pmatrix} 4 \\ 1 \\ 0 \\ 2 \end{pmatrix}}_{B} + t_0 \underbrace{\begin{pmatrix} -3 \\ -2 \\ 0 \\ 0 \end{pmatrix}}_{\vec{BA}} + t_2 \underbrace{\begin{pmatrix} -2 \\ -2 \\ 1 \\ -1 \end{pmatrix}}_{\vec{BC}} \middle| \; t_0, t_2 \in \mathbb{R} \right\}$$

← $\vec{BA}, \vec{BC}$ LIN. NEZÁVISLÉ

tj. $\dim \mathcal{B} = 2$

- Rovnicové

$$\mathcal{B} = \left\{ \begin{array}{l} x_3 + x_4 = 2 \\ \underbrace{2x_1 - 3x_2}_{5 + 0 \cdot t_0 + 2t_2} \underbrace{- x_3 + x_4}_{2 - 2t_2} = 7 \end{array} \right\}$$

← HLEDÁME 2 $\underbrace{\text{(NEZÁVISLÉ)}}$ ROVNICE

← $4 - 2$

← 2 HLAVY

$$B = \left\{ \begin{aligned} x_1 &= 4 + t_0 \cdot (-3) + t_2 \cdot (-2) \\ x_2 &= 1 + t_0 \cdot (-2) + t_2 \cdot (-2) \\ x_3 &= 0 + t_0 \cdot 0 + t_2 \cdot 1 \\ x_4 &= 2 + t_0 \cdot 0 + t_2 \cdot (-1) \end{aligned} \;\middle|\; t_0, t_2 \in \mathbb{R} \right\}$$

$$\underbrace{\begin{pmatrix}4\\1\\0\\2\end{pmatrix}}_{B} \quad \underbrace{\begin{pmatrix}-3\\-2\\0\\0\end{pmatrix}}_{\vec{BA}} \quad \underbrace{\begin{pmatrix}-2\\-2\\1\\-1\end{pmatrix}}_{\vec{BC}}$$

rovnice

ELIMINACE

$$\begin{pmatrix} x_1 - 4 = -3t_0 - 2t_2 \\ x_2 - 1 = -2t_0 - 2t_2 \\ x_3 = 0 + t_2 \\ x_4 - 2 = 0 - t_2 \end{pmatrix} \sim \begin{pmatrix} x_1 - 4 = -3t_0 - 2t_2 \\ 2x_1 - 3x_2 - 5 = 0 + 2t_2 \\ \vdots \qquad \vdots \qquad \vdots \end{pmatrix}$$

ATD.

SUBDETERMINANTY

$$\begin{pmatrix} x_1 - 4 & -3 & -2 \\ x_2 - 1 & -2 & -2 \\ x_3 & 0 & 1 \\ x_4 - 2 & 0 & -1 \end{pmatrix}$$

det $\rightarrow$ $-2(x_1 - 4) + 6x_3 - 4x_3 + 3(x_2 - 1) \overset{!}{=} 0$

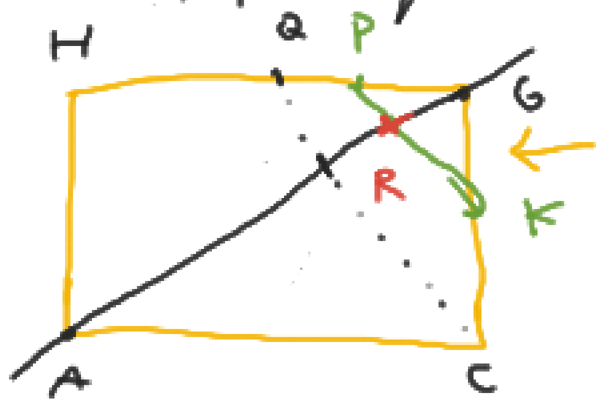
$\boxed{-2x_1 + 3x_2 + 2x_3 + 5 = 0}$ ✓

det $\rightarrow$ $-2(x_4 - 2) - 2x_3 \overset{!}{=} 0$

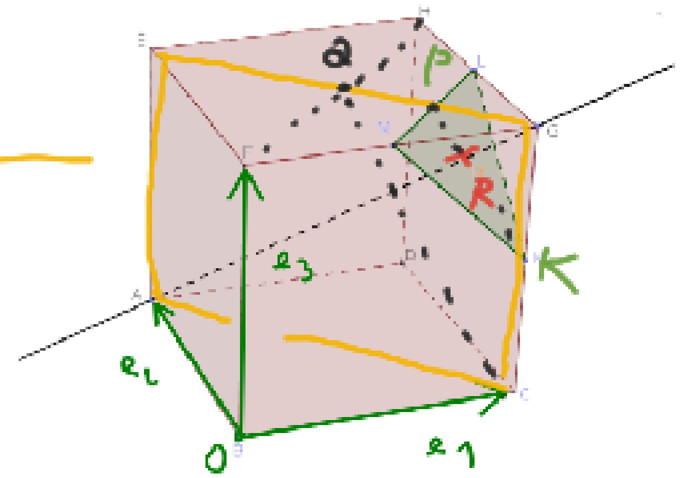
$\boxed{-2x_3 - 2x_4 + 4 = 0}$ ✓

cv. (13) Určete PRŮNIK p : AG a α = KLM ...

– Bez počítání



R ... $1/2 \cdot 1/3 = 1/6$ mezi AG



a) parametricky

AG = (...

$p = \{A + t\,\overrightarrow{AG} \mid t \in \mathbb{R}\} = \left\{ \begin{array}{l} x_1 = 0 + t \cdot 1 \\ x_2 = 1 + t\,(-1) \\ x_3 = 0 + t \cdot 1 \end{array} \;\middle|\; t \in \mathbb{R} \right\}$

$\alpha = \{K + r\,\overrightarrow{KL} + s\,\overrightarrow{KM} \mid r, s \in \mathbb{R}\} = \left\{ \begin{array}{l} x_1 = 1 + r \cdot 0 + s\,(-1/2) \\ x_2 = 0 + r \cdot 1/2 + s \cdot 0 \\ x_3 = 1/2 + r \cdot 1/2 + s \cdot 1/2 \end{array} \;\middle|\; \dots \right\}$

$\overrightarrow{KL}$ $\overrightarrow{KM}$

b) rovnicemi

$p = \left\{ \begin{array}{l} 1x_1 + 1x_2 + 0x_3 = 1 \\ 0x_1 + 1 \cdot x_2 + 1 \cdot x_3 = 1 \end{array} \right\}$ ← z kterých bychom eliminovali t resp. r, s

$\alpha = \{(+1)x_1 + (-1)x_2 + 1x_3 = 3/2\}$ ←

pozn ... (1, −1, 1) = "normála α"

– početně

1) p ... param, α ... param:

$$\begin{aligned} 0 + t &= 1 \quad - 1/2\,s \\ 1 - t &= \quad 1/2\,r \\ 0 + t &= 1/2 + 1/2\,r + 1/2\,s \end{aligned}$$

3 rovnice
3 neznámé

2) p ... rov, α ... rov:

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_2 + x_3 &= 1 \\ x_1 - x_2 + x_3 &= -1/2 \end{aligned}$$

3 rovnice
3 neznámé

3) p .. param, α ... rov

1 rov.
1 neznámá

$$\underbrace{(t)}_{x_1} - \underbrace{(1 - t)}_{x_2} + \underbrace{(t)}_{x_3} = -1/2$$

$3t - 1 = 3/2$

$3t = 5/2, \quad t = 5/6$ ✓

4) p ... rov., α ... param

2 rovnice
2 neznámé

$R = p \cap \alpha = [5/6, 1 - 5/6, 5/6] = [5/6, 1/6, 5/6]$ ✓

5) $p = \{ X = t_0 A + t_1 G \mid \boxed{t_0 + t_1 = 1} \}$ ← af. kombinace

$\alpha = \{ X = s_0 K + s_1 L + s_2 M \mid \boxed{s_0 + s_1 + s_2 = 1} \}$ ←

$p \cap \alpha$ ...

$$
\begin{array}{l}
x_1 = \\
x_2 = \\
x_3 =
\end{array}
\left[
\begin{array}{rcl}
t_1 & = & s_0 + s_1 + \tfrac{1}{2} s_2 \\
t_0 & = & \tfrac{1}{2} s_1 \\
t_1 & = & \tfrac{1}{2} s_0 + s_1 + s_2
\end{array}
\right]
$$

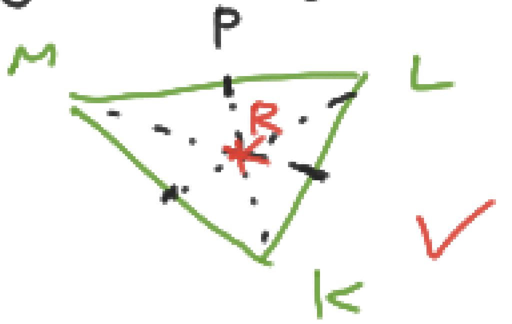
$$
\left[
\begin{array}{rcl}
t_0 + t_1 & = & 1 \\
1 & = & s_0 + s_1 + s_2
\end{array}
\right]
$$

$3 + 2 = \underline{5}$ rovnic

$\underline{5}$ neznámých

musí vyjít totéž ....

... tedy $R = \frac{1}{6} A + \frac{5}{6} G = \frac{1}{3} K + \frac{1}{3} L + \frac{1}{3} M$



cv. (14) VZÁJEMNOU POLOHU podpr. . . v závislosti na $a \in \mathbb{R}$

$$\beta = \left\{ \begin{pmatrix} -4 \\ 4 \\ 8 \end{pmatrix} + s \underbrace{\begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix}}_{u} \,\middle|\, s \in \mathbb{R} \right\}, \quad \mathcal{E} = \left\{ \begin{pmatrix} a \\ 6 \\ -5 \end{pmatrix} + t \underbrace{\begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}}_{v} \,\middle|\, t \in \mathbb{R} \right\}$$

$\beta \cap \mathcal{E}$ . . .

$$\begin{aligned} -4 + 2s &= a + t \\ 4 + s &= 6 - 3t \\ 8 - 4s &= -5 + 3t \end{aligned}$$

3 rovnice, 2 neznámé . . . vzhledem k a

např. $a = 0$ . . . nemá řeš., $\beta \cap \mathcal{E} = \emptyset$, tj. // nebo

↑ vektory u, v NEZÁVISLÉ

obecně

$$\begin{aligned} 2s - t &= a + 4 \\ s + 3t &= 2 \\ -4s - 3t &= -13 \end{aligned} \sim \begin{aligned} \text{I.}\ \ 2s - t &= a + 4 \\ \text{II.}\ \ s + 3t &= 2 \\ \text{III.}\ \ -3s + \boxed{0} &= -11 \end{aligned} \sim \ldots$$

$\text{III.}\ s = \frac{11}{3}$ $\qquad$ $\text{II.}\ 3t = 2 - \frac{11}{3} = -\frac{5}{3}$

$t = -\frac{5}{9}$

$\text{I.}\ \frac{22}{3} + \frac{5}{9} = a + 4 \iff \boxed{a = \frac{22}{3} + \frac{5}{9} - 4 = \frac{35}{9}}$

MEZISHRNUTÍ

$\beta \cap \mathcal{E} \neq \emptyset$

$\iff a = 35/9$

SHRNUTÍ

a) vektory $u = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \in \vec{\mathcal{B}}$ a $v = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} \in \vec{\mathcal{C}}$ NEZÁVISLÉ

$\Longrightarrow \vec{\mathcal{B}} \cap \vec{\mathcal{C}} = \{o\} \Longrightarrow$ NIKDY $\mathcal{B} = \mathcal{C}$ ani $\mathcal{B} \parallel \mathcal{C}$

b) $\mathcal{B} \cap \mathcal{C}$ neprázdný $\iff a = \frac{35}{9}$

Pro $a = \frac{35}{9}$ .... $\mathcal{B} \times \mathcal{C}$ ... RŮZNOBĚŽNÉ ($\mathcal{B} \cap \mathcal{C} =$ bod)

Pro $a \neq \frac{35}{9}$ .... $\mathcal{B} \,\dot{-}\, \mathcal{C}$ ... MIMOBĚŽNÉ

POZN. K SOUSTAVĚ:

$$\begin{bmatrix} 2s - t = a + 4 \\ s + 3t = 2 \\ -4s - 3t = -13 \end{bmatrix} \text{ má řešení} \iff \det \begin{pmatrix} 2 & -1 & a+4 \\ 1 & 3 & 2 \\ -4 & -3 & -13 \end{pmatrix} = \dots = -35 + 9a = 0$$

$$\iff a = \frac{35}{9}$$

cv. (15) VZÁJEMNÁ POLOHA . . .

$$\mathcal{B} = \left\{ \begin{array}{|l} x_1 + 4x_2 - x_3 = 10 \\ 2x_2 + x_4 = 11 \end{array} \right\}_{\dim 2}, \quad \mathcal{C} = \left\{ \begin{bmatrix} 0\\1\\1\\4 \end{bmatrix} + t_1 \begin{pmatrix} 0\\2\\1\\1 \end{pmatrix} + t_2 \begin{pmatrix} 2\\0\\2\\0 \end{pmatrix} \middle| \ t_1, t_2 \in \mathbb{R} \right\}_{\dim 2}$$

$$\boxed{\begin{array}{cc} = & \times \\ \parallel & \not\times \end{array}} \ ?$$

$\mathcal{B} \cap \mathcal{C}$ . . .
$$\begin{array}{|l} (2\cancel{t_2}) + 4(1+2t_1) - (1+t_1+2\cancel{t_2}) = 10 \\ 2(1+2t_1) + (4+t_1) = 11 \end{array} \quad \leftarrow \text{rovn./neznámé}$$

(a) $\left(\begin{array}{cc|c} * & * & * \\ 0 & 0 & 0 \end{array} \right)$

$$\sim \ldots\ldots \sim \boxed{\begin{array}{l} 7t_1 = 7 \\ 5t_1 = 5 \end{array}} \sim \begin{array}{l} t_1 = 1 \\ t_2 = \text{lib.} \end{array}$$

závěry:

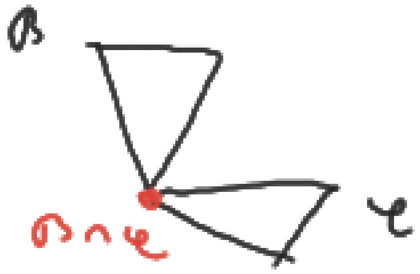
. . . má řeš. $\Rightarrow \mathcal{B} \cap \mathcal{C} \neq \emptyset$
. . . $\dim \mathcal{B} \cap \mathcal{C} = 1$

$\Big\} \ \mathcal{B} \times \mathcal{C}$

CO KDYBY

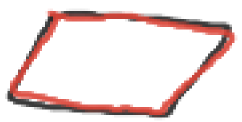
(b) $\left(\begin{array}{cc|c} * & * & * \\ 0 & * & * \end{array} \right) \Rightarrow t_1, t_2$ . . . jednoznačné
tj. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$, dim 0 $\Big\} \ \mathcal{B} \times \mathcal{C}$

$\mathcal{B} \cap \mathcal{C} = $ bod



(c) $\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \Rightarrow t_1, t_2$ . . . lib.
tj. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$, dim 2

$\mathcal{B} \cap \mathcal{C} = \mathcal{B} = \mathcal{C}$



$\mathcal{B} \cap \mathcal{C}$ (d) $\left(\begin{array}{cc|c} x & x & x \\ 0 & 0 & x \end{array}\right) \Rightarrow$ není řeš. $\Rightarrow \mathcal{B} \cap \mathcal{C} = \emptyset$

$\begin{cases} \parallel \Leftrightarrow \vec{\mathcal{B}} = \vec{\mathcal{C}} \\ \not\parallel \Leftrightarrow \vec{\mathcal{B}} \neq \vec{\mathcal{C}} \end{cases}$

?

$\vec{\mathcal{B}} \cap \vec{\mathcal{C}}$ $\left(\begin{array}{cc|c} x & x & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \dim \vec{\mathcal{B}} \cap \vec{\mathcal{C}} = 1 \Rightarrow \underbrace{\vec{\mathcal{B}} \neq \vec{\mathcal{C}}}$

$\mathcal{B} \not\parallel \mathcal{C}$

se společným směrem

("částečná $\parallel$")

$\left(\vec{\mathcal{B}} = \vec{\mathcal{C}} \Longleftrightarrow \dim \vec{\mathcal{B}} \cap \vec{\mathcal{C}} = 2 \right)$

tj. $\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$

Pozměňte zadání tak, abyste vyčerpali všechny možnosti ...

(e)

$\mathcal{B}\cap\mathcal{C}$ $\left(\begin{array}{cc|c} 0 & 0 & * \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$ není řeš. $\Rightarrow \mathcal{B}\cap\mathcal{C}=\emptyset$

$\vec{\mathcal{B}}\cap\vec{\mathcal{C}}$ $\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \dim \vec{\mathcal{B}}\cap\vec{\mathcal{C}} = 2 \Rightarrow \vec{\mathcal{B}} = \vec{\mathcal{C}}$

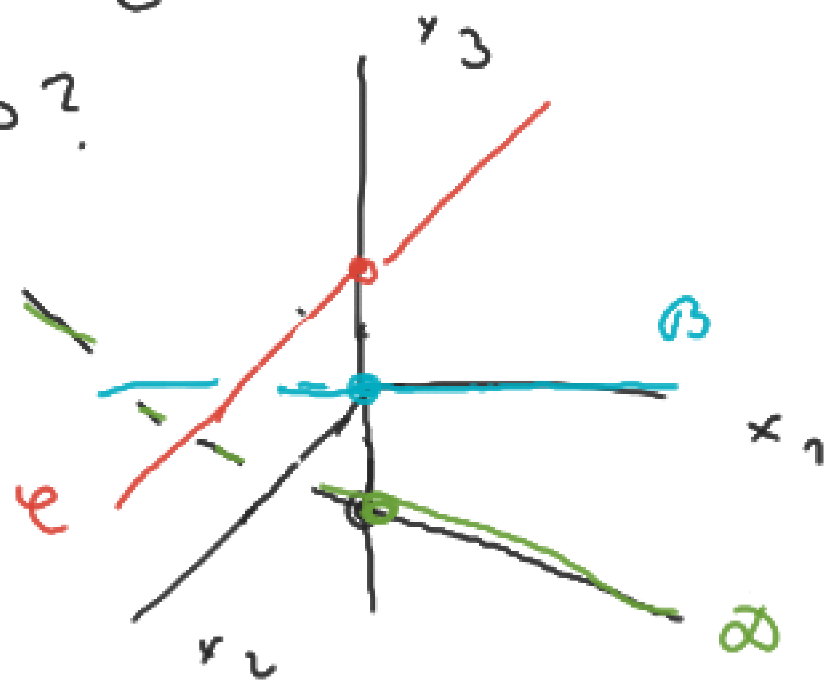
$\Bigg\}\ \mathcal{B} \parallel \mathcal{C}$

---

cv (16) | Př. tři navzájem mimob. podpr. v dim 3

přímky?

Např:



BODY NESTAČÍ

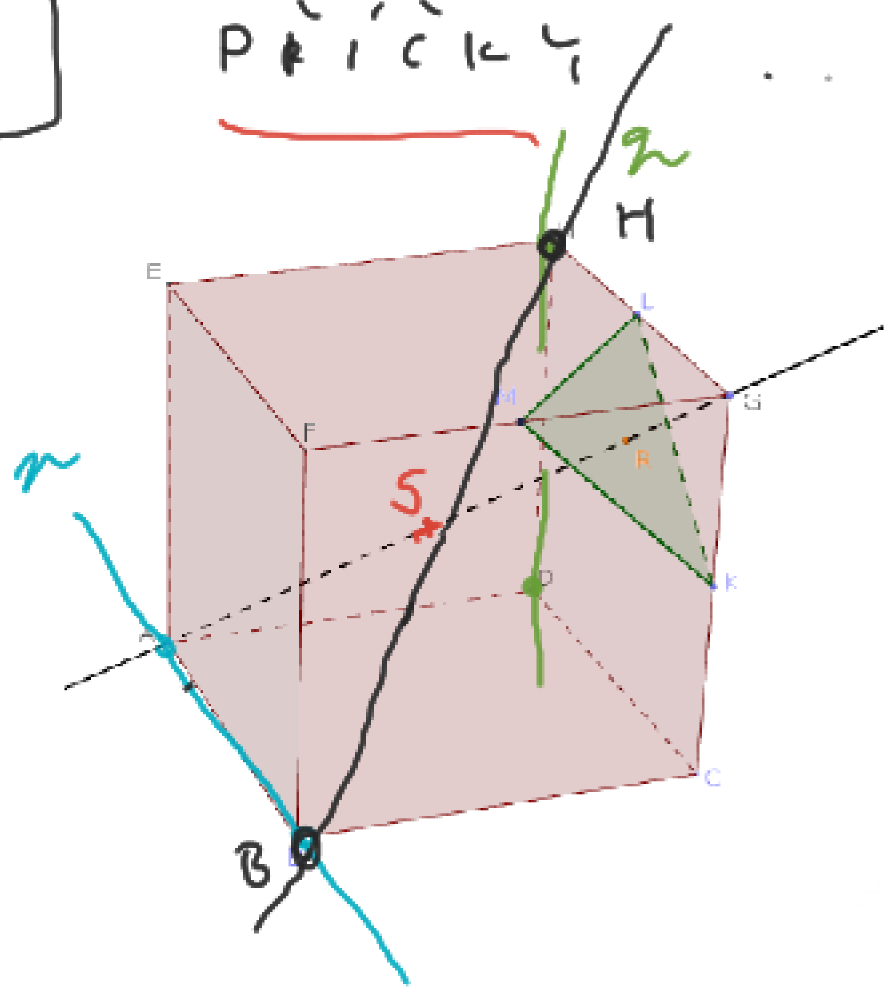
$\mathcal{B}\ \cdot$ $\quad \cdot\ \mathcal{C}$

různé $\Rightarrow \mathcal{B}\cap\mathcal{C}=\emptyset$

rovnob. $\Leftarrow \underset{\{0\}}{\vec{\mathcal{B}}} \overset{!}{=} \underset{\{0\}}{\vec{\mathcal{C}}}$

cv (17)

PŘÍČKY ......



p: AB, q: DH

+ a) STŘED KRYCHLE
  b) resp. SMĚR LM

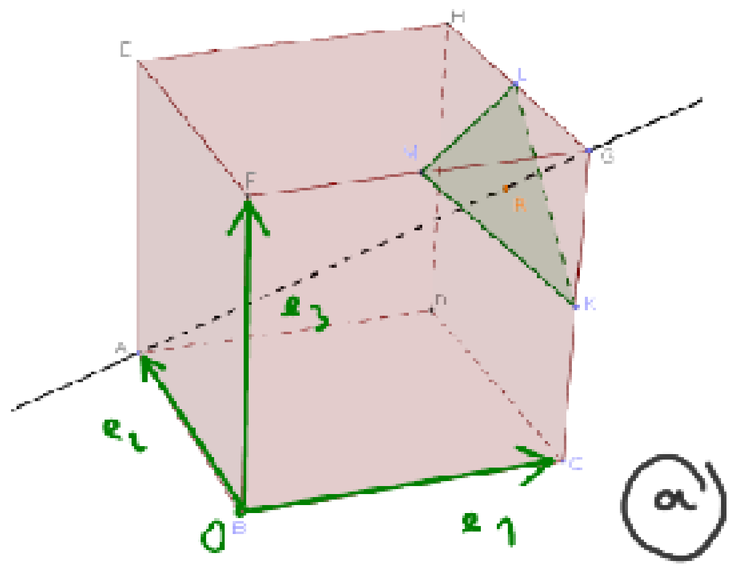
MYŠLENKA

PQ ... příčka proch. S

$\Updownarrow$

$\vec{SP}$ a $\vec{SQ}$ LIN. ZÁVISLÉ

POČÍTÁNÍ:



$$\left.\begin{array}{l} A = [0,1,0] \\ B = [0,0,0] \end{array}\right\} p \ni P = [0, t, 0] \leftarrow \{A + t\vec{AB}\}$$

$$\left.\begin{array}{l} D = [1,1,0] \\ H = [1,1,1] \end{array}\right\} q \ni Q = [1, 1, s] \leftarrow \{D + s\vec{DH}\}$$

a) $S = [\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}]$

$$\vec{SP} = \left(-\tfrac{1}{2}, t - \tfrac{1}{2}, -\tfrac{1}{2}\right)$$

$$\vec{SQ} = \left(\tfrac{1}{2}, \tfrac{1}{2}, s - \tfrac{1}{2}\right)$$

$\vec{SP} = \left(-\frac{1}{2},\ t-\frac{1}{2},\ \frac{1}{2}\right)$

$\vec{SQ} = \left(\frac{1}{2},\ \frac{1}{2},\ s-\frac{1}{2}\right)$

LIN. ZÁVISLÉ $\Leftrightarrow$??

