Repetitorium SŠ matematiky – 6 cvičení – řešení

1

a) $\frac{2-a}{a} = \frac{2}{x-1}$ $\quad a \in \mathbb{R}, \; a \neq 0, \; x \neq 1$

$$(2-a)(x-1) = 2a$$

$$2x - 2 - ax + a = 2a$$

$$x(2-a) = a+2 \quad / : (2-a)$$

$a \neq 2$

$$x = \frac{a+2}{2-a}$$

$a = 2$

$$x \cdot (2-2) = 2+2$$

$$0x = 4$$

$$K = \emptyset$$

$$1 = \frac{a+2}{2-a}$$

$$2-a = a+2$$

$$0 = 2a$$

$$a = 0$$

Výsledky:

$a=0, a=2 \; \ldots \ldots \; K = \emptyset$

$a \neq 0, a \neq 2 \; \ldots \; K = \left\{ \frac{a+2}{2-a} \right\}$

b) $\sqrt{x^2+b^2} - b = x$ $\quad b \in \mathbb{R}$

$$\sqrt{x^2+b^2} = x + b \quad /^2$$

$$x^2 + b^2 = x^2 + 2bx + b^2$$

$$0 = 2bx$$

$$bx = 0 \quad / : b$$

$b \neq 0$

$$x = 0$$

$b = 0$

$$0 \cdot 0 = 0$$

$$K = \mathbb{R}$$

ZK: $x=0, b \neq 0$

$$\sqrt{b^2} - b = 0$$

$$\sqrt{b^2} = b$$

$$|b| = b$$

$$b \geq 0$$

ZK: $b=0$

$$\sqrt{x^2} = x$$

$$|x| = x$$

$$x \geq 0$$

Výsledky:

$b = 0 \; \ldots \; K = \langle 0; \infty)$

$b < 0 \; \ldots \; K = \emptyset$

$b > 0 \; \ldots \; K = \{0\}$

c) pro které $a \in \mathbb{R}$ záporný kořen?

$$\frac{x}{x-a} = a+1 \qquad x \neq a$$

$$x = (a+1)(x-a)$$

$$x = ax + x - a^2 - a$$

$$ax = a^2 + a$$

$$ax = a(a+1) \qquad /: a$$

$a \neq 0$

$\underline{\underline{x = a+1}}$

kdy platí $x = a$?

$a = a+1$

$0 = 1$

nikdy nenastane

$a = 0$

$0 \cdot x = 0 \cdot (0+1)$

$0 = 0$

$\underline{K = \mathbb{R} - \{0\}}$ $\rightarrow x \neq a$

$\boxed{\text{Záporný kořen existuje pro } a < -1 \text{ a pro } a = 0}$

2

$x + (b-1)y = 1 \rightarrow x = 1 - by + y$

$\underline{(b+1)x + 3y = -1}$

$(b+1)(1 - by + y) + 3y = -1$

$b - b^2 y + by + 1 - by + y + 3y = -1$

$y(4 - b^2) = -2 - b$

$y(b^2 - 4) = 2 + b$

$y(b-2)(b+2) = b+2 \qquad /: (b+2)$

$b = -2$

$0 = 0$

$y \in \mathbb{R}$

$x = 1 + 2y + y$

$x = 1 + 3y$

$b \neq -2$

$y(b-2) = 1 \qquad /: (b-2)$

$b = 2$

$0 = 1$

$K = \emptyset$

$b \neq 2$

$y = \frac{1}{b-2}$

$x = 1 - b\frac{1}{b-2} + \frac{1}{b-2}$

$x = \frac{b - 2 - b + 1}{b-2}$

$x = \frac{-1}{b-2} = \frac{1}{2-b}$

Výsledky

$b = 2$ .... $K = \emptyset$

$b = -2$ .... $K = \{[1+3y; y],\ y \in \mathbb{R}\}$

$b \neq \pm 2$ .... $K = \{[\frac{1}{2-b}; \frac{1}{b-2}]\}$

$px^2 + (2p+3)x + p + \frac{3}{4} = 0 \qquad p \in \mathbb{R}$

→ je-li $p = 0$, není rovnice kvadratická

$$0\,x^2 + (2\cdot 0+3)x + 0 + \frac{3}{4} = 0$$
$$3x = -\frac{3}{4}$$
$$\underline{\underline{x = -\frac{1}{4}}}$$

→ pro ostatní hodnoty p je rovnice kvadratická

$p \neq 0 \qquad D = (2p+3)^2 - 4p\left(p+\frac{3}{4}\right) = 4p^2 + 12p + 9 - 4p^2 - 3p =$
$$= 9p + 9 = 9(p+1)$$

1. $D > 0 \Rightarrow p > -1$

$$x_{1,2} = \frac{-2p-3 \pm 3\sqrt{p+1}}{2p}$$

2. $D = 0 \Rightarrow p = -1$

$$x = \frac{-2p-3}{2p} = \frac{2-3}{-2} = \underline{\underline{\frac{1}{2}}}$$

3. $D < 0 \Rightarrow p < -1$

$K = \emptyset$

Výsledky:

$$\boxed{\begin{array}{ll} p = 0 \;\ldots\ldots & K = \left\{-\frac{1}{4}\right\} \\ p = -1 \;\ldots\ldots & K = \left\{\frac{1}{2}\right\} \\ p < -1 \;\ldots\ldots & K = \emptyset \\ p > -1;\ p \neq 0 \;\ldots & K = \left\{\frac{-2p-3 \pm 3\sqrt{p+1}}{2p}\right\} \end{array}}$$

4 Pro která $p \in \mathbb{R}$ má rovnice $x^2 + 2(p-4)x + p^2 + 6p = 0$

a) reálné kořeny

$D = 4p^2 - 32p + 64 - 4p^2 - 24p =$
$= -56p + 64$

$D \geq 0$

$-56p + 64 \geq 0$

$p \leq \frac{64}{56}$

$\underline{\underline{p \leq \frac{8}{7}}}$

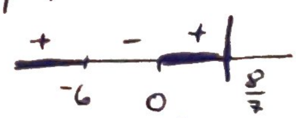
b) oba kořeny kladné

$x_1 + x_2 = -2p + 8$

$x_1 \cdot x_2 = p^2 + 6p$

$-2p + 8 > 0 \quad \wedge \quad p(p+6) > 0$

$p < 4 \quad \wedge$

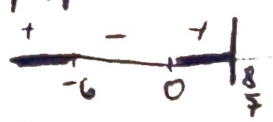


$\underline{\underline{p \in (-\infty; -6) \cup (0; \tfrac{8}{7})}}$

c) oba kořeny záporné

$-2p + 8 < 0 \quad \wedge \quad p(p+6) > 0$

$p > 4 \quad \wedge$



$\underline{\underline{p \in \emptyset}}$

d) jeden kořen kladný a jeden záporný

$p(p+6) < 0$

$\underline{\underline{p \in (-6; 0)}}$