(a) Z HLAVY .... $\vec{SP} = -1 \cdot \vec{SQ} \rightsquigarrow$ $t-\frac{1}{2} = -\frac{1}{2}$ $\rightsquigarrow$ $t = 0$

$\frac{1}{2} = s - \frac{1}{2}$ $\quad s = 1$

($t = 0 \rightsquigarrow P = B$ ... souhlasí s obrázkem ✓)
($s = 1 \rightsquigarrow Q = H$)

(b) VÝPOČET ... $\vec{SP} = k \cdot \vec{SQ}$ ← 3 neznámé

$$\left(\begin{aligned} -\tfrac{1}{2} &= k \cdot \tfrac{1}{2} \\ t-\tfrac{1}{2} &= k \cdot \tfrac{1}{2} \\ -\tfrac{1}{2} &= k \cdot \left(s-\tfrac{1}{2}\right) \end{aligned}\right)$$

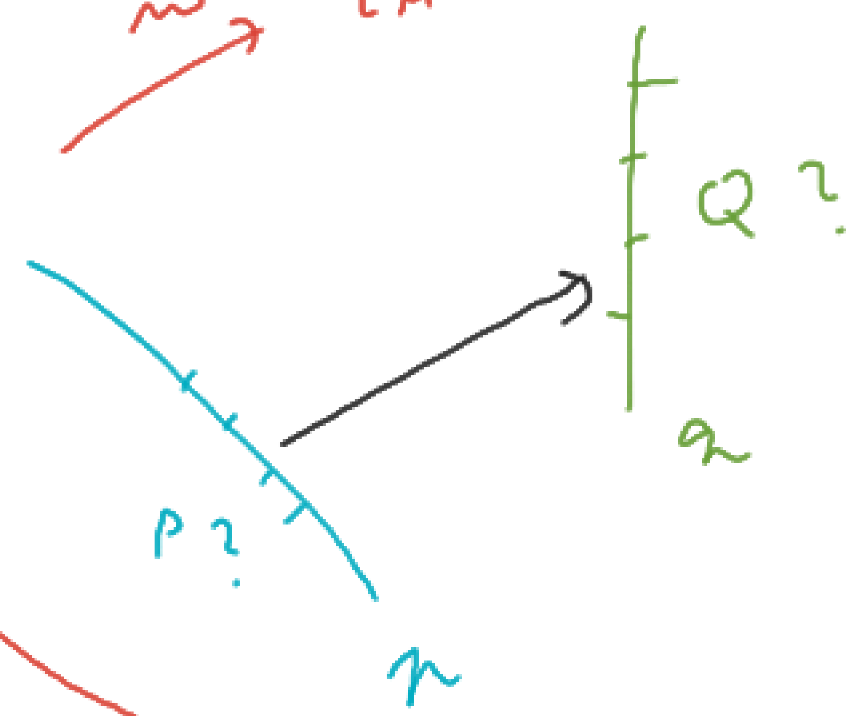
NELINEÁRNÍ vzhledem t, s, k,

ale LINEÁRNÍ vzhledem $t, s, k \cdot s$

alt. $\begin{pmatrix} -\frac{1}{2} & t-\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & s-\frac{1}{2} \end{pmatrix}$ $\xrightarrow{\det}$ $\boxed{\begin{aligned} \ldots &= 0 \\ \ldots &= 0 \end{aligned}}$ ← 2 neznámé ...

(b)

$w = \vec{l_m}$



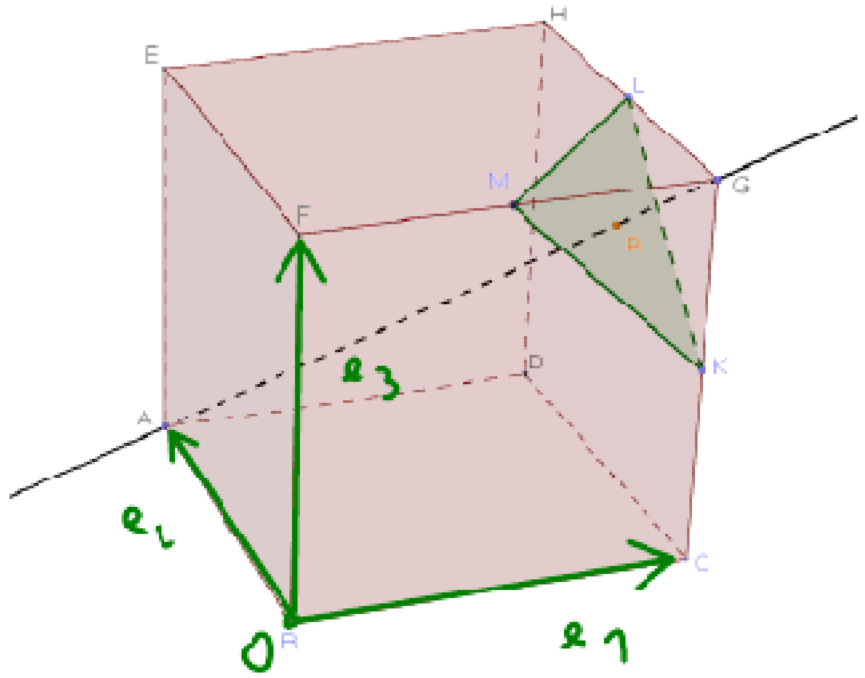
PQ = příčka $\parallel w$

$\Uparrow$

$\underline{\vec{PQ},\ w \text{ lin. závislé}}$

MYŠLENKA OBDOBNÁ

VÝPOČTY TAKY ...



$A =$ —''—
$B =$ —''— } P —''— (t)

$O =$ —''—
$M =$ —''— } Q —''— (s)

$\underline{w = (1,1,0)}$ ←

$\boxed{\vec{PQ} = k \cdot w}$?

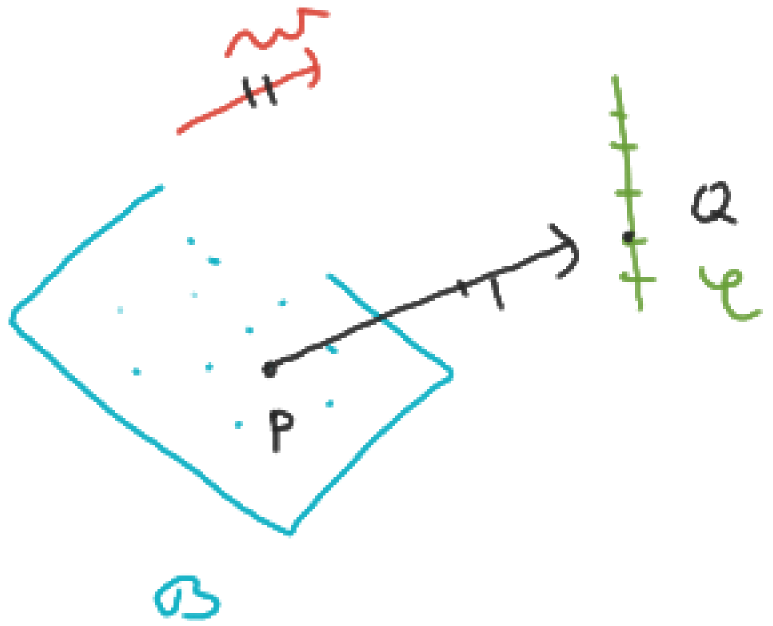
LINEÁRNÍ vzhledem $\underline{t, s, k}$!

cv (19) PŘÍČKY ...

dim 2 dim 1

$$\mathcal{B} = \left\{ \begin{matrix} x_2 - x_4 = 2 \\ x_3 = 1 \end{matrix} \right\}, \quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \middle| \, t \in \mathbb{R} \right\} \quad + \text{ SMĚR } \quad \underline{w = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$$

(a)



PŘEDCHOZÍ MYŠLENKA:

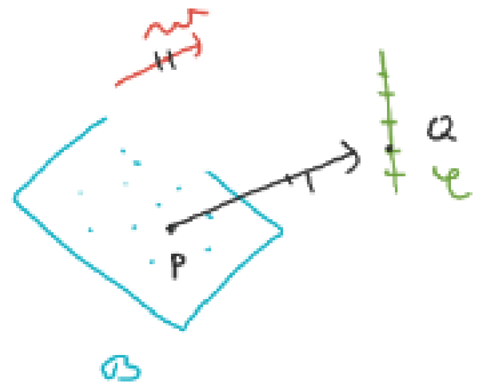
- param. vyjádření $\mathcal{B} = \{ [\vdots] + s_1 (\vdots) + s_2 (\vdots) \}$
- soustava $\boxed{\overrightarrow{PQ} = k \cdot w}$ ← 4 rovnice, 4 neznámé

(b) JINÝ NÁPAD ... viz prezentace / příště ...

cv (19) PŘÍČKY . . .

$\mathcal{B} = \left\{ \begin{array}{r} x_1 - x_4 = 2 \\ x_3 = 1 \end{array} \right\}$ (dim 2), $\quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \;\middle|\; t \in \mathbb{R} \right\}$ (dim 1) + SMĚR $\quad w = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

(a)



PŘEDCHOZÍ MYŠLENKA:
- param. vyjádření $\mathcal{B} = \{ [\vdots] + s_1 (\vdots) + s_2 (\vdots) \}$
- soustava $\boxed{\overrightarrow{PQ} = k \cdot w}$ ← 4 rovnice, 4 neznámé

(b) JINÝ NÁPAD . . . viz prezentace ~~příště . . .~~ → další str. . .

- "$r = \mathcal{C} + w$" . . . . $\boxed{\begin{array}{ll} x_1 = 1 + t & x_3 = 3 + s \\ x_2 = t & x_4 = 3 \end{array}}$
- $P = \mathcal{B} \cap r$ . . . . $\boxed{\begin{array}{l} t - 3 = 2 \\ 3 + s = 1 \end{array}} \Rightarrow \begin{array}{l} t = 5 \\ s = -2 \end{array} \Rightarrow P = \begin{bmatrix} 6 \\ 5 \\ 1 \\ 3 \end{bmatrix}$
- příčka "$p = P + w$" $= \left\{ \begin{bmatrix} 6 \\ 5 \\ 1 \\ 3 \end{bmatrix} + s \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \;\middle|\; s \in \mathbb{R} \right\}$

(• druhý bod $Q \in \mathcal{C}$ . . . $Q = p \cap \mathcal{C}$)

⟶ vyjde: $Q = \begin{bmatrix} 6 \\ 5 \\ 3 \\ 3 \end{bmatrix}$

OBECNÉ NÁPADY

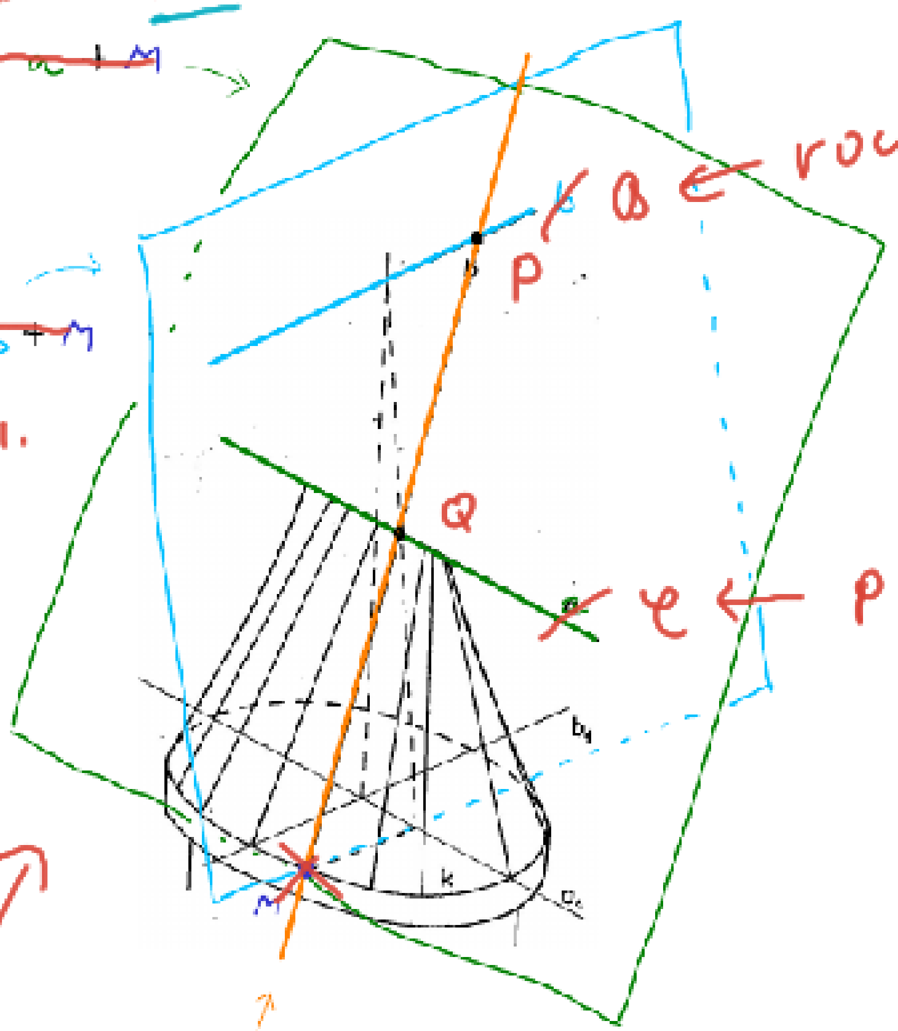
(b) ~~(a)~~ průnik nadprostorů:

„$\beta = \mathcal{B} + w$"

~~$\alpha = a + M$~~

~~$\beta = b + M$~~

„$\gamma = e + w$"

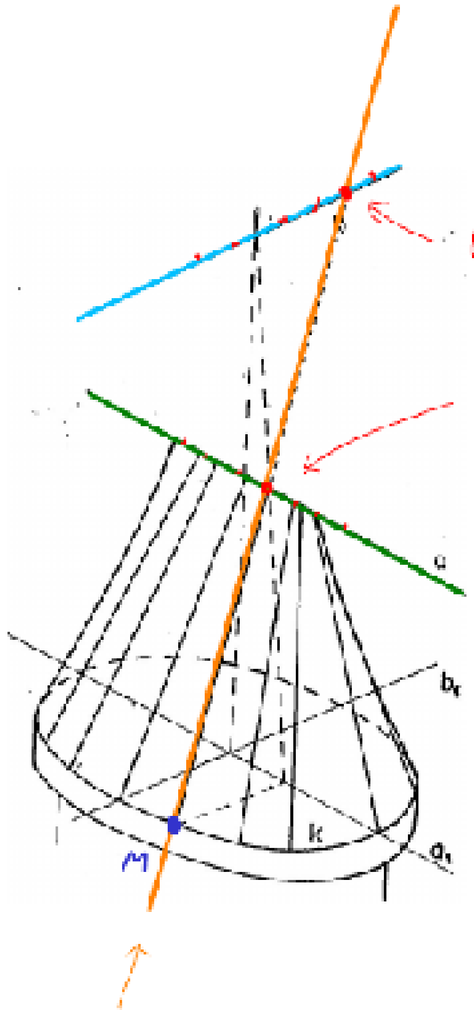


~~$p = \alpha \cap \beta$~~ $\boxed{p = \mathcal{B} \cap \gamma}$

resp. $\boxed{P = \mathcal{B} \cap \gamma}$ ← 2 rovnice / 2 neznámé

$Q = p \cap e$

(a) ~~(b)~~ spojnice koncových bodů:

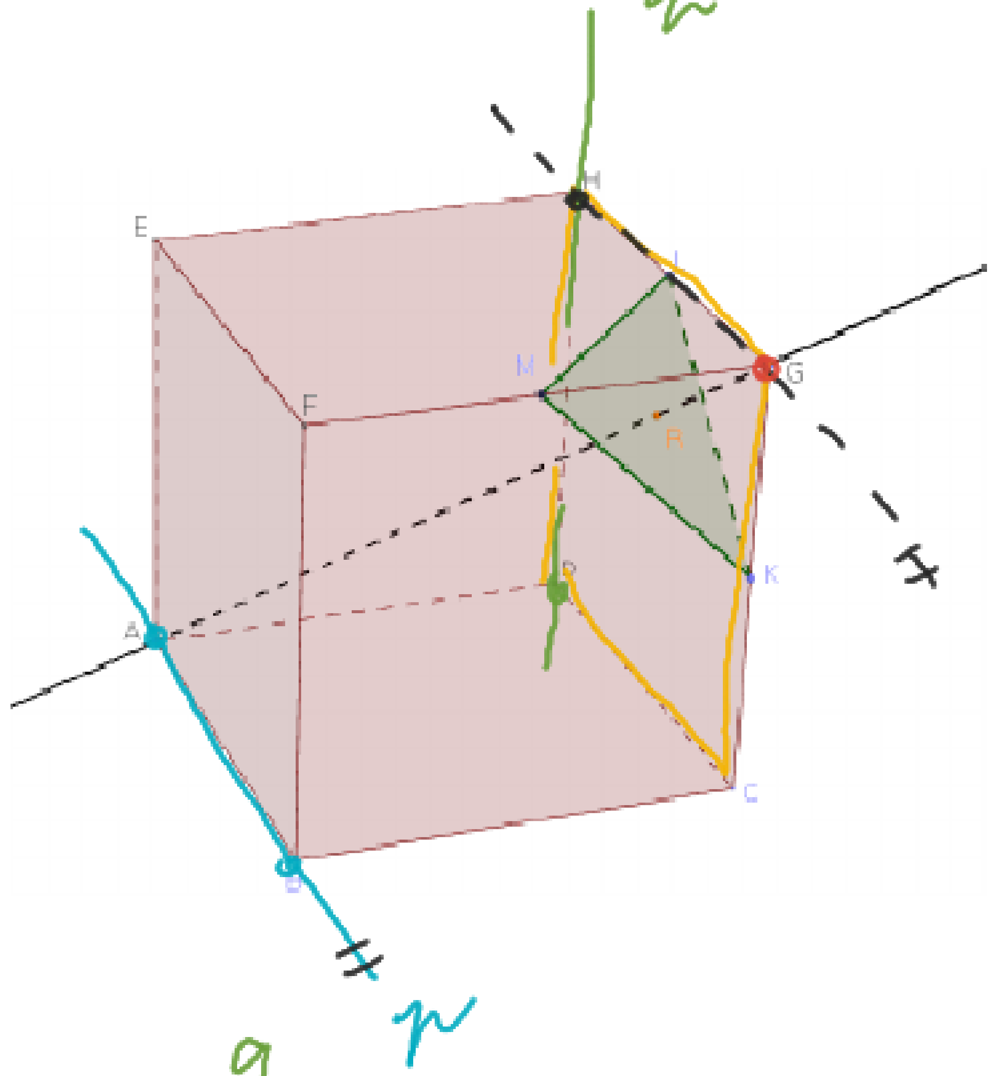


$\left.\begin{array}{l} B \in b \\ A \in a \end{array}\right\}$ tak, aby A, B, M kolin.

tj. $\boxed{\overrightarrow{MA} = k \cdot \overrightarrow{MB}}$

$p = A + B$

cv (17)



$\|$ $p = AB$

$q = DH$

Genericky 1 řešení, ale MŮŽE se stát cokoli...

$M = G$

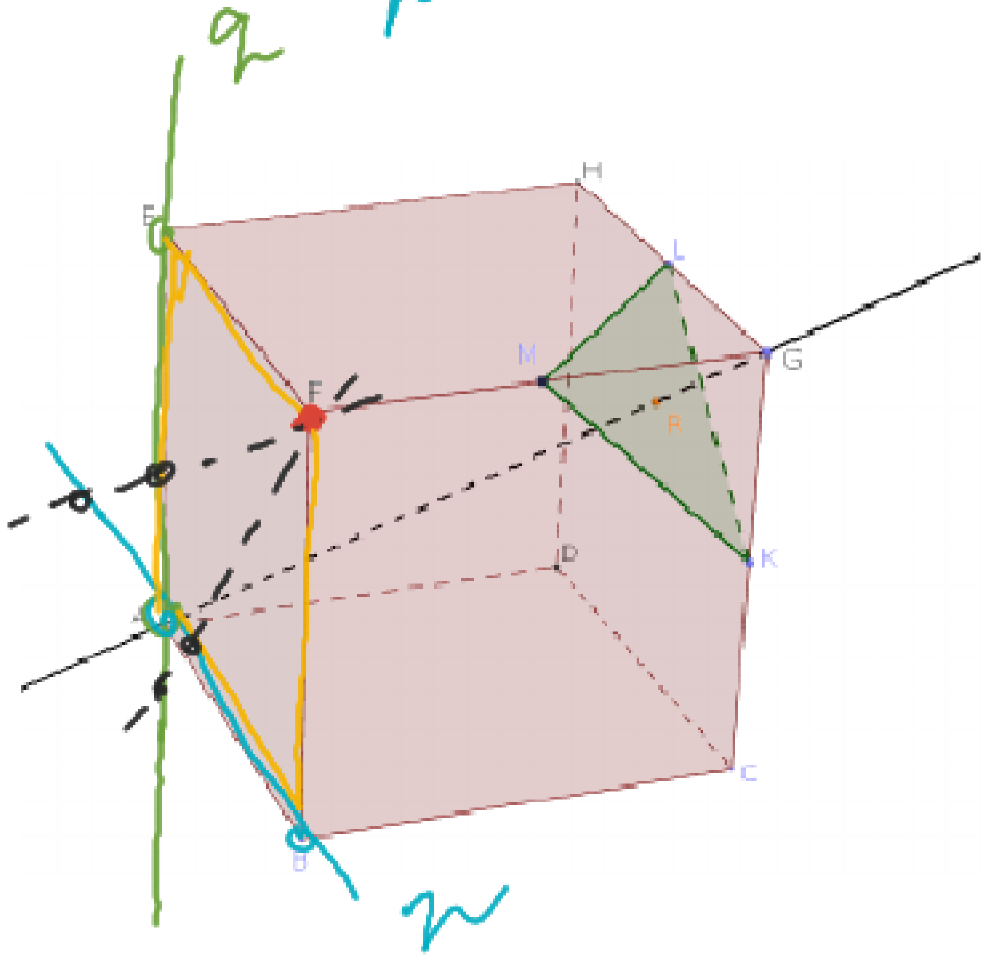
resp. $w = \vec{GH}$

tak, aby

– neex. příčka

(rovina $\beta = q + G$ je $\parallel p$)

resp. GH je "příčka" s nevl. průsečíkem s p



$\|$ $p = AB$

$q = AE$

$M = F$

resp. $w = \vec{BE}$

– ∞ příček ($F \in$ rovině $p + q$)

cv. (20)

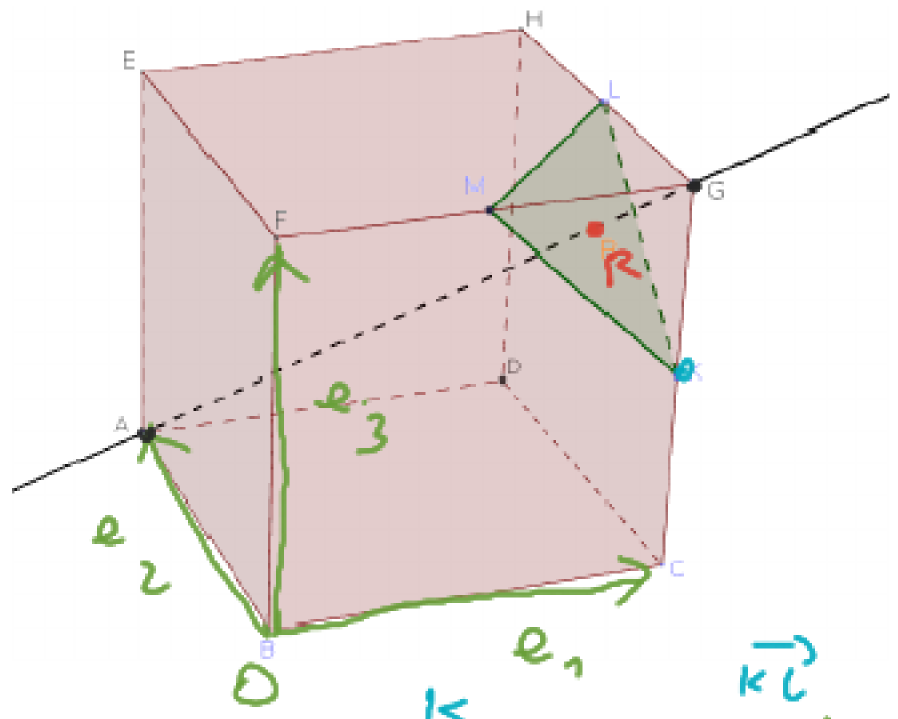
$\alpha = KLM$, $p = AG$

(a) A, G v opač. poloprostorech vzhledem k α?

(b) $R = p \cap \alpha$ = těžiště $\triangle KLM$

⋮

$$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \quad L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$



$$\alpha = \left\{ \overset{K}{\begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}} + r \overset{\overrightarrow{KL}}{\begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}} + s \overset{\overrightarrow{KM}}{\begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix}} \right\} = \left\{ \overset{(*)}{\boxed{x_1 - x_2 + x_3 = \frac{3}{2}}} \right\}$$

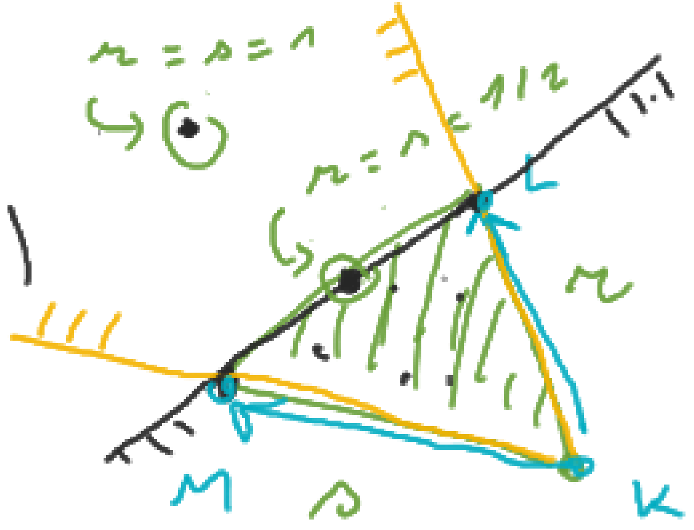
$$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad R = \begin{bmatrix} 5/6 \\ 1/6 \\ 5/6 \end{bmatrix}$$

(a) "dosaď A, G do rov. (*)" ⇝ $A: \; 0 - 1 + 0 = -1 < \frac{3}{2}$

$G: \; 1 - 0 + 1 = 2 > \frac{3}{2}$

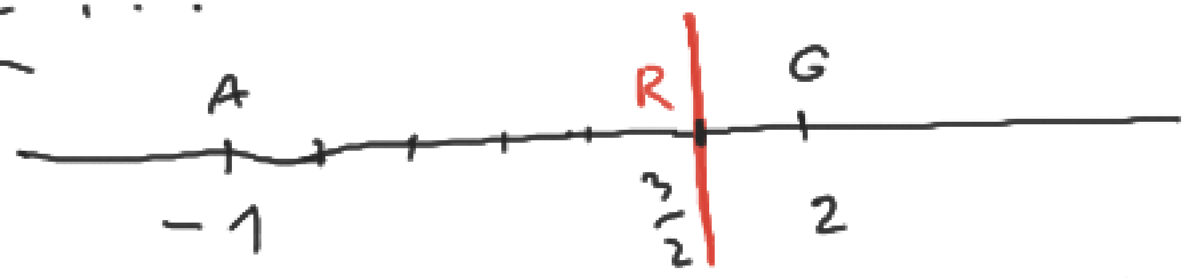
$\Rightarrow$ **A, G v OPAČ. POLOPR.**

(b)



$$\boxed{\begin{matrix} \underline{0 \le r, s \le 1} \\ \underline{r + s \le 1} \end{matrix}}$$

NAVÍC ...



... POMĚRY souhlasí ✓

(b) $\left\{ \overset{K}{\begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}} + r \overset{\overrightarrow{KL}}{\begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}} + s \overset{\overrightarrow{KM}}{\begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix}} \;\middle|\; \boxed{\begin{matrix} 0 \le r, s \le 1 \\ 0 \le r+s \le 1 \end{matrix}} \right\}$ = trojúhelník KLM

$$R = \begin{bmatrix} 5/6 \\ 1/6 \\ 5/6 \end{bmatrix}$$

$R \in \triangle KLM \iff \boxed{\begin{aligned} 5/6 &= 1 - 1/2\, s \\ 1/6 &= 1/2\, r \\ 5/6 &= 1/2 + 1/2\, r + 1/2\, s \end{aligned}}$ a $\boxed{\begin{matrix} 0 \le r, s \le 1 \\ 0 \le r+s \le 1 \end{matrix}}$

I. $\underbrace{5/6 - 1}_{-1/6} = -1/2\, s \rightsquigarrow s = 1/3$

II. $r = 1/3$

III. $5/6 \overset{?}{=} 1/2 + 1/6 + 1/6$ ✓

má řešení
(tj. $R \in$ rovině KLM)

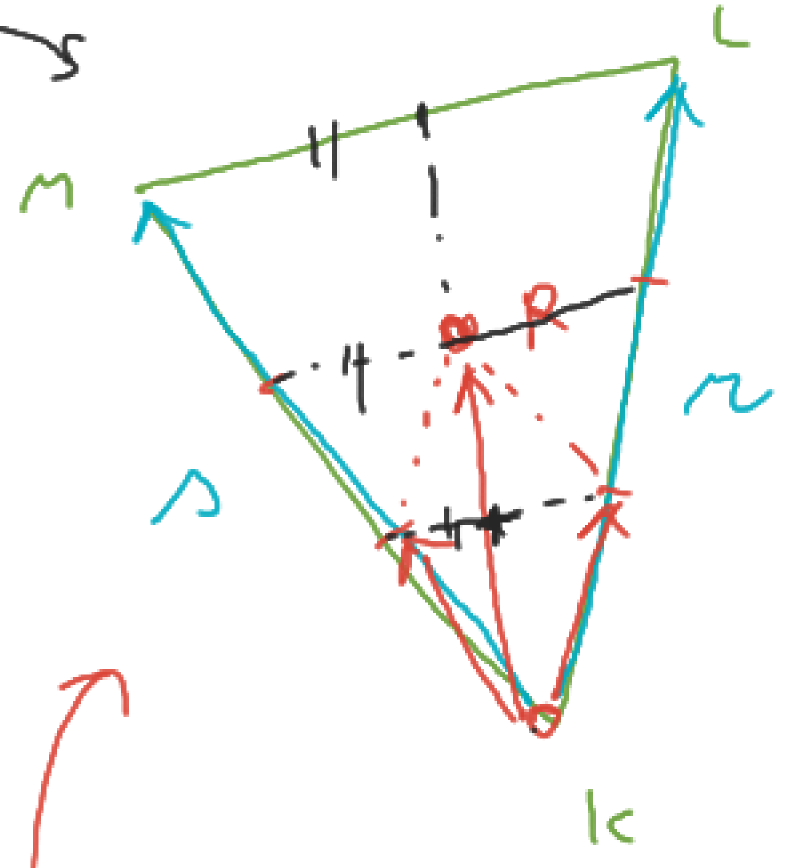
a $\left[\begin{matrix} 0 \le 1/3 \le 1 \\ 0 \le 1/3 + 1/3 \le 1 \end{matrix} \right.$ ✓

$\Downarrow$

$R \in \triangle KLM$

(b) TĚŽIŠTĚ △ KLM ?

vyšlo nám



pomocí param. vyjádř...

$$R = K + \overset{r}{\frac{1}{3}}\,\overrightarrow{KL} + \overset{s}{\frac{1}{3}}\,\overrightarrow{KM}$$

$$= L + \frac{1}{3}\,\overrightarrow{LK} + \frac{1}{3}\,\overrightarrow{LM}$$

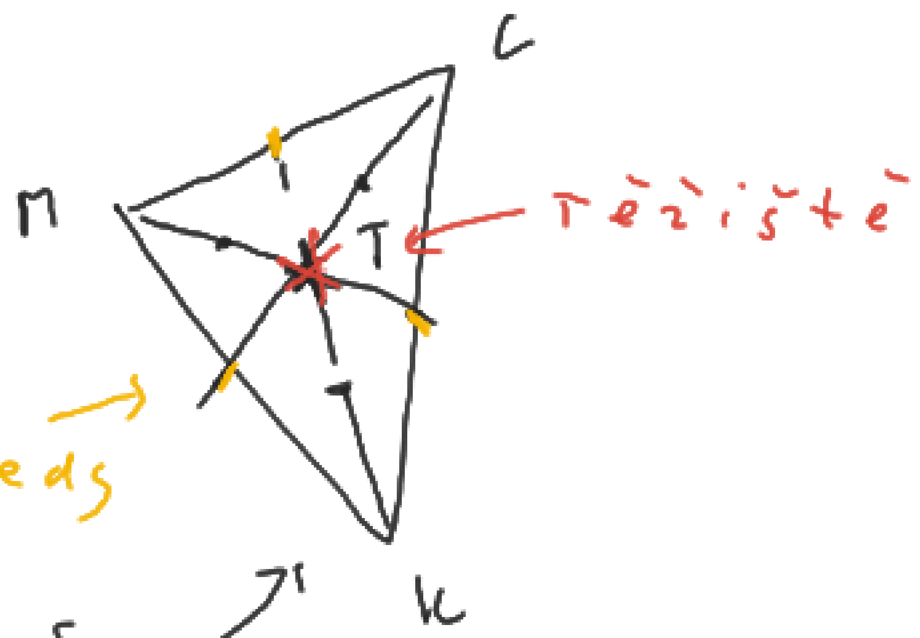
$$= M + \frac{1}{3}\,\overrightarrow{ML} + \frac{1}{3}\,\overrightarrow{MK}$$

TOTÉŽ ✓

pomocí afinních souř.

$$\text{"}\, R = \overset{1-r-s}{\frac{1}{3}}K + \overset{r}{\frac{1}{3}}L + \overset{s}{\frac{1}{3}}M \,\text{"}$$

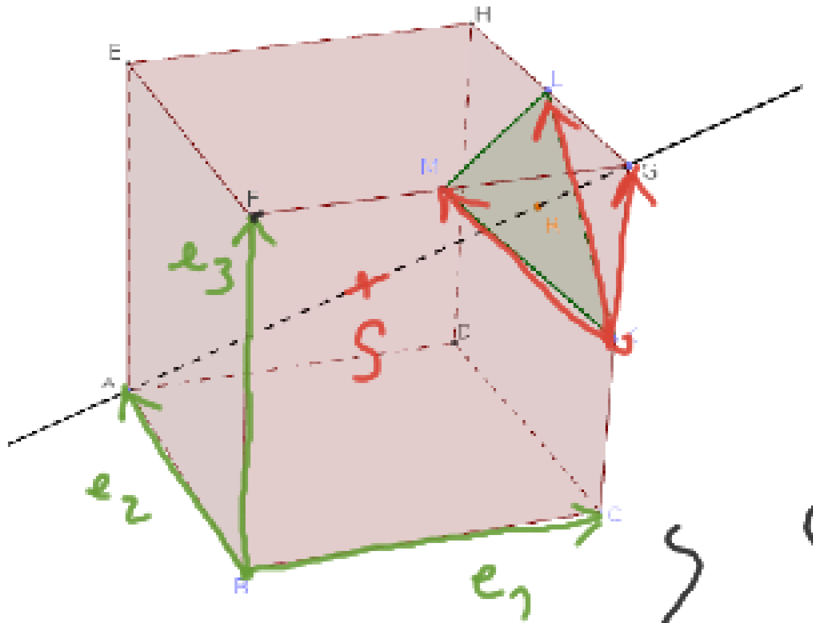
$$\text{"}\, R = K + \underset{r}{\frac{1}{3}}(L-K) + \underset{s}{\frac{1}{3}}(M-K) \,\text{"}$$



těžiště def

cv. (20)

$K = \begin{bmatrix}1\\0\\1/2\end{bmatrix},\ L = \begin{bmatrix}1\\1/2\\1\end{bmatrix},\ M = \begin{bmatrix}1/2\\0\\1\end{bmatrix},\ G = \begin{bmatrix}1\\0\\1\end{bmatrix} \qquad S = \begin{bmatrix}1/2\\1/2\\1/2\end{bmatrix}$



(c) $S \in$ konvexní obal bodů K, L, M, G ?

σ

① param. vyjádř. ... $\sigma = \{ K + r\vec{KL} + s\vec{KM} + t\vec{KG} \mid \boxed{0 \le r,s,t \le 1,\ 0 \le r+s+t \le 1} \}$

② rovnicové vyjádř. ... $\sigma = \left\{ \boxed{x_3 \le 1},\ \boxed{x_1 - x_2 + x_3 \ge \tfrac{3}{2}},\ \boxed{x_2 \ge 0},\ \boxed{x_1 \le 1} \right\}$

③ af. kombinace ... $\sigma = \{ t_K K + t_L L + t_M M + t_G G \mid t_K + t_L + t_M + t_G = 1,\ \boxed{t_K, t_L, t_M, t_G \ge 0} \}$

$\{x_3 = 1\}$ = rovina MLG

$\{x_3 \le 1\}$ = poloprostor ... obs. bod K

$\{x_1 - x_2 + x_3 = \tfrac{3}{2}\}$ = rovina KLM

$\{$ — " — $\ge \tfrac{3}{2}\}$ = poloprostor ... obs. G

např. ② ... $x_3 = \tfrac{1}{2} \le 1$ ✓ $\quad x_1 - x_2 + x_3 = 1/2 \not\ge 3/2 \implies S \notin \sigma$

$x_2 = \tfrac{1}{2} \ge 0$ ✓ $\quad x_1 = \tfrac{1}{2} \le 1$ ✓

ODBOČKA

$X = -1/3\,K + 4/3\,L \iff X = (1 - 4/3)K + 4/3\,L = K + \frac{4}{3}(L - K) = K + \frac{4}{3}\overrightarrow{KL}$ ✓

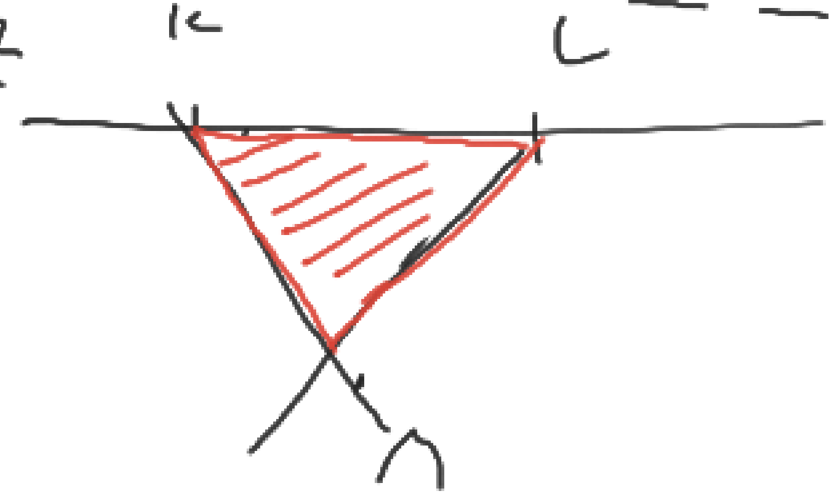
dim 1

	K		L	
t_K	1	1/3	0	-1/3
t_L	0	2/3	1	4/3

- $\{ t_K K + t_L L \mid t_K + t_L = 1 \}$ = přímka KL
- $\{$ —"— $\mid$ —"— , $\boxed{t_K \geq 0}$ $\}$ = polopř. LK

- $\{$ —"— $\mid$ —"— , $\boxed{t_K, t_L \geq 0}$ $\}$ = úsečka KL

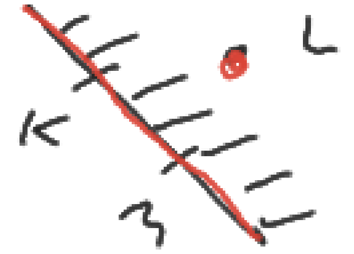
dim 2



- $\{ t_K K + t_L L + t_M M \mid t_K + t_L + t_M = 1 \}$ = rovina KLM

⋮

- $\{$ —"— $\mid$ —"— , $\boxed{t_L \geq 0}$ $\}$ = polorovina



- $\{$ —"— $\mid$ —"— , $\boxed{t_K, t_L, t_M \geq 0}$ $\}$ = $\triangle KLM$

dim 3

obdobně ⟶ čtyřstěn KLMG .... $\boxed{t_K, t_L, t_M, t_G \geq 0}$

Jak by se řešilo pomocí vyjádření ③ ?

⇝ SOUSTAVA pro $S \in \sigma$:

$$\left.\begin{aligned}
\tfrac{1}{2} &= t_K + t_L + \tfrac{1}{2} t_M + t_G \\
\tfrac{1}{2} &= + \tfrac{1}{2} t_L \\
\tfrac{1}{2} &= \tfrac{1}{2} t_K + t_L + t_M + t_G \\
\boxed{1} &\boxed{= t_K + t_L + t_M + t_G}
\end{aligned}\right\} \quad \ldots \quad
\begin{aligned}
t_K &= 1 \geq 0 \checkmark \\
t_L &= 1 \geq 0 \checkmark \\
t_M &= 1 \geq 0 \checkmark \\
t_G &= -2 \not\geq 0
\end{aligned}$$

$$\Downarrow$$

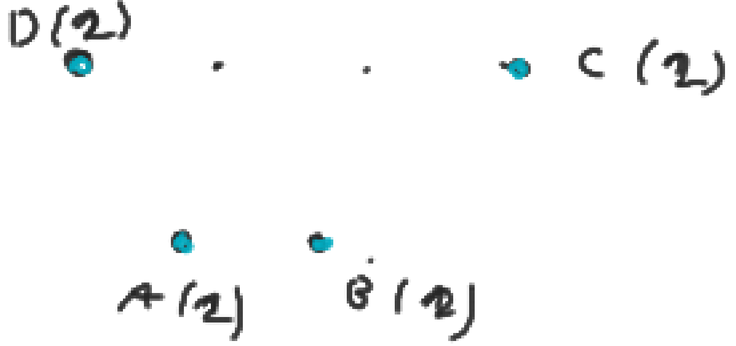
$$S \notin \sigma$$

$$\sigma = \left\{ t_K K + t_L L + t_M M + t_G G \;\middle|\; \begin{array}{l} \boxed{t_K + t_L + t_M + t_G = 1} \\ \boxed{t_K, t_L, t_M, t_G \geq 0} \end{array} \right\}$$

$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$, $L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}$, $M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\qquad S = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$

cv (21)

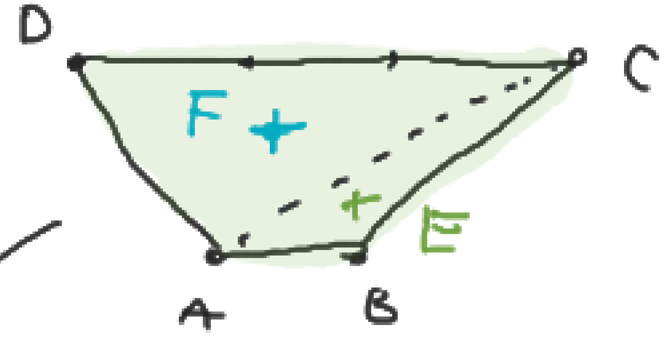
(a) těžiště



$$T_1 = \frac{1}{4}A + \frac{1}{4}B + \frac{1}{4}C + \frac{1}{4}D$$

$$\frac{2}{2+2+2+2}$$

(b) těžiště



těžiště ACD

těžiště ABC

poměr VAH
||
poměr OBSAHů

T = těžiště E(1), F(3)

(1) $\left[\begin{array}{l} F = \frac{1}{3}A + \frac{1}{3}C + \frac{1}{3}D \\ E = \frac{1}{3}A + \frac{1}{3}B + \frac{1}{3}C \end{array}\right]$ "redukce"

(2) $\left[T = \frac{1}{4}E + \frac{3}{4}F \right]$ "bodová soustava"

$4 = 1 + 3$

(3) DOSAZENÍ ... $T_2 = \frac{1}{4}\cdot\frac{1}{3}(A+B+C) + \frac{3}{4}\cdot\frac{1}{3}(A+C+D)$

$$T_2 = \frac{1}{3}A + \frac{1}{12}B + \frac{1}{3}C + \frac{1}{4}D = \ldots\ldots = \ldots\ldots$$

$T_1 \neq T_2$

A, B, C, D NEJSOU V OB. POLOZE

⇒ "SOUŘ." NEJSOU JEDNOZNAČNÉ !!

$$T_1 = \frac{1}{4}A + \frac{1}{4}B + \frac{1}{4}C + \frac{1}{4}D = 1\cdot A - \frac{1}{2}B + \frac{1}{2}C$$

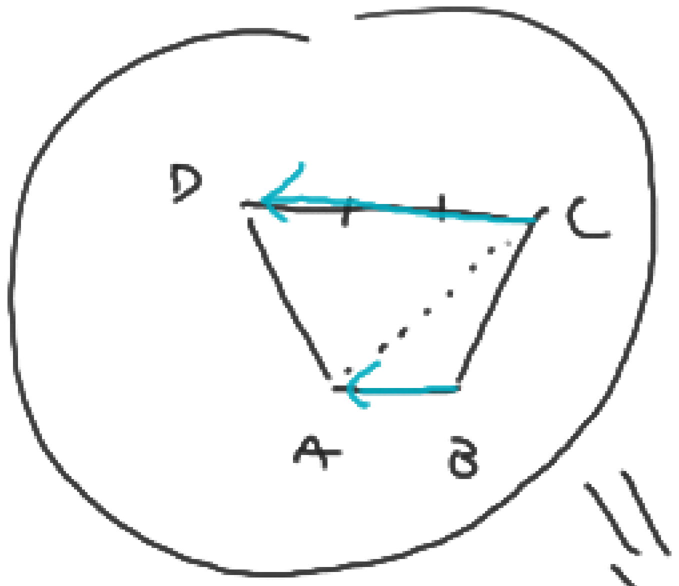
$$T_2 = \frac{1}{3}A + \frac{1}{12}B + \frac{1}{3}C + \frac{1}{4}D = \frac{13}{12}A - \frac{2}{3}B + \frac{7}{12}C$$

$$\Rightarrow T_1 \neq T_2$$

VSKUTKU

JEDNOZNAČNÁ vyjádření!

(A, B, C jsou v obecné poloze)



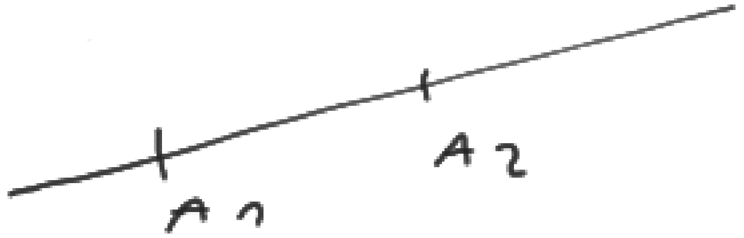
$$\boxed{D = 3A - 3B + 1C}$$

$$\overrightarrow{CD} = 3\overrightarrow{BA} \Rightarrow D - C = 3(A - B)$$

cv (22)

$A_1 = [1,-1,0,2]$, $A_2 = [3,-1,2,4]$, $A_3 = [3,1,0,0]$,
$A_4 = [5,1,2,2]$, $A_5 = [3,0,1,2]$, $A_6 = [4,-1,0,2]$

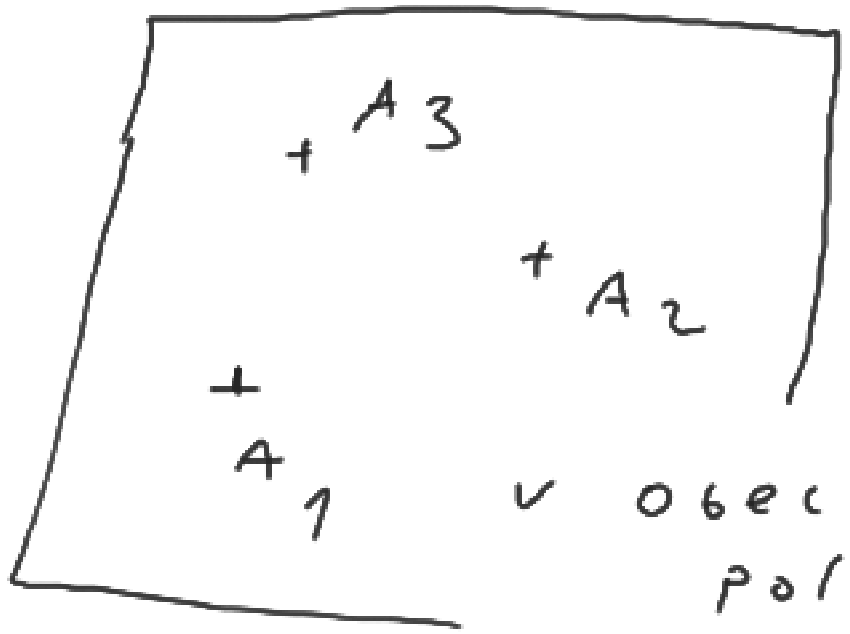
- $A_2 \overset{?}{=} A_1$ $\rightsquigarrow$ JISTĚ <u>NE</u>
- $\boxed{A_3 \overset{?}{=} t_1 A_1 + t_2 A_2, \quad t_1 + t_2 = 1}$ $\iff$ $A_3 \in$



$\Updownarrow$

$$\boxed{\begin{aligned} 3 &= t_1 + 3t_2 \\ 1 &= -t_1 - t_2 \\ 0 &= + 2t_2 \\ 0 &= 2t_1 + 4t_2 \\ 1 &= t_1 + t_2 \end{aligned}}$$

$\rightsquigarrow$ NEMÁ řešení, tj.



v obecné poloze

- $\boxed{A_4 \overset{?}{=} t_1 A_1 + t_2 A_2 + t_3 A_3, \quad t_1 + t_2 + t_3 = 1}$

$\rightsquigarrow$ má řešení . . . . . . PŘÍŠTĚ DOPLNÍME

cv (22) | $A_1 = [1,-1,0,2]$, $A_2 = [3,-1,2,4]$, $A_3 = [3,1,0,0]$,
$A_4 = [5,1,2,2]$, $A_5 = [3,0,1,2]$, $A_6 = [4,-1,0,2]$

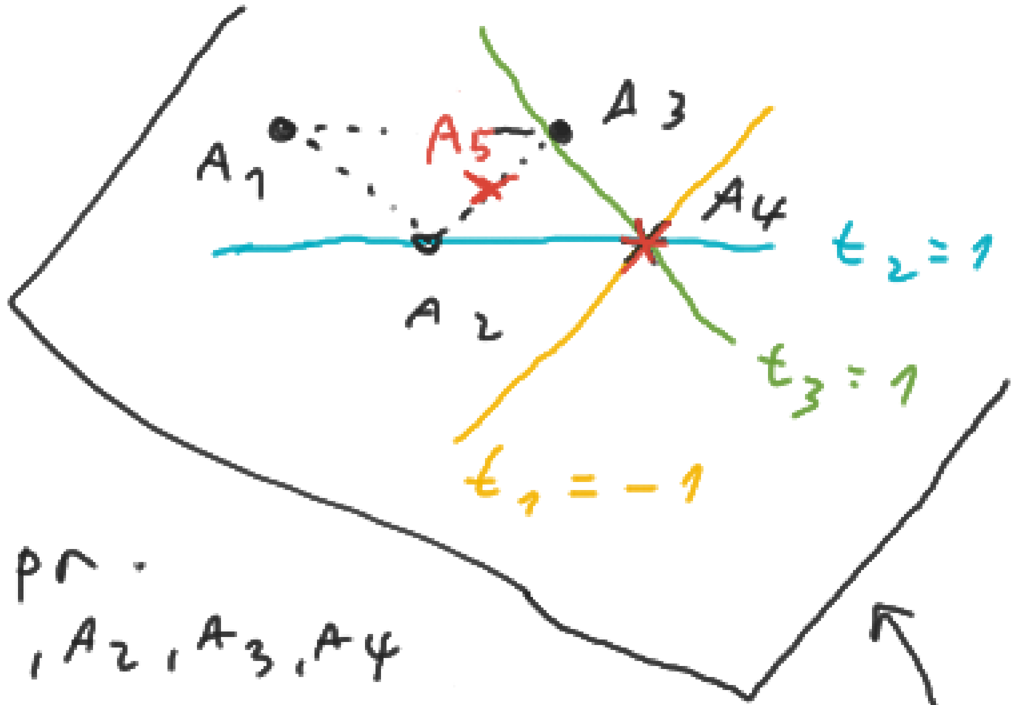
MINULE (a) $A_2 \neq A_1$
(b) $A_3 \neq t_1 A_1 + t_2 A_2$

(c) $A_4 = t_1 A_1 + t_2 A_2 + t_3 A_3$, $t_1 + t_2 + t_3 = 1$

$$\left[\begin{array}{c|ccc} 5 & 1 & 3 & 3 \\ 1 & -1 & -1 & 1 \\ 2 & 0 & 2 & 0 \\ 2 & 2 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \Longleftarrow \boxed{\begin{array}{l} t_1 = -1 \\ t_2 = 1 \\ t_3 = 1 \end{array}}$$

, tj. $\boxed{A_4 = -1 A_1 + 1 A_2 + 1 A_3}$

zejména $A_4 \in$ rovině A_1, A_2, A_3 ✓



(d) $A_5 = \sum_{i=1}^{4} t_i A_i$, $\sum_{i=1}^{4} t_i = 1$

$$\left[\begin{array}{c|cccc} 3 & . & . & . & 5 \\ 0 & . & . & . & 1 \\ 1 & . & . & . & 2 \\ 2 & . & . & . & 2 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right] \Longleftarrow \dots \Longleftarrow \boxed{\begin{array}{l} t_1 = \text{lib.} \\ t_2 = 1/2 - t_1 \\ t_3 = 1/2 - t_1 \\ t_4 = t_1 \end{array}}$$

, tj. $A_5 \in$ podpr. A_1, A_2, A_3, A_4

z předch. $\Longrightarrow$ A_1, A_2, A_3, A_4 v rovině

$\Longrightarrow$ nejednoznačnost!

$\boxed{t_1 = 0} \rightsquigarrow \underbrace{t_2 = \frac{1}{2} = t_3,\ t_4 = 0}$

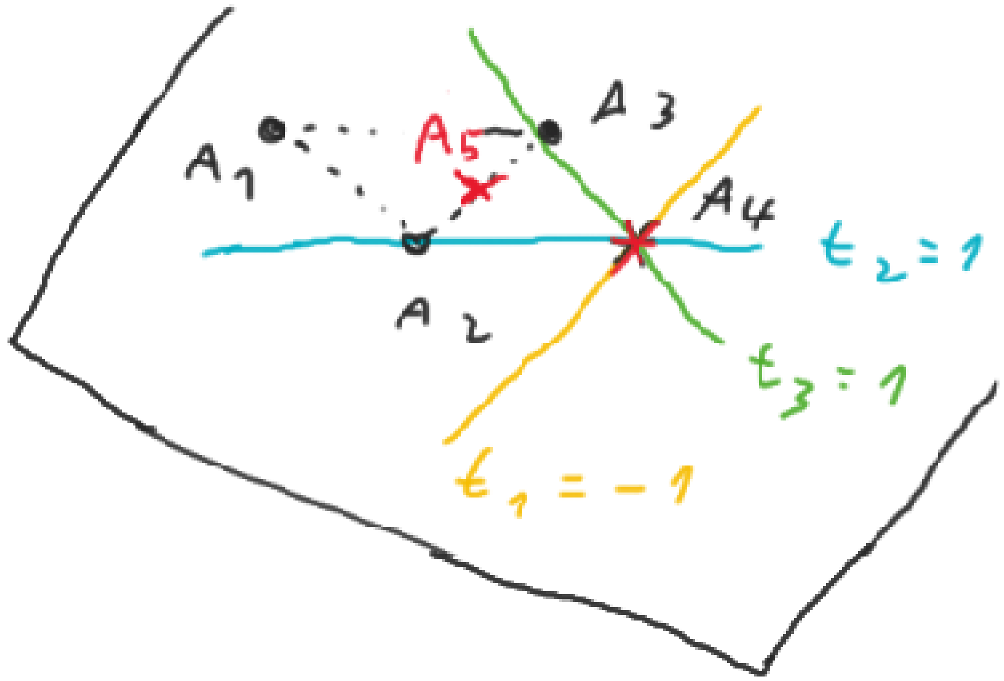
$\boxed{A_5 = \frac{1}{2} A_2 + \frac{1}{2} A_3}$

(e) $A_6 = \sum_{i=1}^{5} \dots\dots\dots$,

$$\left[\begin{array}{c|ccccc} 4 & . & . & . & . & 3 \\ -1 & . & . & . & . & 0 \\ 0 & . & . & . & . & 1 \\ 2 & . & . & . & . & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \rightsquigarrow \boxed{\text{NEMÁ ŘEŠENÍ}}$$

tj. .... $A_6 \notin$ rovině $A_1, \dots, A_5$

DOPLNĚNÍ

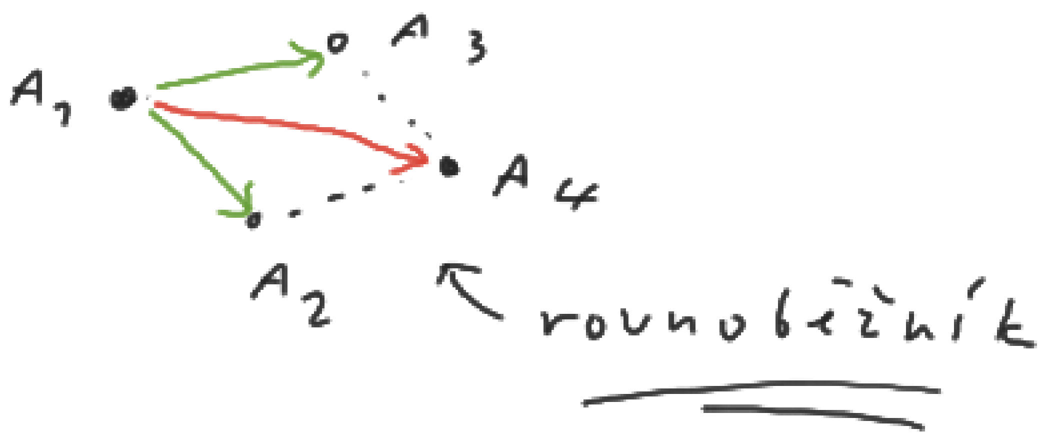


(c) $A_4 = -A_1 + A_2 + A_3 = A_1 + (A_2 - A_1) + (A_3 - A_1)$

t_1, t_2, t_3 JEDNOZN.

a $t_1 \cancel{\geq} 0$

$A_4 \cancel{\in}$ konvex. obalu A_1, A_2, A_3 ✓



(d) $A_5 = t_1 A_1 + (1/2 - t_1) A_2 + (1/2 - t_1) A_3 + t_1 A_4$ & $A_4 = -A_1 + A_2 + A_3$

koef. NEJEDNOZN.
a znaménka
mohou být LIB!

$A_5 = \frac{1}{2} A_2 + \frac{1}{2} A_3$ ... střed $A_2 A_3$

(totéž co subs. $t_1 = 0$)

(subs. $t_1 = 1/2$ ⇝ $A_5 = \frac{1}{2} A_1 + \frac{1}{2} A_4$ ... střed $A_1 A_4$) ✓

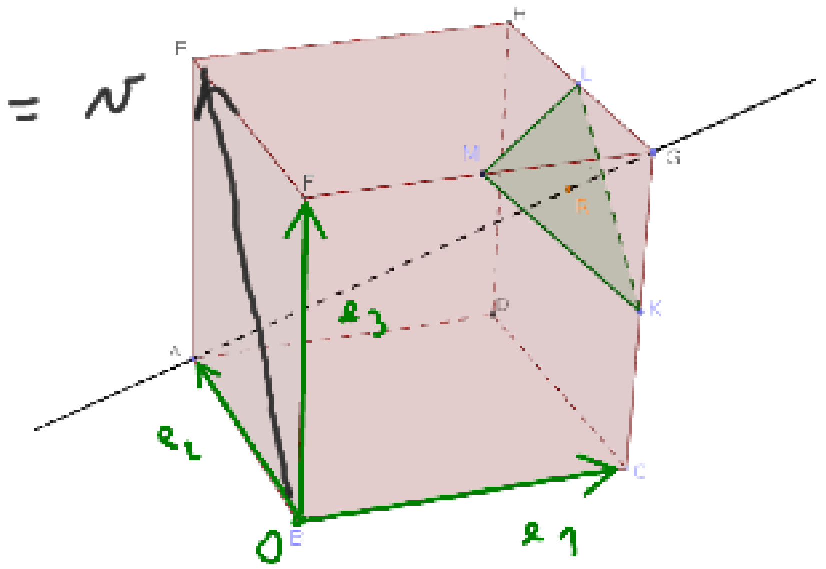
Ale EX. vyjádření:

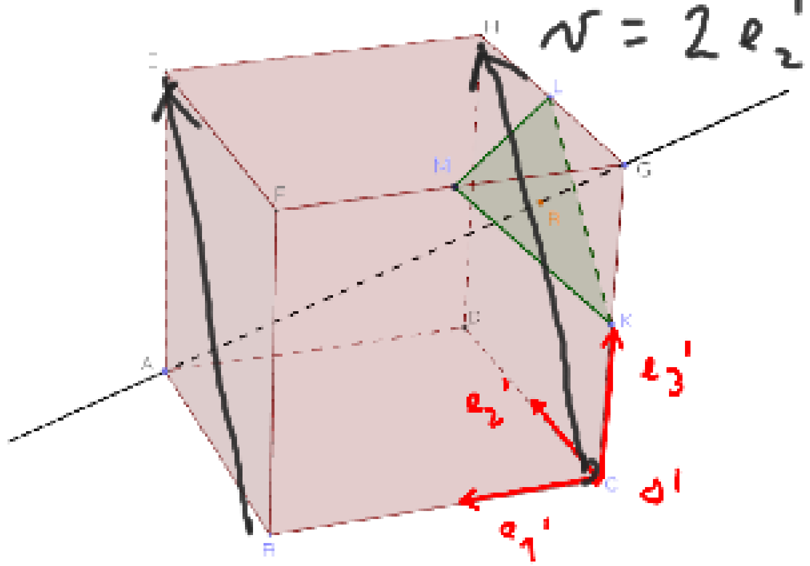
$t_1, t_2, t_3, t_4 \geq 0$

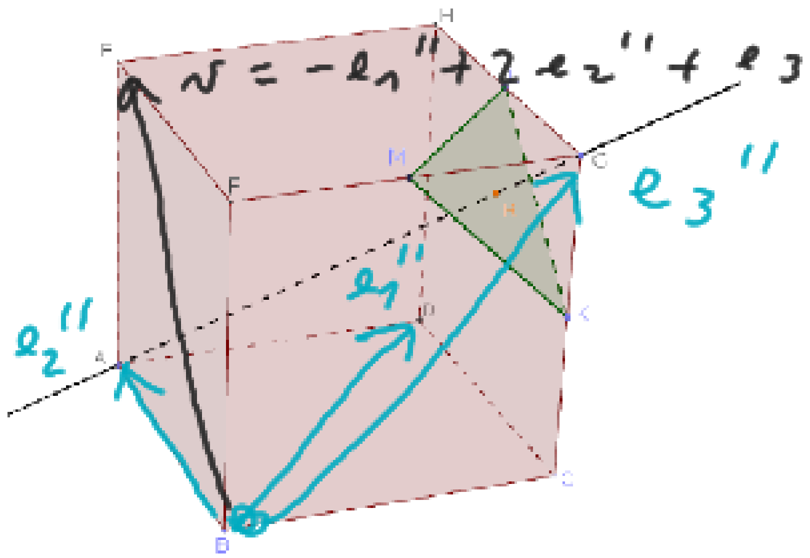
TEDY $A_5 \in$ konvex. obalu A_1, A_2, A_3, A_4 ✓

cv (23) | KARTÉZSKÁ SOUŘ. SOUSTAVA

KRYCHLE S HRANOU 1







ANO

$\|e_1\| = \|e_2\| = \|e_3\| = 1$

$e_1 \perp e_2,\ e_2 \perp e_3,\ e_1 \perp e_3$

tj.

$$\boxed{\begin{array}{l} e_1 \cdot e_1 = e_2 \cdot e_2 = e_3 \cdot e_3 = 1 \\ e_1 \cdot e_2 = e_2 \cdot e_3 = e_1 \cdot e_3 = 0 \end{array}}$$

$$\left.\begin{array}{l} u = (1, 2, -1) \\ v = (0, 1, 1) \end{array}\right\} \quad \begin{array}{l} u \cdot u = 1 + 4 + 1 = 6 \\ u \cdot v = 0 + 2 - 1 = 1 \\ v \cdot v = 0 + 1 + 1 = 2 \end{array}$$

NE

$\|e_1'\| = \|e_2'\| = \|e_3'\| = \frac{1}{2} \neq 1$ $k = 1/2$

$e_1 \perp e_2,\ e_2 \perp e_3,\ e_3 \perp e_4$

$u = (-2, 4, -2)$

$v = (0, 2, 2)$

$k^2 = \frac{1}{4}$

$u \cdot u = \frac{1}{4}(4 + 16 + 4) = \frac{1}{4} 24 = 6$

$u \cdot v = \frac{1}{4}(0 + 8 - 4) = \frac{1}{4} 4 = 1$

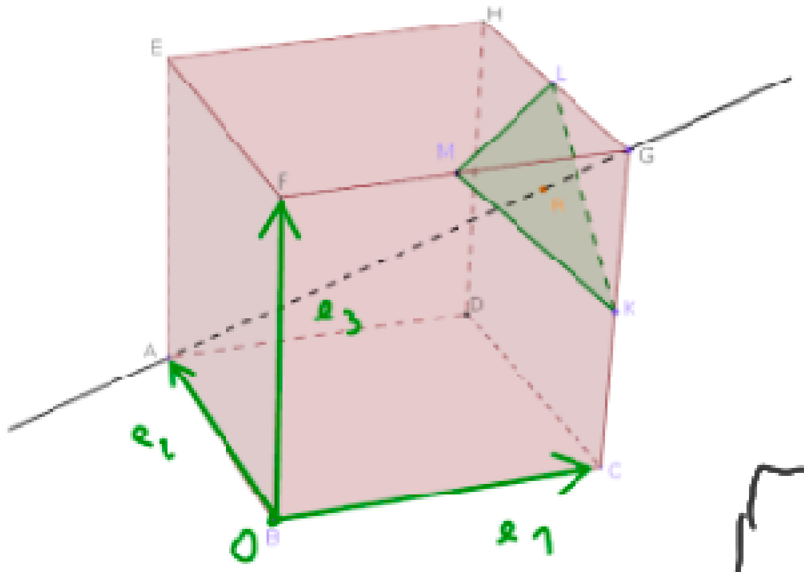
$v \cdot v = \frac{1}{4}(0 + 4 + 4) = \frac{1}{4} 8 = 2$

NE ← $e_1'' \not\perp e_2''$ ...

skal. součin obecně ...

... potřeba víc sčítanců

KARTÉZSKÁ



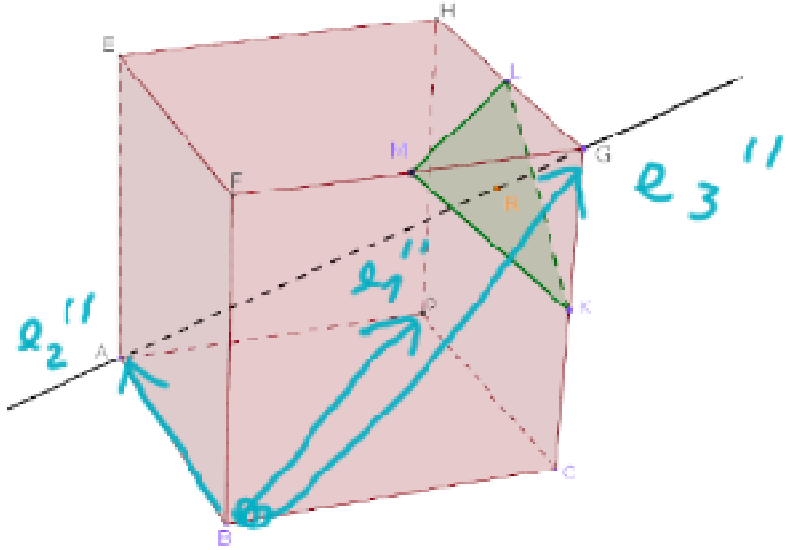
POZN. $\underbrace{(1,2,-1)}_{u} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{skal. souč.}} \cdot \underbrace{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}_{v} = 2 - 1 = \underline{\underline{1}}$

TOTÉŽ

$$\left.\begin{aligned} u &= e_1 + 2e_2 - e_3 \\ v &= e_2 + e_3 \end{aligned}\right\} \quad u \cdot v = 1\, \underline{e_1 \cdot e_2}^{\,0} + 1\, \underline{e_1 \cdot e_3}^{\,0} + 2\, \underline{e_2 \cdot e_2}^{\,1} + 2\, \underline{e_2 \cdot e_3}^{\,0} - \underline{e_3 \cdot e_2}^{\,0} - \underline{e_3 \cdot e_3}^{\,1} = 2 - 1 = \underline{\underline{1}}$$

$$\boxed{\begin{aligned} e_1'' &= e_1 + e_2 \\ e_2'' &= e_2 \\ e_3'' &= e_1 + e_3 \end{aligned}} \rightleftarrows \boxed{\begin{aligned} e_1 &= e_1'' - e_2'' \\ e_2 &= e_2'' \\ e_3 &= -e_1'' + e_2'' + e_3'' \end{aligned}}$$

OBECNÁ



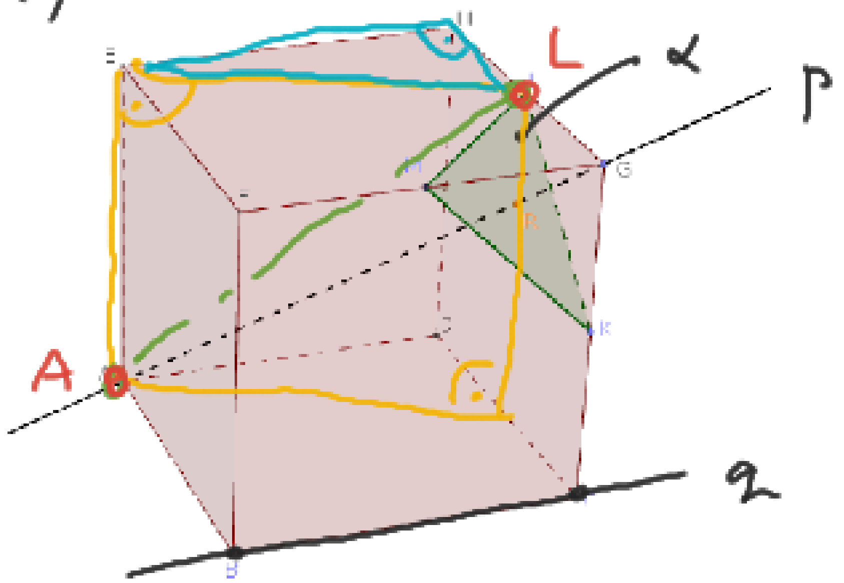
$$\left.\begin{aligned} u &= 2e_1'' - e_3'' \\ v &= -e_1'' + 2e_2'' + e_3'' \end{aligned}\right\} \quad u \cdot v = -2\, \underline{e_1'' \cdot e_1''}^{\,2} + 4\, \underline{e_1'' \cdot e_2''}^{\,1} + 2\, \underline{e_1'' \cdot e_3''}^{\,1} + \underline{e_3'' \cdot e_1''}^{\,1} - 2\, \underline{e_3'' \cdot e_2''}^{\,0} - \underline{e_3'' \cdot e_3''}^{\,2}$$

$$= -4 + 4 + 2 + 1 - 2 = \underline{\underline{1}}$$

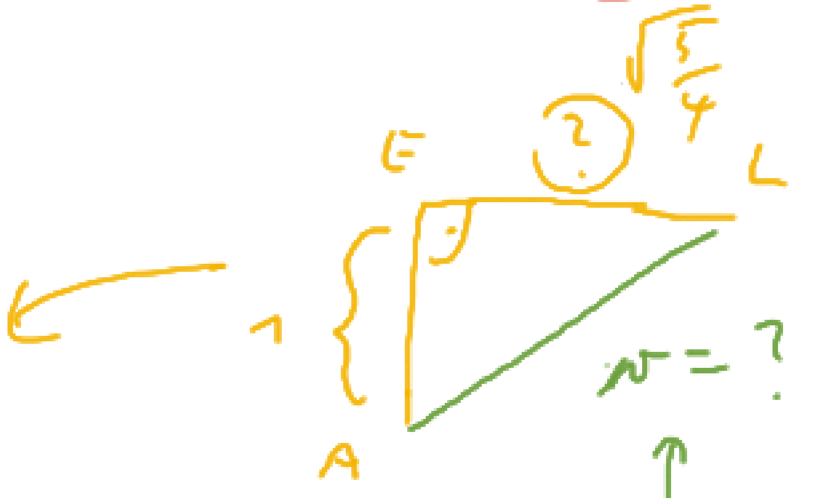
$$\underbrace{(2,0,-1)}_{u} \cdot \underbrace{\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}}_{\text{skal. souč.}} \cdot \underbrace{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}}_{v} = \ldots = \underline{\underline{1}}$$

cv (24) VZDÁLENOST $v(A,L)$, $v(L,\alpha)$, $v(L,p)$, $v(p,q) = ?$

(a)



KRYCHLE
s hranou 1

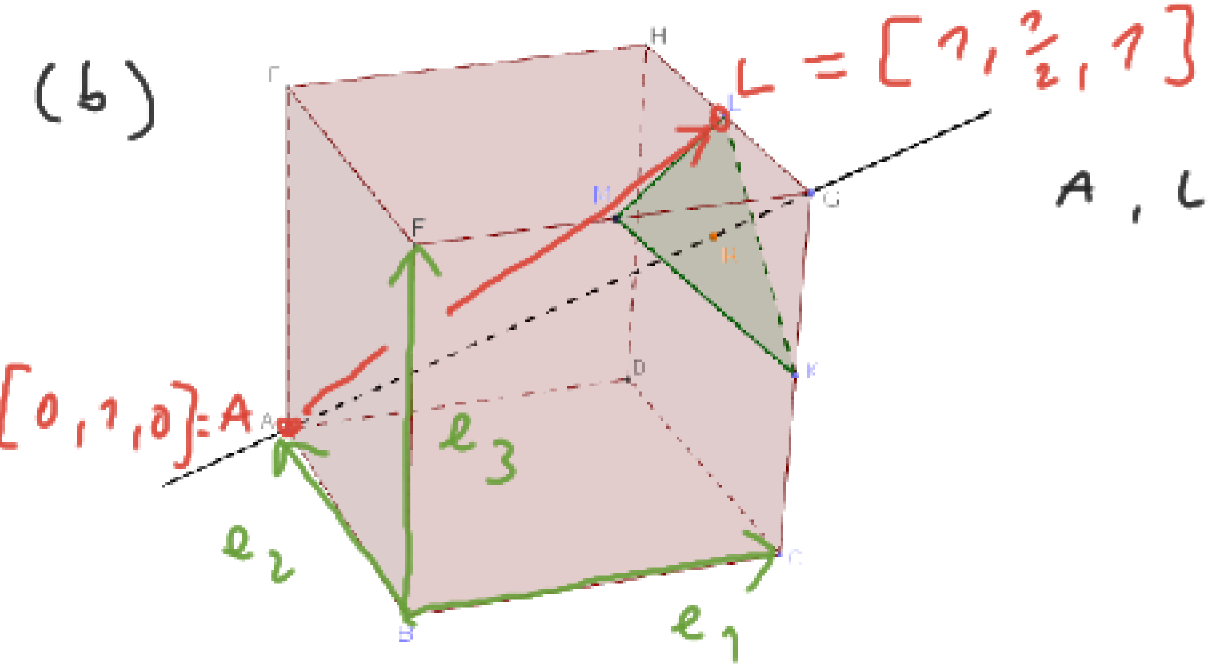


$$\sqrt{1+\frac{5}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$\sqrt{1+\frac{1}{4}} = \sqrt{\frac{5}{4}}$$

2 × Pythagorova věta

(b)



Analyticky (totéž)

$$A, L \rightsquigarrow \vec{AL} = \left(1, -\frac{1}{2}, 1\right) = u$$

$$\rightsquigarrow |AL| = \|u\| = \sqrt{u \circ u}$$

$$= \sqrt{\underbrace{1+\frac{1}{4}}_{\frac{5}{4}}+1} = \sqrt{\frac{9}{4}} = \frac{3}{2} \checkmark$$

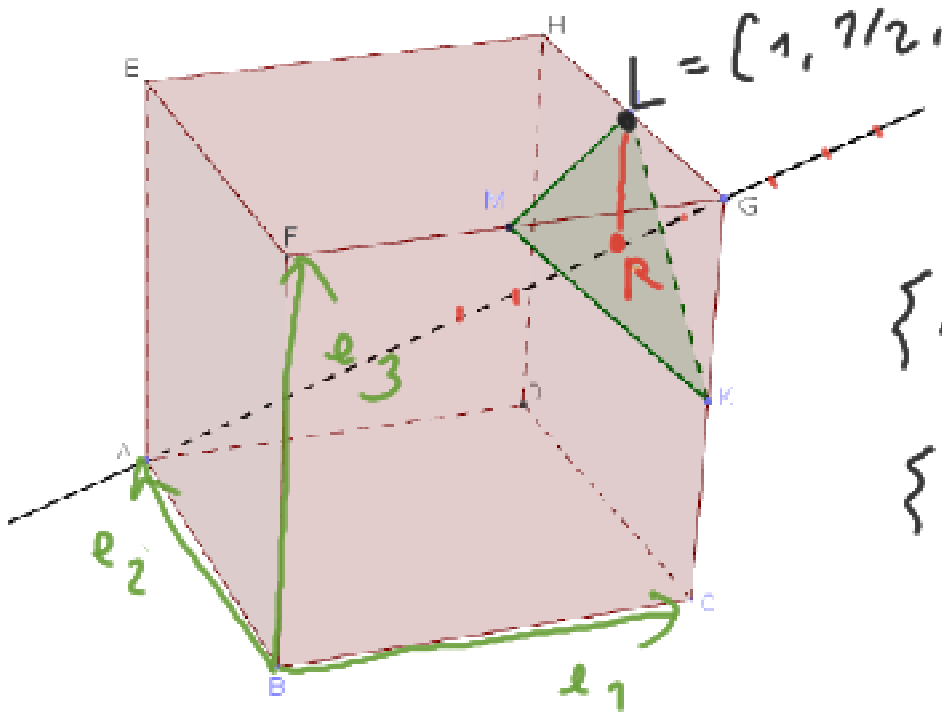
Definice ("minimum")

• $v(L,\alpha) = 0 \iff L \in \alpha$

$\alpha = KLM$

• $v(L,p) \neq 0$?

(a) $v(L,p) = \min\{|LP|,\ P \in p\}$



$p = \{A + t\,\vec{AG} \mid t \in \mathbb{R}\} = \{[t, 1-t, t]\}$

$\vec{LP} = (t-1, 1/2 - t, t-1)$

$|LP| = \|\vec{LP}\| = \sqrt{(t-1)^2 + (1/2 - t)^2 + (t-1)^2}$

$= \sqrt{3t^2 - 5t + 9/4}$

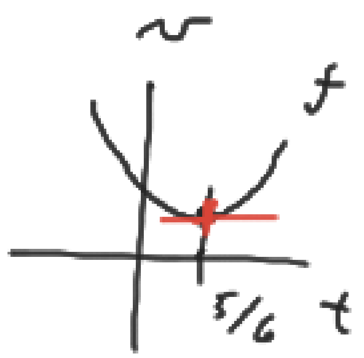
$v = \min\{f(t) = \sqrt{3t^2 - 5t + \frac{9}{4}} \mid t \in \mathbb{R}\}$

derivace $\longrightarrow$ $f'(t) = \frac{1}{2}\frac{6t-5}{\sqrt{\ldots}} = 0$

$\Updownarrow$

$\boxed{6t - 5 = 0}$

$t = 5/6$



$P = R = [5/6, 1/6, 5/6]$ $\longleftarrow$

$\vec{LR} = (-\frac{1}{6}, -\frac{1}{3}, -\frac{1}{6}) = -\frac{1}{6}(1,2,1) \rightsquigarrow v = \frac{1}{6}\sqrt{6}$

(nutné $f''(5/6) > 0$ tj. minimum)

(b) $v(L, p) = |LP| \iff LP \perp p$, tj. $\boxed{\vec{LP} \cdot \vec{AG} = 0}$

Geom. charakterizace ("pata kolmice")

$P = [t, 1-t, t]$, $\vec{AG} = (1, -1, 1)$

$\vec{LP} = (t-1, \frac{1}{2}-t, t-1)$

$$\boxed{(t-1) - (\tfrac{1}{2}-t) + (t-1) = 0}$$

$3t - \frac{5}{2} = 0$

$t = 5/6$

totéž ... $P = R = \dots$

$v = \dots$

(a) $\boxed{6t - 5 = 0}$ ✓

POZN

(a)–(b) ... velmi OBECNÉ postupy!

Další (SPEC.) nápady příště...

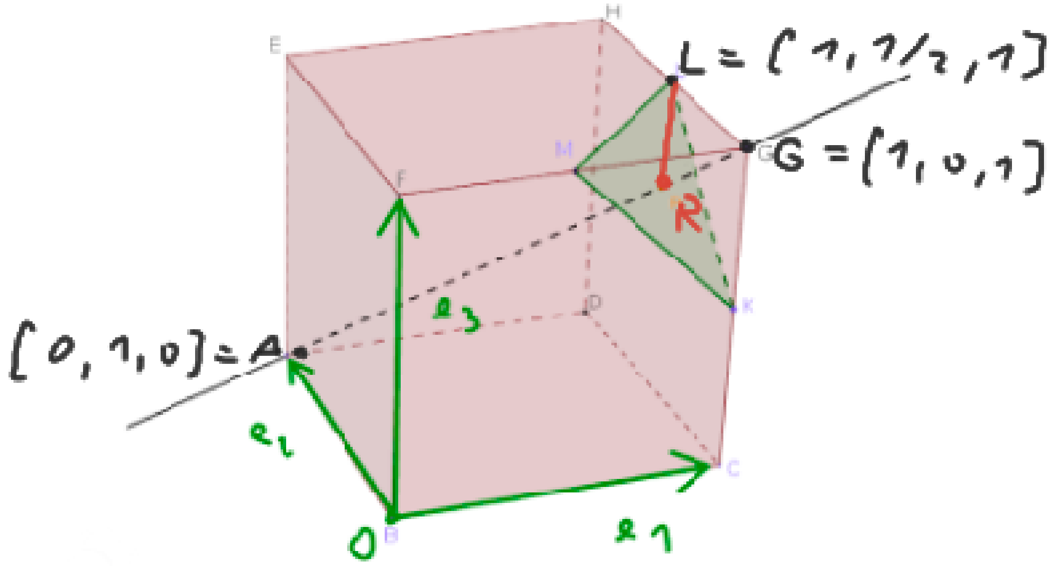
cv (24) | minule $v(L, p)$ . . .

(a) minimum

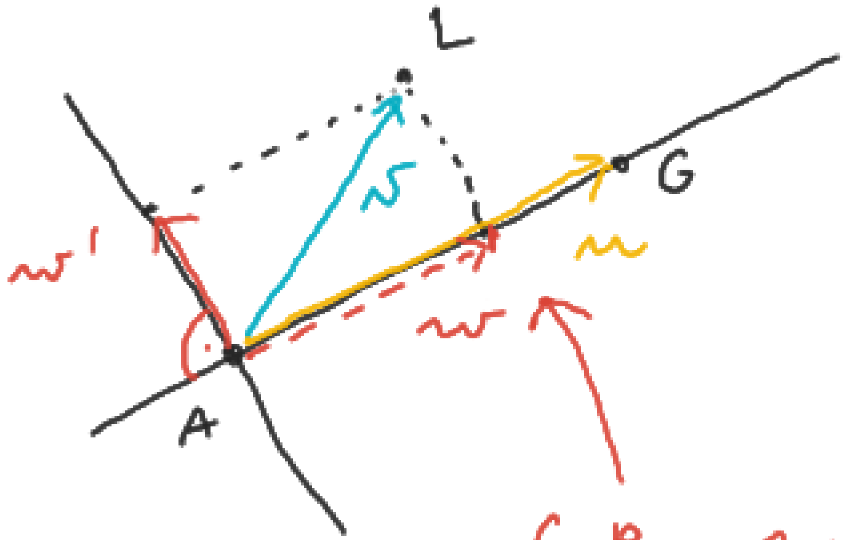
(b) pata kolmice

$$\left.\begin{array}{l}\text{(a) minimum}\\ \text{(b) pata kolmice}\end{array}\right\} \quad R = A + \tfrac{5}{6}\overrightarrow{AG}$$

$$\rightsquigarrow \text{vzdál} = |LR| = \frac{\sqrt{6}}{6}$$



(c) kolmý průmět



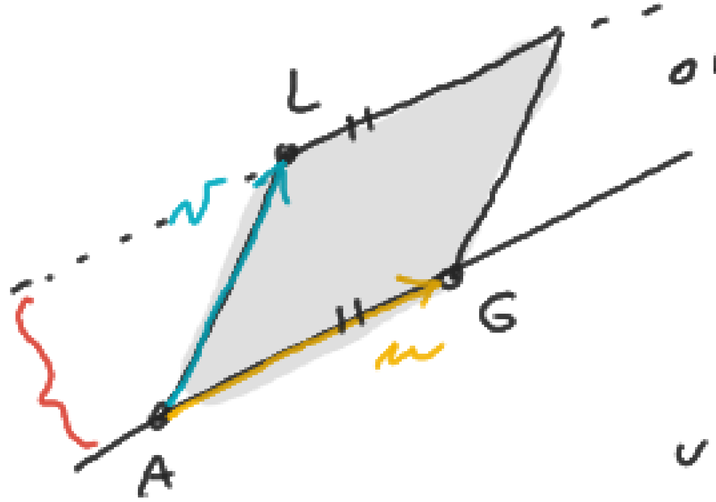
$w = \perp$ průmět v do u

$w' = v - w$

vzdál $= \|w'\|$

$(R = A + w)$

(d) výška rovnoběžníku

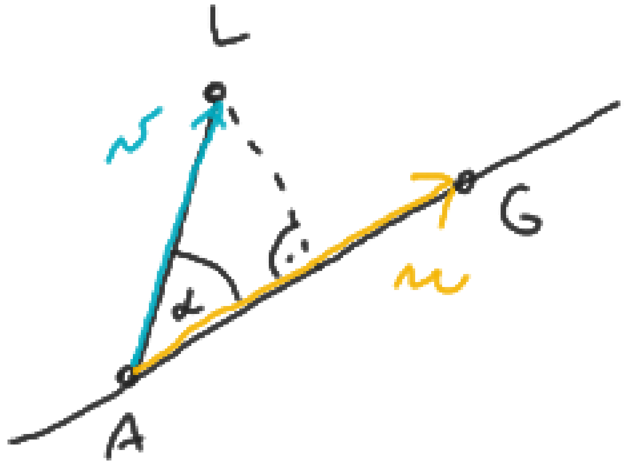


$$\text{obsah} \ ▱ = \underbrace{\text{zákl.} \ \overrightarrow{AG} \times \text{výška}}$$

$$\text{výška} = \frac{\text{obsah} \ ▱}{\|u\|}$$

vzdál = výška

pozn



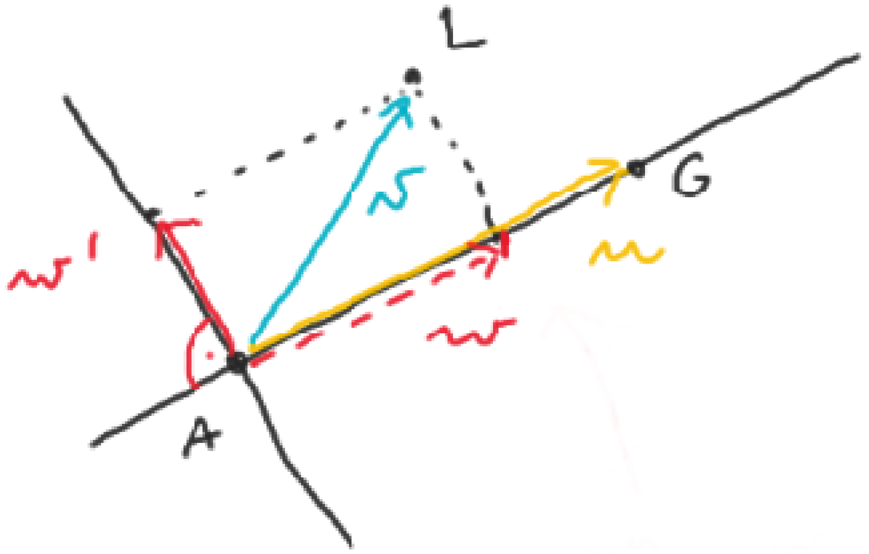
$$\cos\alpha = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha} \quad \leftarrow \sin^2\alpha + \cos^2\alpha = 1$$

$$\text{vzdál} = \|v\| \cdot \sin\alpha$$

(c) KOLMÝ PRŮMĚT OBECNĚ

w = ⊥ průmět v do $\langle u\rangle$, resp. $\langle u\rangle^\perp$



$$\boxed{\begin{aligned} w &= a\cdot u \qquad a = ? \\ w' &= v - w \perp u \end{aligned}}$$

$$(v - a\cdot u)\cdot u = 0$$

$$v\cdot u - a(u\cdot u) = 0$$

$$a = \frac{v\cdot u}{u\cdot u}$$

$$\boxed{\begin{aligned} w &= \left(\frac{v\cdot u}{u\cdot u}\right) u \\ w' &= v - w \end{aligned}}$$

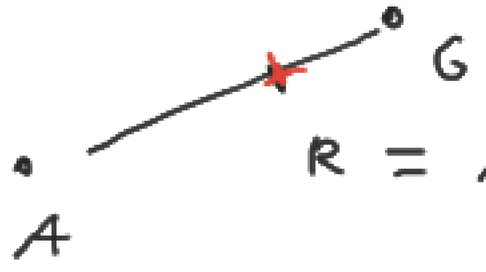
$$w\cdot w = \left(\frac{v\cdot u}{u\cdot u}\right)^2 (u\cdot u) = \frac{(v\cdot u)^2}{u\cdot u}$$

$$w'\cdot w' = (v-w)\cdot(v-w) = v\cdot v - 2\, v\cdot w + w\cdot w = \dots = v\cdot v - \frac{(v\cdot u)^2}{u\cdot u}$$

(c) DOSAZENÍ

$$A = [0, 1, 0]$$
$$G = [1, 0, 1]$$
$$C = [1, 1/2, 1]$$

$$u = \overrightarrow{AG} = (1, -1, 1)$$
$$v = \overrightarrow{AC} = (1, -1/2, 1)$$



$$R = A + \frac{5}{6}\overrightarrow{AG}$$

totéž co (a)-(b) ✓

$$w = \left(\frac{v \cdot u}{u \cdot u}\right) u \overset{!}{=} \left(\frac{5/2}{3}\right) \cdot u = \frac{5}{6} u$$

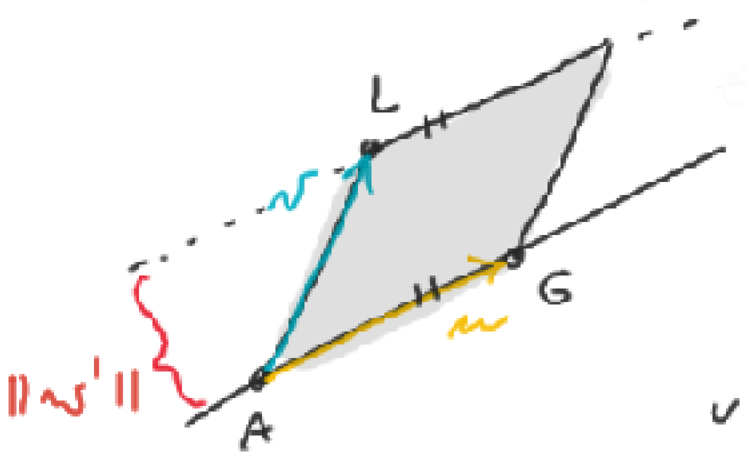
$$w' = v - \frac{5}{6} u = \ldots\ldots$$

RESP.

$$w' \cdot w' = v \cdot v - \frac{(v \cdot u)^2}{u \cdot u} = \frac{9}{4} - \frac{(5/2)^2}{3} = \frac{9}{4} - \frac{25}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\text{vzdál.} = \|w'\| = \sqrt{\frac{1}{6}}$$ ✓

(d) výška rovnoběžníku



$$\text{výška} = \frac{\text{obsah}}{\|u\|}$$

vzdál = výška

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}$$
$$u = (1, -1, 1)$$
$$v = (1, -1/2, 1)$$

det $\rightsquigarrow$ $x_1 \underbrace{\begin{vmatrix} -1 & 1 \\ -1/2 & 1 \end{vmatrix}}_{-1/2} - x_2 \underbrace{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}_{0} + x_3 \underbrace{\begin{vmatrix} 1 & -1 \\ 1 & -1/2 \end{vmatrix}}_{+1/2}$

TRIK

① "VEKTOROVÝ SOUČIN"

$$u \times v = \left(-\tfrac{1}{2}, 0, \tfrac{1}{2}\right) \rightsquigarrow \|u \times v\| \overset{!!}{=} \text{obsah}$$

$$\|u \times v\| = \frac{\sqrt{2}}{2} \rightsquigarrow \text{vzdál} = \text{výška} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{3}} = \frac{1}{\sqrt{2 \cdot 3}} = \frac{1}{\sqrt{6}}$$

POSTŘEH

② Z PŘEDCHOZÍHO:

$$\text{vzdál} = \|w'\| = \sqrt{w' \cdot w'} = \sqrt{v \cdot v - \frac{(v \cdot u)^2}{u \cdot u}} =$$

$$= \sqrt{\frac{(v \cdot v)(u \cdot u) - (v \cdot u)^2}{u \cdot u}} = \frac{\sqrt{(v \cdot v)(u \cdot u) - (v \cdot u)^2}}{\|u\|} \overset{!!}{=} \frac{\text{obsah}}{\|u\|}$$

NAVÍC:

$$\sqrt{(u\cdot u)(v\cdot v)-(u\cdot v)^2} = \text{OBSAH}$$

rovno-běžník

$$\det\begin{pmatrix} u\cdot u & u\cdot v \\ v\cdot u & v\cdot v \end{pmatrix}$$

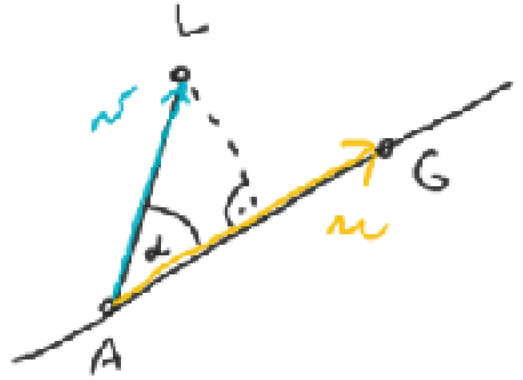
! ... tzv. GRAMOVA MATICE (DETERMINANT)

OBECNĚJI:

$$\boxed{\sqrt{\det\begin{pmatrix} u\cdot u & u\cdot v & u\cdot w \\ v\cdot u & v\cdot v & v\cdot w \\ w\cdot u & w\cdot v & w\cdot w \end{pmatrix}} = \text{OBJEM}}$$

rovno-běžno-stěn

POZN



$$\cos\alpha = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$

$$\text{vzdál} = \|v\| \cdot \sin\alpha$$

$$u = \overrightarrow{AG} = (1, -1, 1)$$

$$v = \overrightarrow{AL} = (1, -1/2, 1)$$

$$\cos\alpha = \frac{5/2}{\sqrt{3}\sqrt{9/4}} = \frac{5}{\sqrt{27}}$$

$$\sin\alpha = \sqrt{1 - \frac{25}{27}} = \sqrt{\frac{2}{27}}$$

$$\text{vzdál} = \sqrt{\frac{9}{4} \cdot \frac{2}{27}} = \underline{\underline{\frac{1}{\sqrt{6}}}} \ \checkmark$$

OBECNĚ:

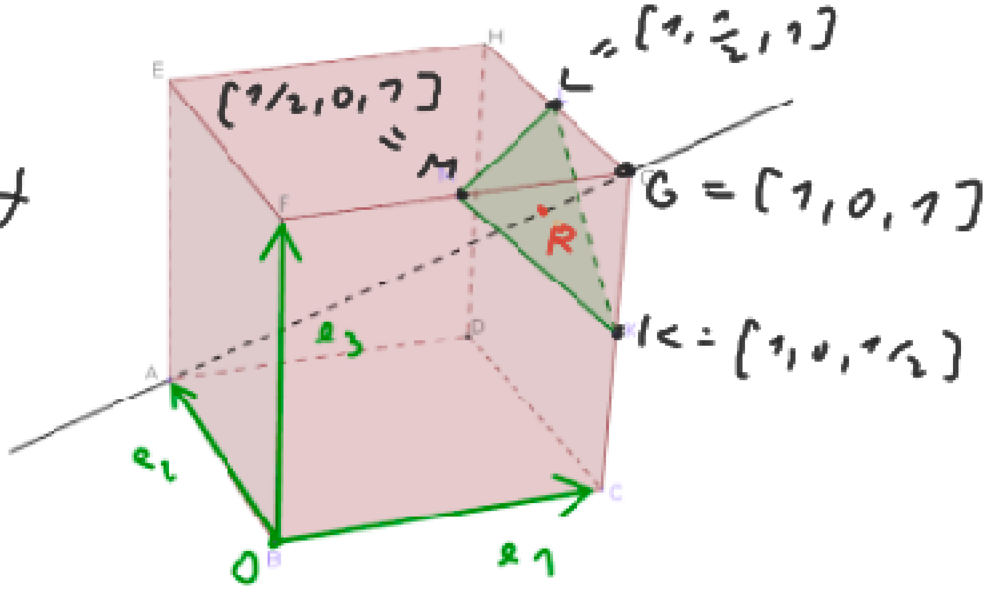
$$\sin\alpha = \sqrt{1 - \frac{(u \cdot v)^2}{\|u\|^2 \|v\|^2}} = \frac{\sqrt{\|u\|^2 \|v\|^2 - (u \cdot v)^2}}{\|u\| \cdot \|v\|}$$

$$\text{vzdál} = \frac{\sqrt{\|u\|^2 \|v\|^2 - (u \cdot v)^2}}{\|u\|}$$

... což nám něco připomíná!

cv (24) $v(G,\alpha) = ?$

$\alpha = \{ K + t\,\vec{KL} + s\,\vec{KM} \mid t, s \in \mathbb{R} \}$
$= \{ x_1 - x_2 + x_3 - \frac{3}{2} = 0 \}$



(a) minimum

$P \in \alpha \rightsquigarrow \vec{GP} = \vec{GK} + t\,\vec{KL} + s\,\vec{KM} = \left(-\frac{s}{2}, \frac{t}{2}, -\frac{1}{2} + \frac{t}{2} + \frac{s}{2}\right)$

$\rightsquigarrow f(t,s) = \vec{GP} \cdot \vec{GP} = \dots$ kvadr. polynom v t a s

$\rightsquigarrow |GP| = \min \iff f(t,s) = \vec{GP} \cdot \vec{GP} = \min$

$\iff \boxed{\frac{\partial f}{\partial t} = \frac{\partial f}{\partial s} = 0}$ parciální derivace podle t, s

2 lin. rovnice / 2 neznámé

$\Updownarrow$

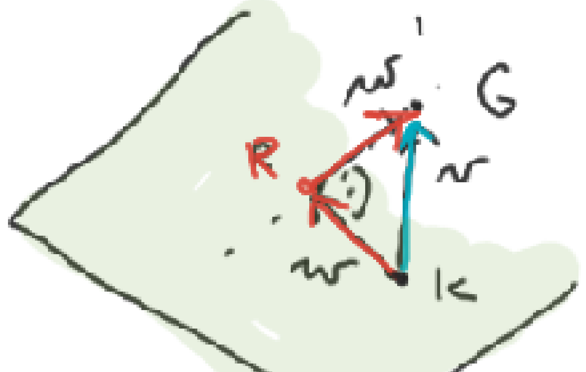
(b) pata kolmice

$P \in \alpha \rightsquigarrow \vec{GP} \perp \alpha \iff \boxed{\vec{GP} \cdot \vec{KL} = \vec{GP} \cdot \vec{KM} = 0}$ (*)

$\dots \rightsquigarrow \begin{bmatrix} \frac{t}{2} + s = \frac{1}{2} \\ t + \frac{s}{2} = \frac{1}{2} \end{bmatrix} \rightsquigarrow t = s = 1/3 \rightsquigarrow P = R = [5/6, 1/6, 5/6]$

$\rightsquigarrow$ vzdál $= |GR| = \dots = \frac{\sqrt{3}}{6}$ ✓

(c) kolmý průmět

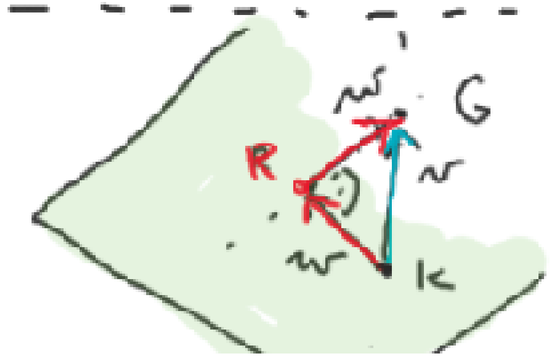


[$w = \perp$ průmět v do $\vec{\alpha} = \langle \vec{KL}, \vec{KM} \rangle$] $\iff$ (*)

[$w' = v - w = \perp$ průmět v do $\vec{\alpha}^{\perp}$] ... otoč

vzdál $= \|w'\|$

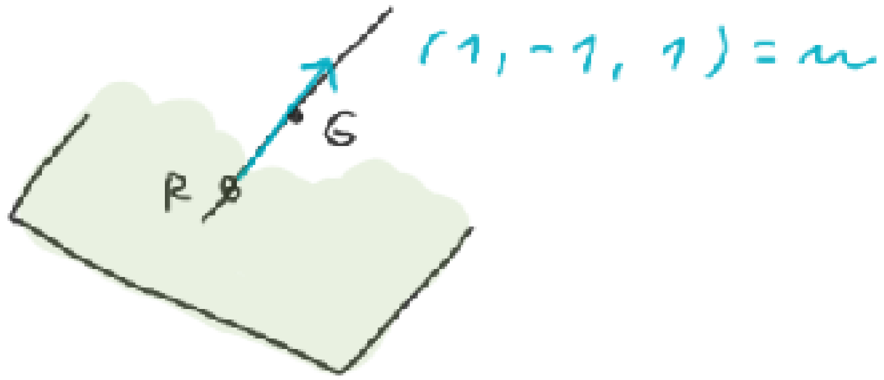
(c) ZKRATKA



[w = ⊥ průmět v do $\vec{\alpha} = \langle \vec{KL}, \vec{KM} \rangle$ ← dim 2]

$w' = v - w$ = ⊥ průmět v do $\vec{\alpha}^{\perp} = \langle n \rangle$ ← dim 1 !

vzdál $= \|w'\|$



TOTÉŽ!

$$w' = \left(\frac{v \cdot n}{n \cdot n}\right) \cdot n$$

$$\|w'\| = \sqrt{\frac{(v \cdot n)^2}{n \cdot n}} = \frac{|v \cdot n|}{\|n\|}$$

$$= \frac{1/2}{\sqrt{3}} = \frac{\sqrt{3}}{3 \cdot 2} = \frac{\sqrt{3}}{6} \checkmark$$

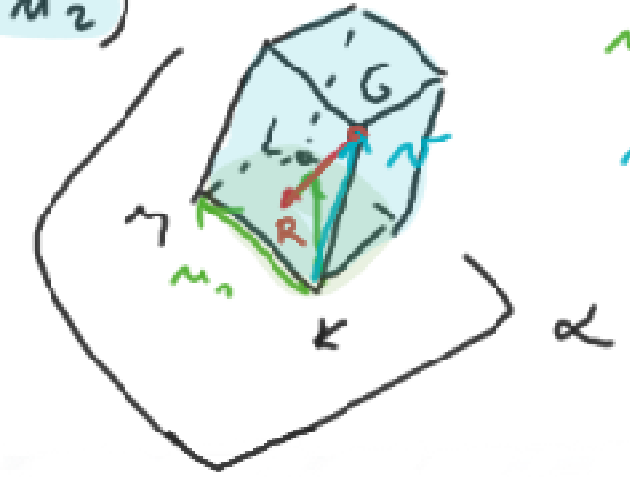
POZN.

$$\alpha = \{ x_1 - x_2 + x_3 - \tfrac{3}{2} = 0 \}$$

$$v(G, \alpha) = \frac{|1 - 0 + 1 - \frac{3}{2}|}{\sqrt{3}} = \frac{1/2}{\sqrt{3}}$$

(d) OBJEM / OBSAH

$$\text{objem}(v, u_1, u_2) = \text{obsah}(u_1, u_2) \times \text{výška}$$



$u_1 = \overrightarrow{KM} = (-1/2,\ 0,\ 1/2)$

$u_2 = \overrightarrow{KC} = (0,\ 1/2,\ 1/2)$

$v = \overrightarrow{KG} = (0,\ 0,\ 1/2)$

$$\frac{1}{4} \cdot \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

OBECNĚ ....

$$\begin{pmatrix} u_1 \cdot u_1 & u_1 \cdot u_2 & u_1 \cdot v \\ u_2 \cdot u_1 & u_2 \cdot u_2 & u_2 \cdot v \\ v \cdot u_1 & v \cdot u_2 & v \cdot v \end{pmatrix}$$

$\xrightarrow{\det}$ objem(u_1, u_2, v)

$\dots = \sqrt{\frac{1}{64}} = \underline{\frac{1}{8}}$

$\xrightarrow{\det}$ obsah(u_1, u_2)

$\dots = \sqrt{\frac{3}{16}} = \underline{\frac{\sqrt{3}}{4}}$

$$\text{vzdál} = \frac{1}{2\sqrt{3}} = \frac{1}{\sqrt{6}} \ ✓$$

SPEC ...

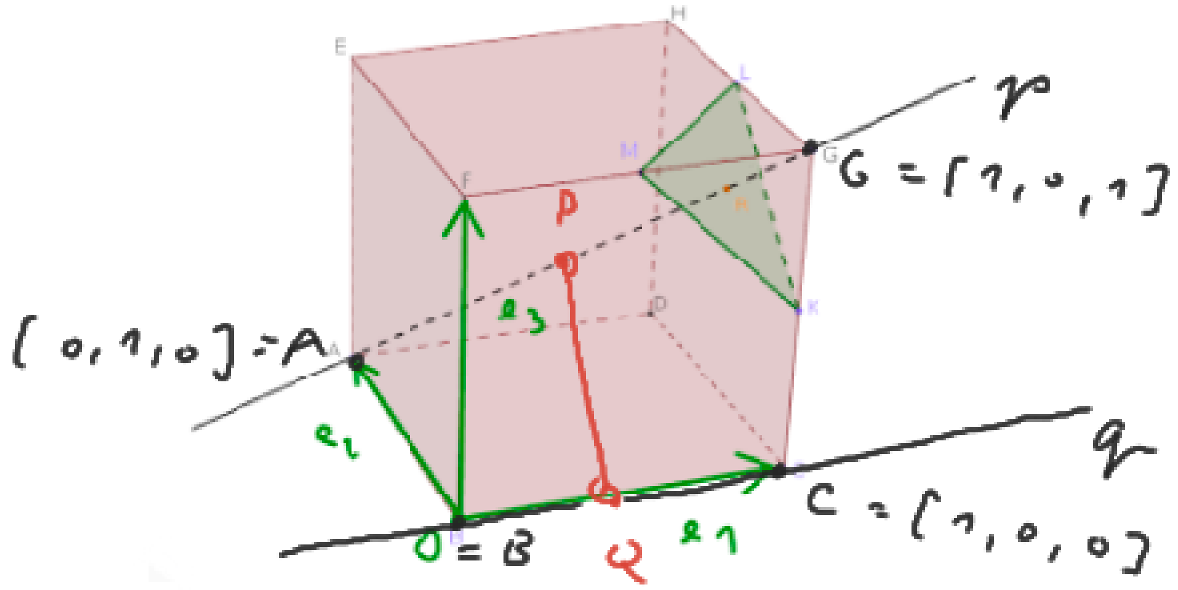
"VNĚJŠÍ SOUČIN"

$$\begin{pmatrix} -1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 \end{pmatrix} \xrightarrow{\det} \boxed{\text{OBJEM}(u_1, u_2, v)}$$

$$\det = -\frac{1}{8} \rightsquigarrow \text{OBJEM} = \underline{\frac{1}{8}} \ ✓$$

cv (14)

$v(p,q) = ?$



(a)-(b)

$P = A + t\vec{AG} \in p \ , \ Q = B + s\vec{BC} \in q$

$\vec{PQ} \perp p$ a $\vec{PQ} \perp q \iff \boxed{\vec{PQ}\cdot\vec{AG} = \vec{PQ}\cdot\vec{BC} = 0}$

$\ldots \rightsquigarrow \begin{bmatrix} -2t + 2s = 0 \\ 6t - 2s = 2 \end{bmatrix} \rightsquigarrow \underline{t = s = \frac{1}{2}} \rightsquigarrow P = [\frac{1}{2}, \frac{1}{2}, \frac{1}{2}]$

$Q = [\frac{1}{2}, 0, 0]$

$\rightsquigarrow$ vzdál $= |PQ| = \underline{\underline{\frac{\sqrt{2}}{2}}}$ ✓

(c) $v = \vec{AB} = (0,1,0)$

$\underbrace{n \in \langle \vec{AG}, \vec{BC} \rangle^{\perp}} \ldots \boxed{n = (0,1,1)}$

$v' = \perp$ průmět v do n ... $v' = \left(\frac{v\cdot n}{n \cdot n}\right) n = \frac{1}{2} n$

$\rightsquigarrow$ vzdál $= \frac{1}{2}\|n\| = \underline{\underline{\frac{\sqrt{2}}{2}}}$ ✓

(d) $n_1 = \vec{AG} = (1,-1,1)$

$n_2 = \vec{BC} = (1,0,0)$

objem $(n_1, n_2, v) = \ldots = 1$

obsah $(n_1, n_2) = \ldots = \sqrt{2}$

$\Bigg\} \rightsquigarrow$ vzdál $= \frac{1}{\sqrt{2}} = \underline{\underline{\frac{\sqrt{2}}{2}}}$ ✓

cv (27) $\mathcal{B} = \left\{ \begin{array}{r} x_2 - x_4 = 2 \\ x_3 = 1 \end{array} \right\}$, $\mathcal{C} = \left\{ \underbrace{\begin{bmatrix}1\\0\\3\\3\end{bmatrix}}_{B} + t \underbrace{\begin{pmatrix}1\\1\\0\\0\end{pmatrix}}_{u} \,\middle|\, t \in \mathbb{R} \right\}$ vzdál. & vzáj. poloha?

$$\mathcal{B} = \left\{ \underbrace{\begin{bmatrix}0\\2\\1\\0\end{bmatrix}}_{C} + r \underbrace{\begin{pmatrix}1\\0\\0\\0\end{pmatrix}}_{v_1} + s \underbrace{\begin{pmatrix}0\\1\\0\\1\end{pmatrix}}_{v_2} \,\middle|\, r, s \in \mathbb{R} \right\}$$

(a)-(b) $P = C + r v_1 + s v_2 \in \mathcal{B}$, $Q = B + t u \in \mathcal{C}$

$\vec{PQ} \perp \mathcal{B}$ a $\vec{PQ} \perp \mathcal{C} \iff \vec{PQ} \cdot v_1 = \vec{PQ} \cdot v_2 = \vec{PQ} \cdot u = 0$

$\underbrace{\ldots}$ $\rightsquigarrow \begin{bmatrix} -2t + 4s = 2 \\ 4t - 2r - 2s = 2 \\ -2t + 2r = 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} r = 6 \\ s = 3 \\ t = 5 \end{bmatrix} \rightsquigarrow$ $P = [6, 5, 1, 3]$, $Q = [6, 5, 3, 3]$ $\rightsquigarrow$ vzdál $= |PQ| = \underline{\underline{2}}$

- vzdál $\neq 0 \implies \mathcal{B} \cap \mathcal{C} = \emptyset$
- dim{řešení} $= 0 \implies \vec{\mathcal{B}} \cap \vec{\mathcal{C}} = \{o\}$

$\Big\}$ $\mathcal{B} \times \mathcal{C}$ … <u>MIMOBĚŽNÉ</u>

(c) $v = \vec{BC} = (-1, 2, -2, -3)$

$\underbrace{w \in (\vec{\mathcal{B}} + \vec{\mathcal{C}})^{\perp} = \langle u, v_1, v_2 \rangle^{\perp}}$ … $\boxed{w = (0, 0, 1, 0)}$

$w' = \perp$ průmět v do w … $w' = \left(\frac{v \cdot w}{w \cdot w} \right) w = -2 w \rightsquigarrow$ vzdál $= 2 \|w\| = \underline{\underline{2}}$ ✓

(d) objem$(v_1, v_2, u, v) = \ldots = 2$

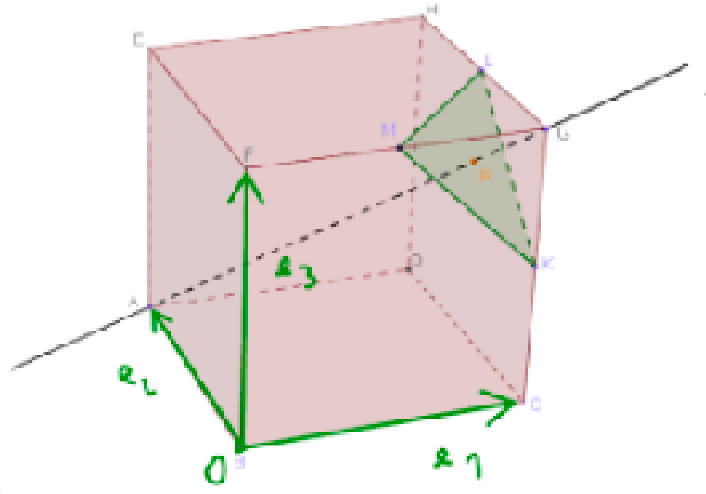
objem$(v_1, v_2, u) = \ldots = 1$

$\rightsquigarrow$ vzdál $= \frac{2}{1} = \underline{\underline{2}}$ ✓

<u>POZN</u> $w = (0, 0, 1, 0)$ je "náhodou" stejný jako ve cv (19)

$\rightsquigarrow$ jiné řešení …

(28) V úloze (c) určete objemy všech rovnoběžnostěnů, jejichž 4 z 8 vrcholů jsou K, L, M, G,



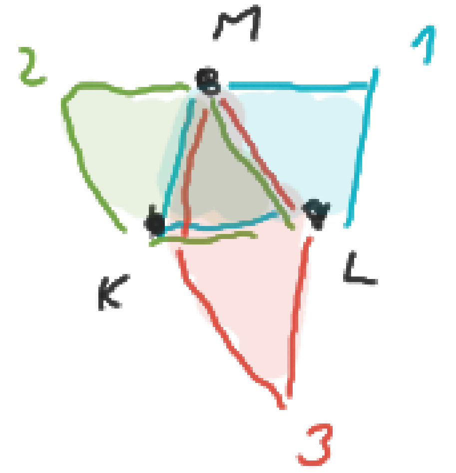
← KRYCHLE S HRANOU 1

$G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

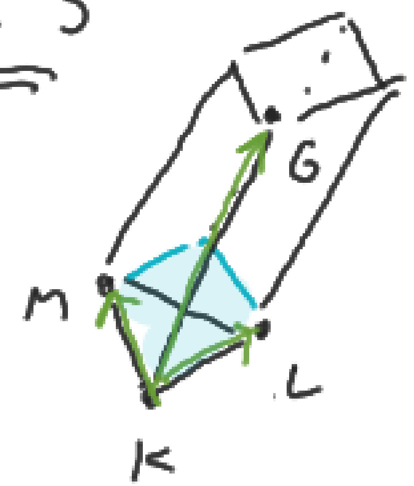
$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \quad L = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

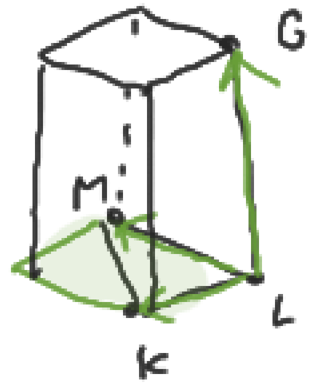
mnoho různých doplnění, OBJEM STÁLE STEJNÝ!!

dim 2



dim 3





atd...

např.

$\vec{KL} = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix} \quad \vec{KM} = \begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix} \quad \vec{KG} = \begin{pmatrix} 0 \\ 0 \\ 1/2 \end{pmatrix} \rightsquigarrow \vec{KL} = \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \vec{KM} = \frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \vec{KG} = \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

(a) Gramův det:

$$\det\begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/4 \end{pmatrix} \overset{!}{=} \det\left(\frac{1}{2^2}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}\right) \overset{!}{=} \frac{1}{4^3}\underbrace{\det\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{1}$$

$$\rightsquigarrow \text{objem} = \sqrt{\frac{1}{4^3}} = \frac{1}{8}$$

(b) VNĚJŠÍ SOUČIN

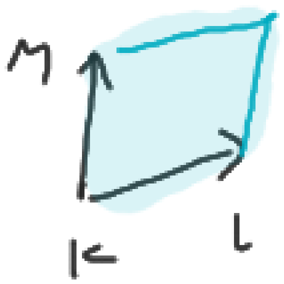
$$\det\begin{pmatrix} 0 & -1/2 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1/2 & 1/2 \end{pmatrix} = \frac{1}{2^3}\det\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{8} = \text{objem} \checkmark$$

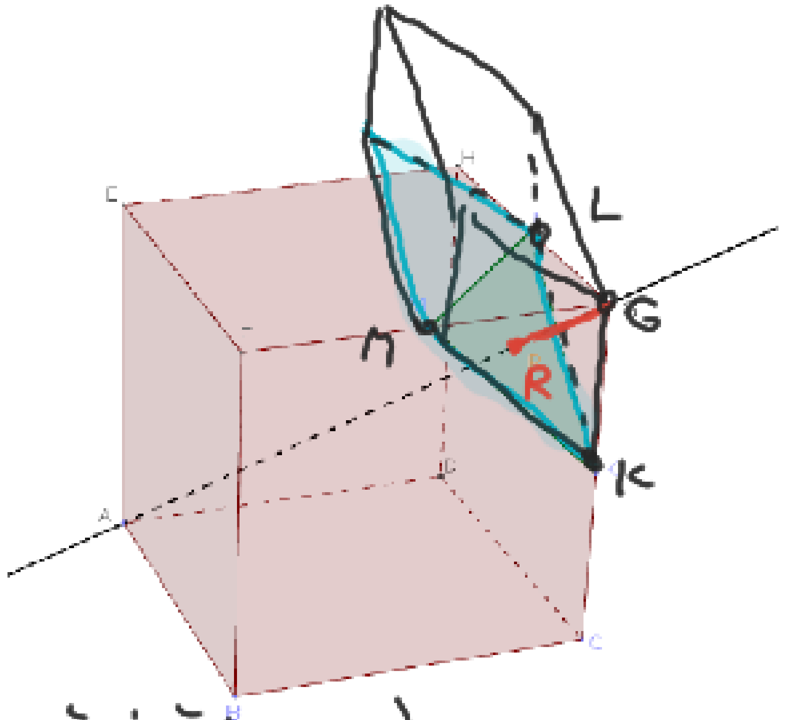
$$\begin{array}{ccc} \overrightarrow{KC} & \overrightarrow{KM} & \overrightarrow{KG} \end{array}$$

(c) "PODSTAVA × VÝŠKA"

kolmý průmět ...

$$\ldots = |RG| = \frac{\sqrt{3}}{6}$$





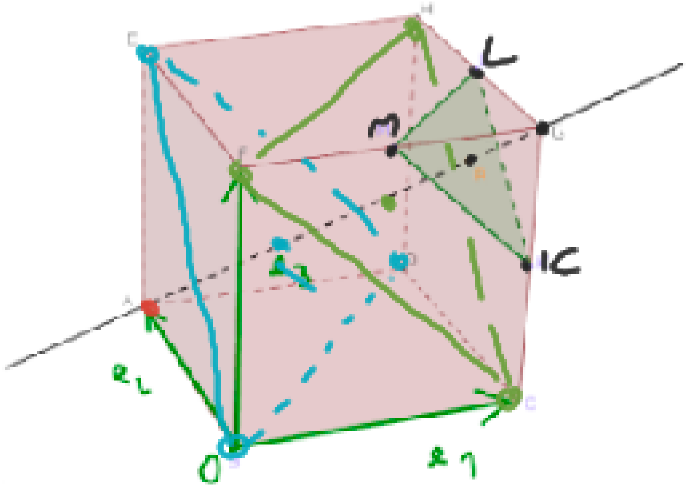
(máme z dřívějška)

$$\text{obsah}(\overrightarrow{KC}, \overrightarrow{KM}) = \ldots = \frac{\sqrt{3}}{4}$$

$$\text{objem}(\overrightarrow{KC}, \overrightarrow{KM}, \overrightarrow{KG}) = \frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{6} = \frac{1}{8} \checkmark$$

(28) V úloze (o) určete objemy všech

- rovnoběžnostěnů, jejichž 4 z 8 vrcholů jsou K, L, M, G,
- čtyřstěnů, jejichž 3 ze 4 vrcholů jsou K, L, M a zbylý vrchol je vrcholem krychle.

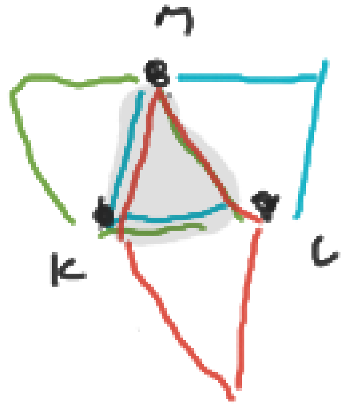


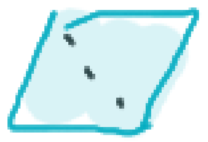
← KRYCHLE S HRANOU 1

$G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

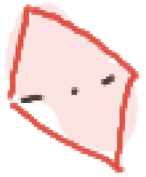
$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \quad L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

dim 2



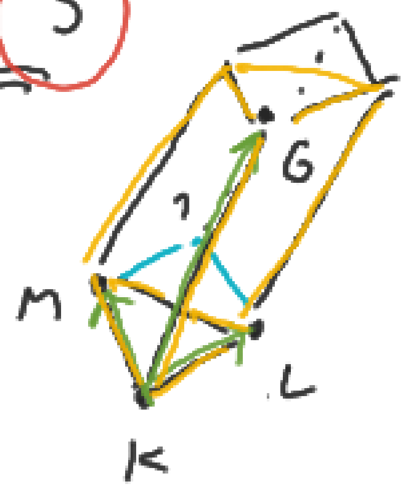
$= \frac{1}{2}$ 

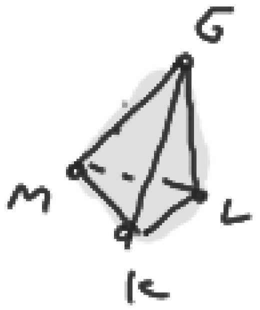
$= \frac{1}{2}$ 

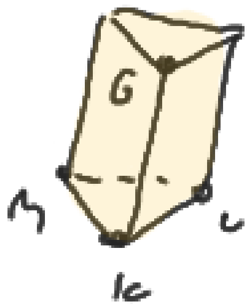
$= \frac{1}{2}$ 

$\frac{1}{2!}$

dim 3





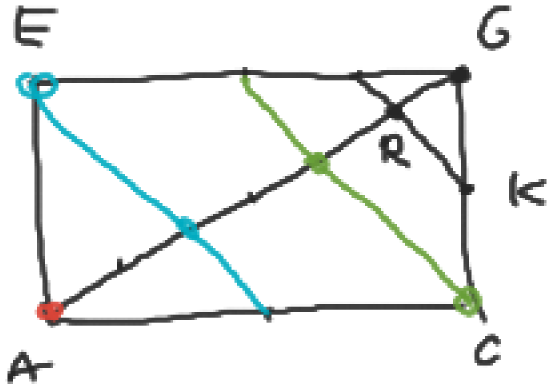
$= \frac{1}{3}$ 

$= \underbrace{\frac{1}{3} \cdot \frac{1}{2}} \cdot$ 

$\frac{1}{6} = \frac{1}{3!}$

- objem KLMG $= \frac{1}{6} \cdot \frac{1}{8} = \frac{1}{48}$
- objem KLMC = objem KLMH = = objem KLMF = ... $= 1/48$
- objem KLME = ... $= 1/16$
- objem KLMA = ... $= 5/48$

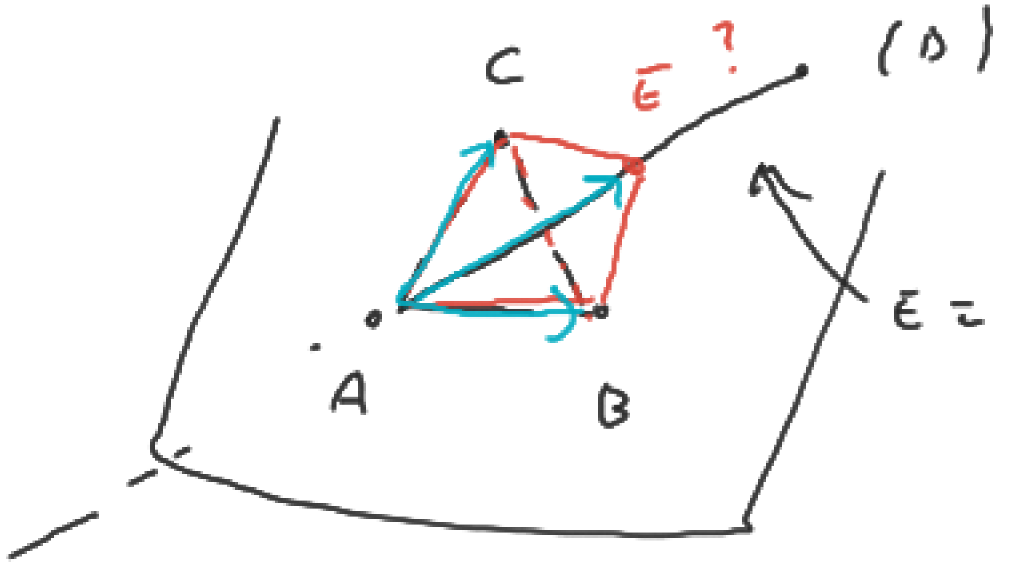
viz též →



(29) V 3-dim prostoru, pro

$A=[0,0,1]$, $B=[2,1,1]$, $C=[1,2,1]$, $D=[1,1,2]$,

- určete bod E na přímce AD tak, aby simplex $ABCE$ měl objem 1.



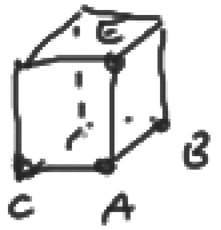
$E = A + t\,\vec{AD}$

$\vec{AB} = (2,1,0)$ $\quad \vec{AC} = (1,2,0)$ $\quad \vec{AE} = (t,t,t) = t \cdot \vec{AD}$

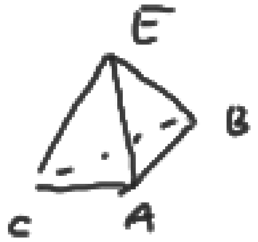
neznámá t ?

(a) vnější součin

$$\det \begin{pmatrix} 2 & 1 & t \\ 1 & 2 & t \\ 0 & 0 & t \end{pmatrix} = 3t = \pm \text{ objem rovnob.}$$



objem simplexu



$$= \pm \frac{1}{6} \cdot 3t = \pm \frac{1}{2} t \overset{!}{=} 1$$

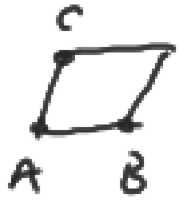
(b) Gram. det

[pracnější]

tj. $t = \pm 2$ ⇝ buď $E = [-2,-2,3]$,

nebo $E = [-2,-2,-1]$.

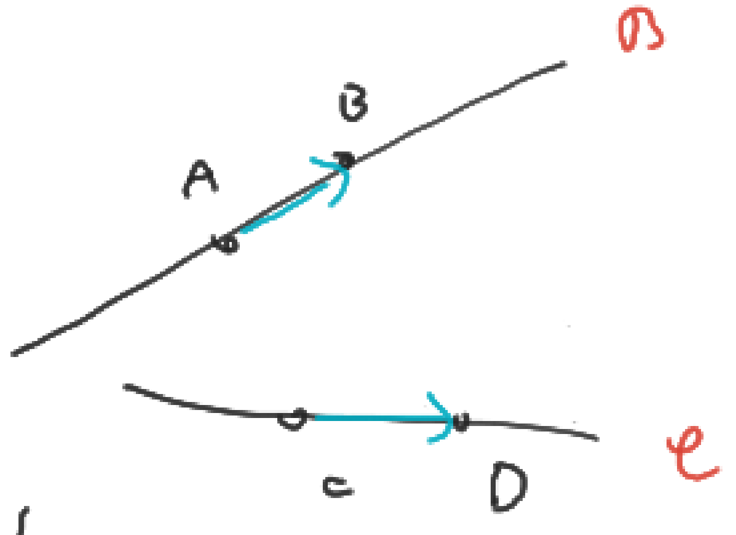
(c) "podstava × výška"

[obsah 

$= \dots = 3$, výška $= \dots = t$]

(31) $A = [2, 1, 0, -3]$, $B = [5, 4, 0, -3]$ ⇝ $\mathcal{B} = A + B$

$C = [2, 0, 1, -1]$, $D = [9, 0, 2, -1]$ ⇝ $\mathcal{C} = C + D$



- vzáj. polohu $\mathcal{B}$ a $\mathcal{C}$
- rovnic. vyjádření $\mathcal{B} + \mathcal{C}$

$\overrightarrow{AB} = (3, 3, 0, 0)$

$\overrightarrow{CD} = (7, 0, 1, 0)$ } nezávislé ⇒ NEJSOU = ANI ∥

$\overrightarrow{AC} = (0, -1, 1, 2)$ } lin. závislé ⇒ ✗ … RŮZNOB.

nezávislé ⇒ $\not\times$ … MIMOB. ?

(a) úpravy …. ✓

(b) "VZOREČKY"

- gramův det = 0 ⇔ ZÁVISLÉ
- vektorový součin = 0 ⇔ ZÁVISLÉ

!

vektorový součin

$$\boxed{[\vec{AB}, \vec{CD}, \vec{AC}, \text{l.b}] = \underline{\underline{w}} \cdot \text{l.b}} \leftarrow \text{def.}$$

$$\det \begin{vmatrix} \vec{AB} = (3,3,0,0) \\ \vec{CD} = (7,0,1,0) \\ \vec{AC} = (0,-1,1,2) \\ \text{l.b} = (x_1,x_2,x_3,x_4) \end{vmatrix} = -x_1 \underbrace{\begin{vmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 2 \end{vmatrix}}_{6} + x_2 \underbrace{\begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix}}_{6} - x_3 \underbrace{\begin{vmatrix} 3 & 3 & 0 \\ 7 & 0 & 0 \\ 0 & -1 & 2 \end{vmatrix}}_{-42} + x_4 \underbrace{\begin{vmatrix} 3 & 3 & 0 \\ 7 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix}}_{-21+3=-18}$$

Laplaceův rozvoj

$$\underline{\underline{w}} = \vec{AB} \times \vec{CD} \times \vec{AC} = (-6, 6, +42, -18) = 6 \cdot (-1, 1, +7, -3)$$

- $w \neq (0,0,0,0) \Rightarrow$ NEZÁVISLÉ ✓
- $w \perp \vec{AB}, \vec{CD}, \vec{AC}$
- $\|w\| =$ objem 

← navíc připomínáme

$\vec{AB}, \vec{CD}, \vec{AC}$ nezávislé $\Rightarrow$ $\mathcal{B}$ a $\mathcal{C}$ MIMOBĚŽNÉ

$\Rightarrow$ $\dim \mathcal{B}+\mathcal{C} = 3$

1 rovnice $(4-3=1)$

rovnicové vyjádření:

$$\mathcal{B}+\mathcal{C} = \{ -x_1 + x_2 + 7x_3 - 3x_4 = 8 \}$$

koef. vlevo:

$\underline{\underline{w}} = (-1, 1, 7, -3)$

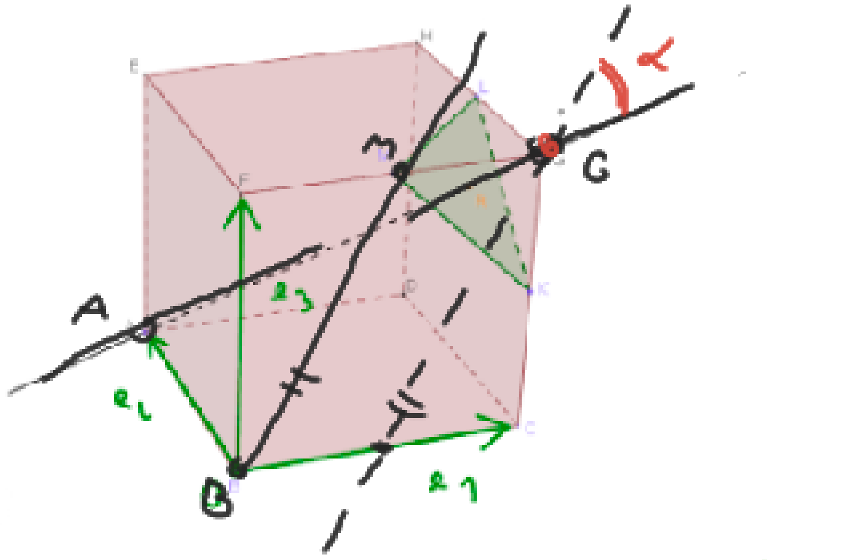
= normála nadroviny

koef. vpravo:

dosaď $A, B, \ldots$

$[2, 1, 0, -3]$

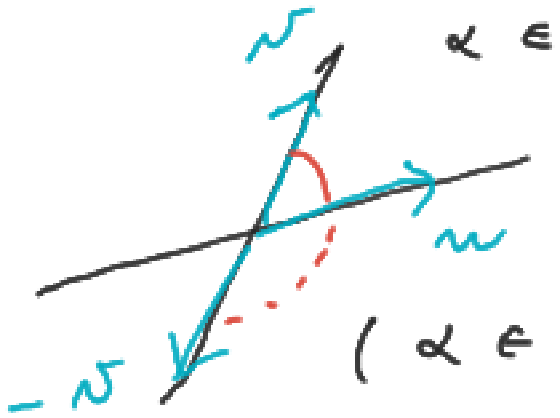
(32) ODCHYLKA přímek BM a AG



všude jenom VEKTORY!

$$\measuredangle(BM, AG) = \measuredangle(\overrightarrow{BM}, \pm\overrightarrow{AG}) =: \alpha \rightsquigarrow \boxed{\cos\alpha = \frac{|u \cdot v|}{\|u\| \cdot \|v\|}}$$

menší ze dvou možností:



$\alpha \in [0°, 90°] \rightsquigarrow \cos\alpha \geq 0$

$(\alpha \in [90°, 180°] \rightsquigarrow \cos\alpha \leq 0)$

$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \; L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}, \; M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \; G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$u = \overrightarrow{AG} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$v = \overrightarrow{BM} = \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}$

$$\cos\alpha = \frac{3/2}{\sqrt{3}\sqrt{5/4}} = \frac{\sqrt{3}}{\sqrt{5}}$$

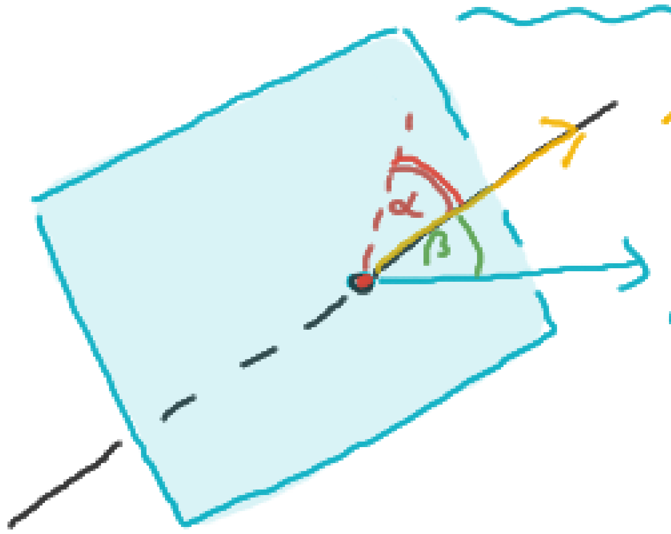
$(\ldots \doteq 0{,}77,$

$\text{tj. } \alpha \doteq 39{,}23°)$

(32) ODCHYLKA přímky AG a roviny BGE

(a) pomocí KOLMÉHO PRŮMĚTU $\rightsquigarrow$ viz cv. (33)

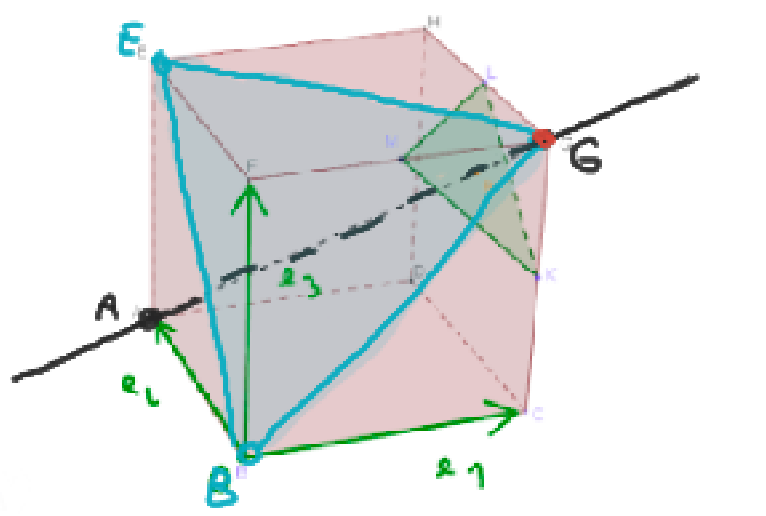
(b) pomocí NORMÁLY:



$m = \overrightarrow{AG}$

n = normála BGE

$$\boxed{\alpha + \beta = 90^\circ}$$



$E = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

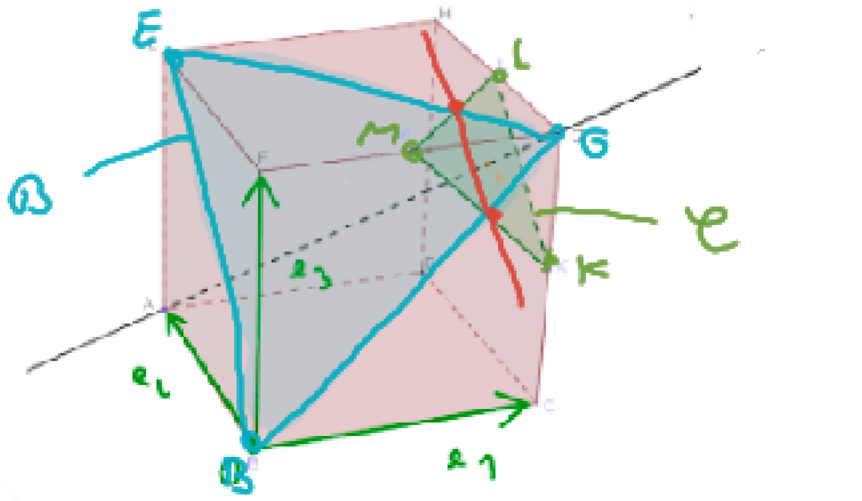
umíme:

- $n = \overrightarrow{BG} \times \overrightarrow{BE}$ ... $(-1, -1, 1)$
- $\sphericalangle(m, n) = \beta$ ... $\cos\beta = \dfrac{|m \cdot n|}{\|m\| \cdot \|n\|} = \ldots = \dfrac{1}{3}$

$\alpha = 90^\circ - \beta \quad \rightsquigarrow \quad \sin\alpha = \cos\beta = \ldots = \dfrac{1}{3}$

(tj. $\alpha \doteq 19{,}47^\circ$)

(32) ODCHYLKA rovin BCE a KLM

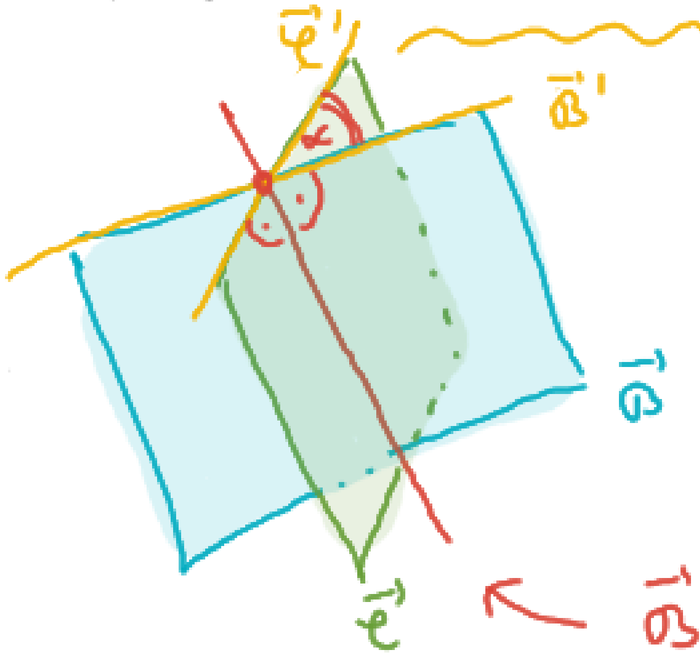


$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$, $L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}$, $M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$E = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

(a) pomocí menších podpr:



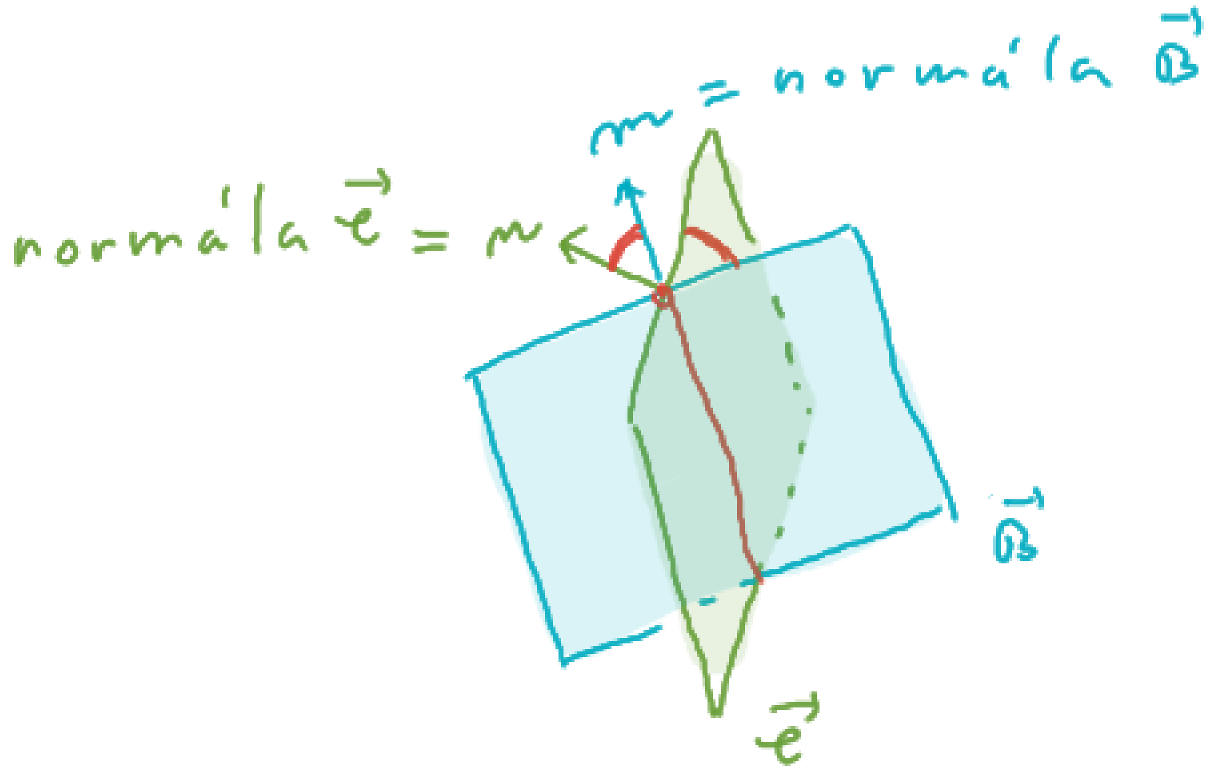
$\vec{\mathcal{B}} \cap \vec{\mathcal{E}} \neq \{o\}$

$\boxed{\begin{array}{l} \vec{\mathcal{B}}' \subset \vec{\mathcal{B}} \text{ a } \vec{\mathcal{B}}' \perp \vec{\mathcal{B}} \cap \vec{\mathcal{E}} \\ \vec{\mathcal{E}}' \subset \vec{\mathcal{E}} \text{ a } \vec{\mathcal{E}}' \perp \vec{\mathcal{B}} \cap \vec{\mathcal{E}} \end{array}}$!

- $\vec{\mathcal{B}} \cap \vec{\mathcal{E}}$ ... dim 1 ... $\langle (0,1,1) \rangle$ ($\overrightarrow{BE}$)
- $\vec{\mathcal{B}}'$ ... dim 1 ... $\langle \underbrace{(2,-1,1)}_{u} \rangle = \langle (a,b,a+b) \rangle \cap \langle (0,1,1) \rangle^{\perp}$ ($a\overrightarrow{BG} + b\overrightarrow{BE}$)
- $\vec{\mathcal{E}}'$ ... dim 1 ... $\langle \underbrace{(2,1,-1)}_{v} \rangle = \langle (c,d,-c+d) \rangle \cap \langle (0,1,1) \rangle^{\perp}$ ($c\overrightarrow{KL} + d\overrightarrow{KM}$)

$\rightarrow \sphericalangle(\mathcal{B}_2, \mathcal{E}_2) \overset{!}{=} \sphericalangle(\vec{\mathcal{B}}'_1, \vec{\mathcal{E}}'_1) = \sphericalangle(u,v) \ldots = \underline{\underline{\arccos \frac{2}{6}}}$

(b) pomocí NORMÁL



POZN:

nápady s normálami lze použít jen u NADROVIN!

$\uparrow \dim \vec{e}^{\perp} = 1$

- $m \propto \overrightarrow{BG} \times \overrightarrow{BE}$ ... $(-1,-1,1)$
- $n \propto \overrightarrow{KL} \times \overrightarrow{KM}$ ... $(1,-1,1)$

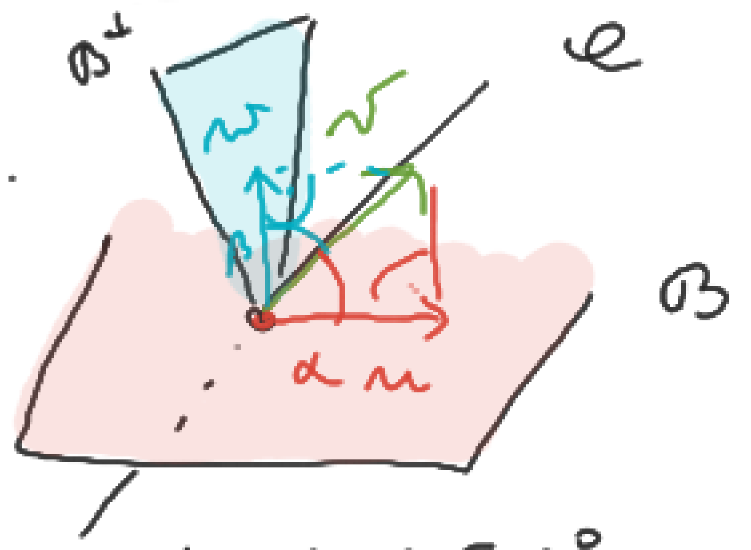
$$\sphericalangle(B, e) \overset{!}{=} \sphericalangle(m, n) \; \dots \; \arccos \frac{1}{3} \checkmark$$

(33) ODCHYLKA $\mathcal{B} = \left\{ \begin{array}{r} x_2 - x_4 = 2 \\ x_3 = 1 \end{array} \right\}$, $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$ ← dim 1

↖ dim 2

$\dim \vec{\mathcal{B}}^{\perp} = 2 \quad (= 4 - 2)$

→ POMOCÍ KOLMÉHO PRŮMĚTU :



u = kolmý průmět v do $\vec{\mathcal{B}}$

w = — " — v do $\vec{\mathcal{B}}^{\perp}$

$$\boxed{\begin{array}{l} \alpha + \beta = 90° \\ u + w = v \end{array}}$$

$v = (1, 1, 0, 0)$

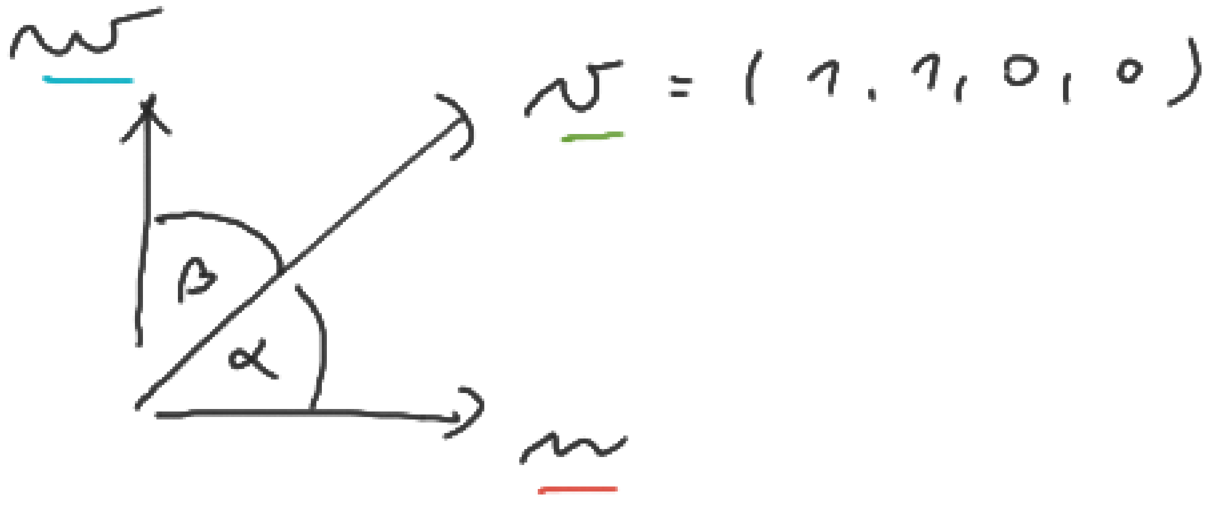
$\vec{\mathcal{B}} = \dots$

$\vec{\mathcal{B}}^{\perp} = \langle \underset{n_1}{(0,1,0,-1)}, \underset{n_2}{(0,0,1,0)} \rangle$ ← čteme SNADNO ze zadání !

→ $w = \overset{?}{a_1} n_1 + \overset{?}{a_2} n_2$ $\quad a_1, a_2 \in \mathbb{R}$??

$$\boxed{\begin{array}{l} v - w \perp n_1 \\ v - w \perp n_2 \end{array}} \iff \boxed{\begin{array}{l} w \circ n_1 = v \circ n_1 \\ w \circ n_2 = v \circ n_2 \end{array}} \iff \left[\begin{array}{l} \overset{2}{(n_1 \circ n_1)}\, a_1 + \overset{0}{(n_2 \circ n_1)}\, a_2 = \overset{1}{v \circ n_1} \\ \overset{0}{(n_1 \circ n_2)}\, a_1 + \overset{1}{(n_2 \circ n_2)}\, a_2 = \overset{0}{v \circ n_2} \end{array} \right.$$

$$\boxed{\begin{array}{l} 2a_1 = 1 \\ a_2 = 0 \end{array}} \longrightarrow \underline{w} = \frac{1}{2} u_1 + 0\, u_2 = \frac{1}{2} (\underbrace{0, 1, 0, -1}_{u_1})$$



$$\cos \beta = \frac{v \circ w}{\|v\| \cdot \|w\|} = \frac{v \circ u_1}{\|v\| \cdot \|u_1\|} = \frac{1}{\sqrt{2}\sqrt{2}} = \frac{1}{2} = \sin \alpha$$

$$\Downarrow$$

$$\underline{\underline{\alpha = 30^\circ}}$$

(34) $u = (1,2,3)$, $v = (2,-1,*)$ <u>dim 3</u>

Doplňte tak, aby $u \in \mathcal{B}$, $v \in \mathcal{C}$ a

(a) $\sphericalangle(\mathcal{B},\mathcal{C}) = 90°$

(b) $\vec{\mathcal{B}} \subseteq \vec{\mathcal{C}}^{\perp}$

(c) (a) NE (b) ANO

(d) (a) ANO (b) NE

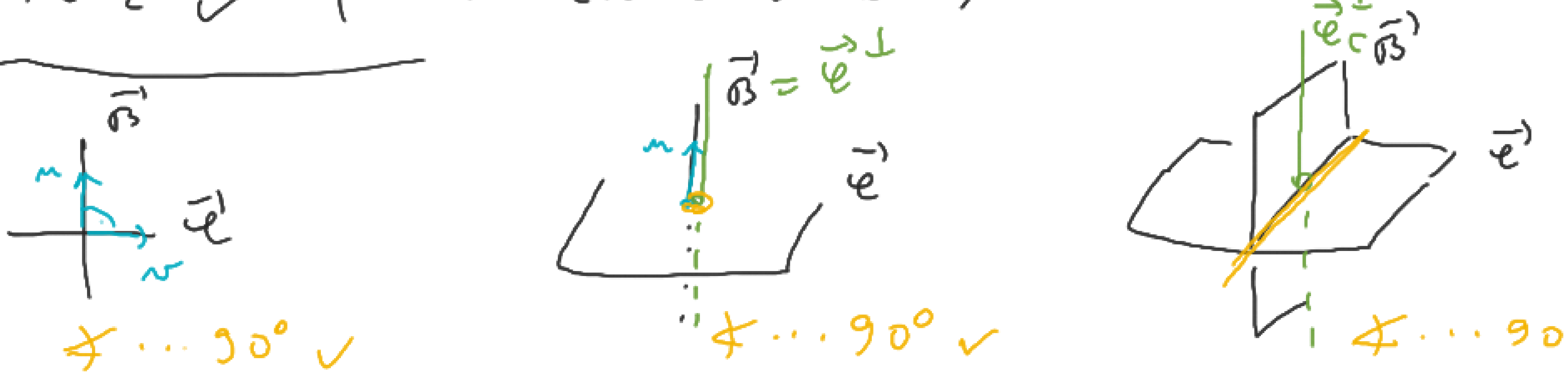
(a) $\dim \mathcal{B} = \dim \mathcal{C} = 1 \rightsquigarrow v = (2,-1,0)$ ✓ (tak, aby $u \circ v = 0$)

<u>např.</u> $\mathcal{B} = \{ [\,\vdots\,] + t \begin{pmatrix}1\\2\\3\end{pmatrix} \}$

$\mathcal{C} = \{ [\,\vdots\,] + s \begin{pmatrix}2\\-1\\0\end{pmatrix} \}$

cokoliv

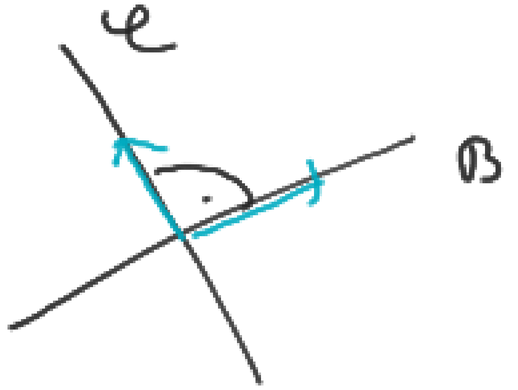
(b) TOTÉŽ ✓ ("všechno v $\vec{\mathcal{B}}$ je $\perp$ na všechno v $\vec{\mathcal{C}}$")



další <u>možnosti</u>

(34) $u = (1, 2, 3, *)$ $\quad v = (2, -1, *, *)$ $\quad$ dim 4

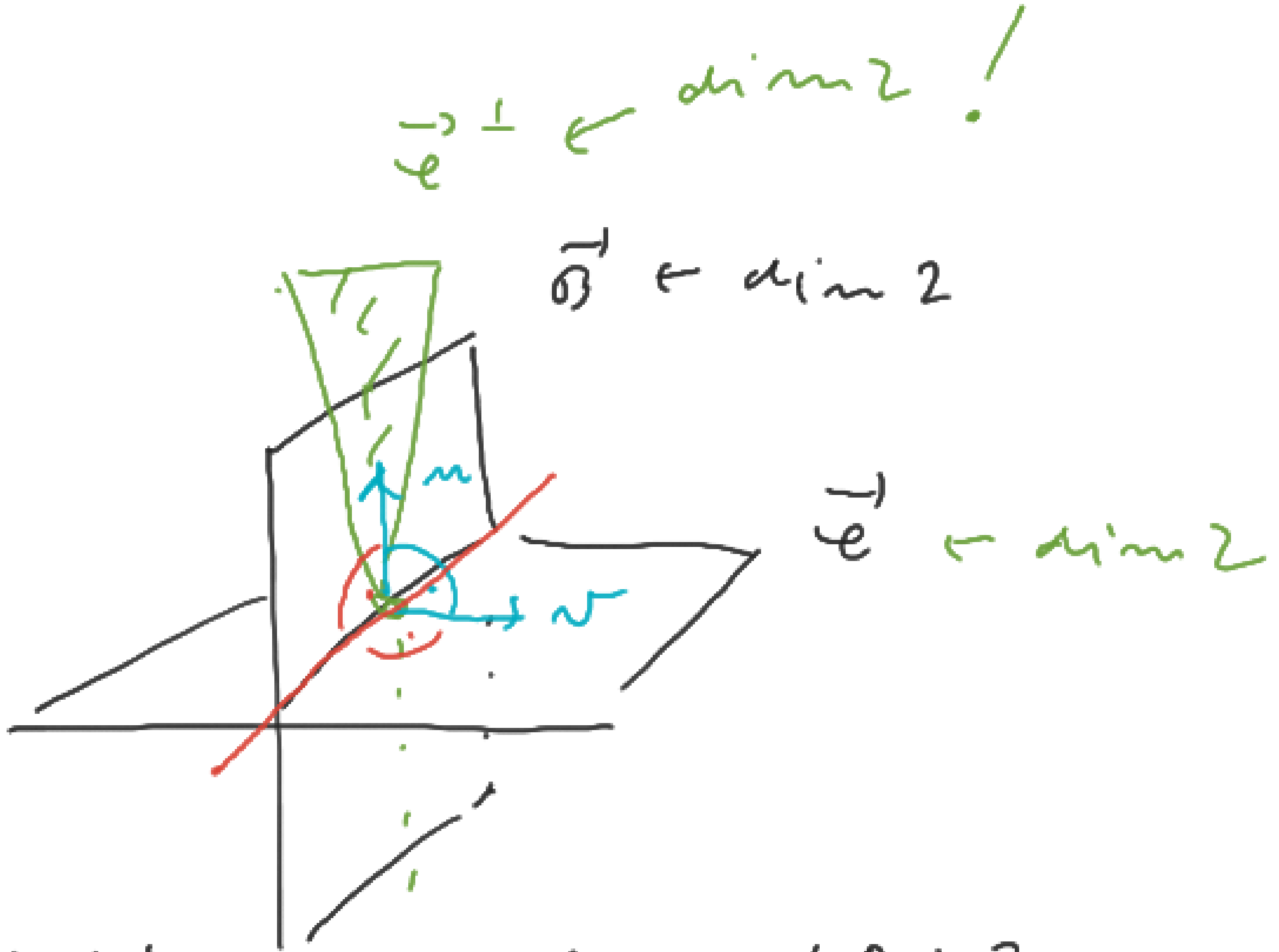
(a)
(b) } obdobně



např. $u = (1, 2, 3, 0)$
$v = (2, -1, 0, 0)$

(c) $\sphericalangle \neq 90^\circ$ a $\vec{B} \subseteq \vec{p}^{\perp}$?

... NENÍ MOŽNÉ!



(d) $\sphericalangle = 90^\circ$ a $\vec{B} \not\subseteq \vec{p}^{\perp}$

$$B = \left\{ [\,\vdots\,] + t \begin{pmatrix} 1 \\ 2 \\ 3 \\ 0 \end{pmatrix} + r \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

x u

$$p = \left\{ [\,\vdots\,] + s \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \check{s} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\} \rightsquigarrow \vec{p}^{\perp} = \left\{ \begin{pmatrix} a \\ 2a \\ b \\ 0 \end{pmatrix} \right\} = \left\{ a \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$$ ... dim 2 ✓

$\sphericalangle = 90^\circ$ ✓

v

cokoliv $\langle u, v \rangle^{\perp}$

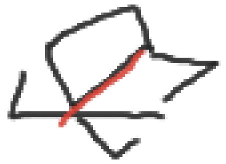
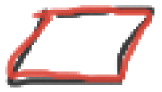
$\vec{B} \not\subseteq \vec{p}^{\perp}$ ✓

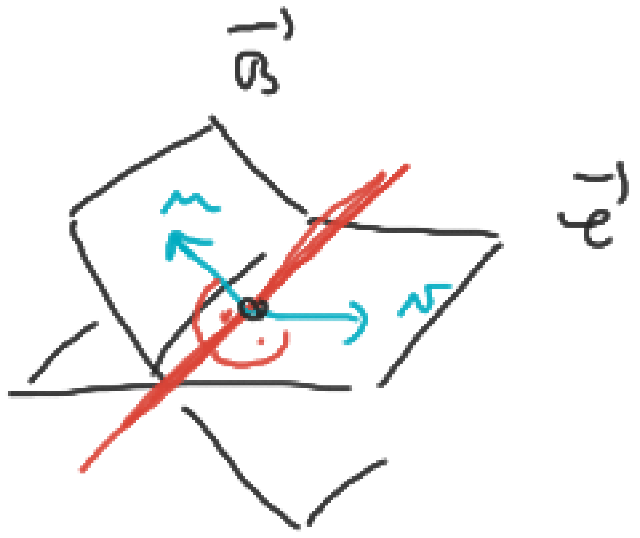
DOPLŇKOVÉ CV. $\qquad \measuredangle(\mathcal{B}, \mathcal{C}) = \,?$

$$\mathcal{B} = \left\{ \begin{bmatrix} \vdots \end{bmatrix} + t\begin{pmatrix}1\\2\\3\\0\end{pmatrix} + r\begin{pmatrix}1\\2\\3\\4\end{pmatrix} \right\} \qquad \mathcal{C} = \left\{ \begin{bmatrix} \vdots \end{bmatrix} + s\begin{pmatrix}2\\-1\\1\\1\end{pmatrix} + \check{s}\begin{pmatrix}-4\\2\\2\\3\end{pmatrix} \right\}$$

MOŽNOSTI

$\vec{\mathcal{B}} \cap \vec{\mathcal{C}}$:

- $\{0\} \rightsquigarrow$  $\rightsquigarrow$ NEUMÍME ☹
- dim 1 $\rightsquigarrow$  $\rightsquigarrow \measuredangle = \measuredangle$ menších podpr... ☺
- dim 2 $\rightsquigarrow$  $\rightsquigarrow \measuredangle = 0$ ☺



$$\vec{\mathcal{B}} \cap \vec{\mathcal{C}} = \left\langle \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\rangle \quad \dots \quad \dim 1$$

$$u \in \vec{\mathcal{B}} \text{ a } u \perp \vec{\mathcal{B}} \cap \vec{\mathcal{C}} \rightsquigarrow u = \begin{pmatrix}1\\2\\3\\0\end{pmatrix}$$

$$v \in \vec{\mathcal{C}} \text{ a } v \perp \vec{\mathcal{B}} \cap \vec{\mathcal{C}} \rightsquigarrow v = \begin{pmatrix}10\\-5\\1\\0\end{pmatrix} \leftarrow \begin{pmatrix}2s-4\check{s}\\-s+2\check{s}\\s+2\check{s}\\s+3\check{s}\end{pmatrix} \cdot \begin{pmatrix}0\\0\\0\\1\end{pmatrix} = 0$$

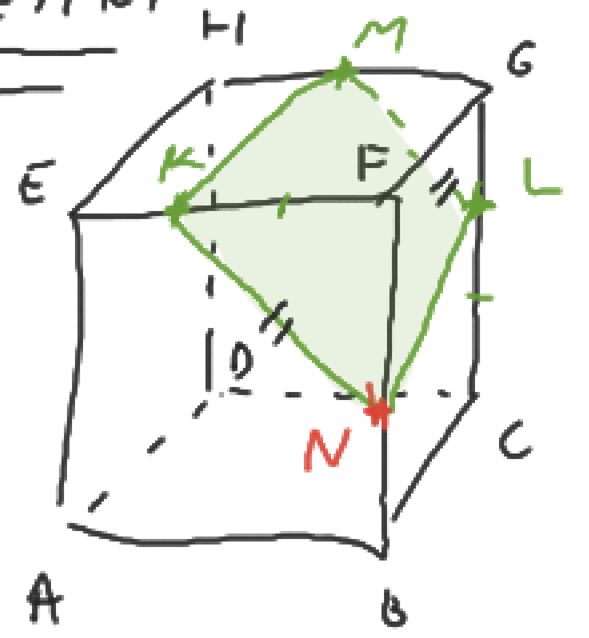
$$s + 3\check{s} = 0$$

např. $\boxed{s = 3 \quad \check{s} = -1}$

$$\measuredangle(\mathcal{B}, \mathcal{C}) = \measuredangle(u, \pm v) = \alpha$$

$$\dots \cos\alpha = \frac{|u \cdot v|}{\|u\| \cdot \|v\|} = \frac{3}{\sqrt{14}\sqrt{126}}$$

2 PÍSEMKY



$K = \frac{2}{3} E + \frac{1}{3} F \qquad L = \frac{1}{3} C + \frac{2}{3} G \qquad M = \frac{1}{2} G + \frac{1}{2} H$

(a) celý řez $\rightsquigarrow$ N

(b) poměr obsahů $\triangle KLM$ a $\triangle KLN$

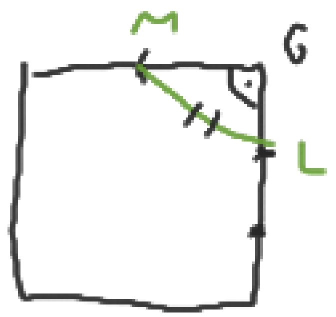
přímka

rovina

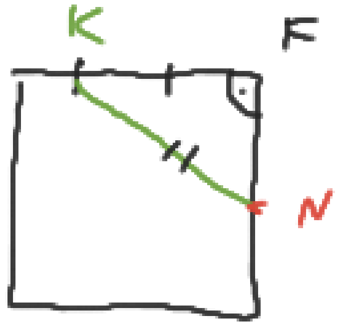
(a) $N = BF \cap KLM \rightsquigarrow N = \frac{4}{9} B + \frac{5}{9} F$

(b)

$\triangle KLM : \triangle KLN = \overline{ML} : \overline{KN} = 3 : 4$



$\sim$



... podobné $\frac{1}{2} : \frac{2}{3} = 3 : 4$

VYMYSLETE PŘÍKLAD...

$\mathcal{B}, \mathcal{C}$ mimob. a spol. směr

$\boxed{\mathcal{B} \cap \mathcal{C} = \emptyset, \text{ tj. } \overrightarrow{BC} \notin \vec{\mathcal{B}} + \vec{\mathcal{C}}}$

$\boxed{\vec{\mathcal{B}} \cap \vec{\mathcal{C}} \neq \{o\}}$

a $\boxed{\vec{\mathcal{B}} \cancel{\not\subseteq} \vec{\mathcal{C}}}$

$$\left(\begin{array}{cccc|c} 1 & \cdot & \cdot & \cdot & \cdot \\ 0 & 1 & \cdot & \cdot & \cdot \\ 0 & 0 & 1 & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 1 \end{array}\right)$$

$\underbrace{\quad}_{\vec{\mathcal{B}}}\ \underbrace{\quad}_{\vec{\mathcal{C}}}\ \underbrace{\quad}_{\overrightarrow{BC}}$

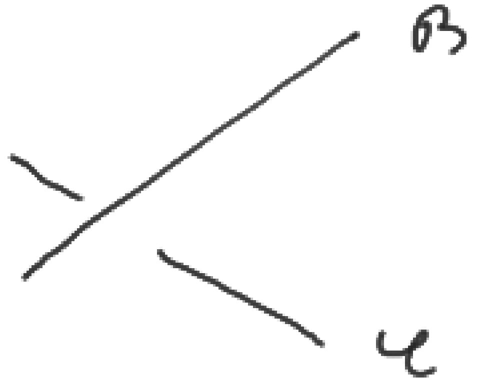
✓

✓

dim $\mathcal{B}$ = 2

dim $\mathcal{C}$ = 2

dim 2, 3



⇝ NENÍ MÍSTO!

dim 4 ?

dim $\mathcal{B}$ = dim $\mathcal{C}$ = 1 : spol. směr ⇒ ROVNOBĚŽNÉ ...