$$F(x)\Big|_a^b = \int_a^b f(x)\,dx = F(b) - F(a)$$

F je prim. f. k f

$$\int_0^1 x(2-x^2)^5\,dx =$$

$2-x^2=t$, $dt=-2x\,dx$ $\rightarrow$ $x\,dx = -\frac{1}{2}dt$

$x=0:\ t=2-0=2$

$x=1:\ t=2-1=1$

$$= -\frac{1}{2}\int_2^1 t^5\,dt = \frac{1}{2}\int_1^2 t^5\,dt = \frac{1}{2}\frac{t^6}{6}\Big|_1^2 = \frac{1}{2}\left(\frac{2^6}{6}-\frac{1}{6}\right) = \frac{2^6-1}{12} = \frac{63}{12} = \frac{21}{4}.$$

$$\int_0^{\frac{\pi}{2}} \cos^3 3x\cdot \sin 6x\,dx = \int_0^{\frac{\pi}{2}} \cos^3 3x\cdot 2\sin 3x\cos 3x\,dx =$$

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$$= 2\int_0^{\frac{\pi}{2}} \cos^4 3x \sin 3x\,dx = -\frac{2}{3}\int_1^0 t^4\,dt = \frac{2}{3}\int_0^1 t^4\,dt =$$

$\cos 3x = t$

$dt = -3\sin 3x\,dx$, $\sin 3x\,dx = -\frac{1}{3}dt$

$x=0:\ t=\cos 0 = 1$

$x=\frac{\pi}{2}:\ t=\cos\frac{3\pi}{2}=0$

$$= \frac{2}{3}\frac{t^5}{5}\Big|_0^1 = \frac{2}{15}t^5\Big|_0^1 = \frac{2}{15}.$$

$$= 2\int_0^{\frac{\pi}{2}} \cos^4 3x\left(-\frac{1}{3}d(\cos 3x)\right) = -\frac{2}{3}\int_0^{\frac{\pi}{2}} \cos^4 3x\, d(\cos 3x) =$$

$$= -\frac{2}{3}\cdot\frac{(\cos 3x)^5}{5}\Big|_0^{\frac{\pi}{2}} = -\frac{2}{15}\Big(\underbrace{\left(\cos 3\cdot\tfrac{\pi}{2}\right)^5}_{0} - (\cos 0)^5\Big) = \frac{2}{15}.$$

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per partes

$$\int_0^{\frac{\pi}{4}} x\,tg^2 x\,dx = \left| \begin{array}{ll} u=x, & u'=1 \\ v'=tg^2 x, & v=\int tg^2 x\,dx = tg x - x \end{array}\right|$$

$1+tg^2 x = 1+\frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x+\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$tg^2 x = \frac{1}{\cos^2 x}-1$

$$\int tg^2 x\,dx = \int\left(\frac{1}{\cos^2 x}-1\right)dx =$$

$$= \int\frac{dx}{\cos^2 x} - \int dx = \int d(tg x - x) =$$

$$= tg x - x$$

$$= x\cdot(tg x - x)\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} (tg x - x)\,dx =$$

$$= \frac{\pi}{4}\left(\underbrace{tg\frac{\pi}{4}}_{1} - \frac{\pi}{4}\right) - 0 - \int_0^{\frac{\pi}{4}} tg x\,dx - \int_0^{\frac{\pi}{4}} x\,dx =$$

$$= \frac{\pi}{4}\left(1-\frac{\pi}{4}\right) - \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x}\,dx - \frac{x^2}{2}\Big|_0^{\frac{\pi}{4}} =$$

$(tg x)' = \frac{1}{\cos^2 x}$

$$= \frac{\pi}{4} - \frac{\pi^2}{16} + \int_0^{\frac{\pi}{4}} \frac{1}{\cos x}\,d(\cos x) - \frac{1}{2}\cdot\frac{\pi^2}{16} =$$

$d(\cos x) = -\sin x\,dx$

$$= \frac{\pi}{4} - \frac{\pi^2}{16} + \ln|\cos x|\Big|_0^{\frac{\pi}{4}} - \frac{1}{2}\cdot\frac{\pi^2}{16} = \frac{\pi}{4} - \frac{\pi^2}{32} + \ln\left(\cos\frac{\pi}{4}\right) - \ln(\cos 0) =$$

$$= \frac{\pi}{4} - \frac{\pi^2}{32} + \ln\frac{1}{\sqrt{2}} = \frac{\pi}{4} - \frac{\pi^2}{32} - \frac{1}{2}\ln 2.$$

$$\ln\frac{1}{\sqrt{2}} = \underbrace{\ln 1}_{0} - \ln\sqrt{2} = -\ln\sqrt{2} = -\ln 2^{\frac{1}{2}} = -\frac{1}{2}\ln 2$$

$$\int_0^{\ln 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\,dx =$$

$dt = e^x dx$

$e^x - 1 = t$, $\sqrt{e^x-1} = \sqrt{t}$

$d(e^x-1) = d(e^x) = \boxed{e^x dx = dt}$

$x = 0 \rightarrow t = e^0 - 1 = 0$

$x = \ln 5 \rightarrow t = e^{\ln 5} - 1 = 5 - 1 = 4.$

$e^x + 3 = t + 1 + 3 = t + 4$

$$= \int_0^4 \frac{\sqrt{t}\,dt}{t+4} =$$

$t = u^2$

$dt = 2u\,du$

$t = 0: \ u = 0$

$t = 4: \ u = 2$

$$= \int_0^2 \frac{u}{u^2+4}\cdot 2u\,du = 2\int_0^2 \frac{u^2\,du}{u^2+4} = 2\int_0^2 \frac{u^2+4-4}{u^2+4}\,du =$$

$$= 2\int_0^2 du - 8\int_0^2 \frac{du}{u^2+4} = 2\cdot 2 - 8\cdot\int_0^2 \frac{du}{4\left(\frac{u^2}{4}+1\right)} = 4 - 2\int_0^2 \frac{du}{\left(\frac{u}{2}\right)^2+1} =$$

$$= 4 - 2\cdot 2\int_0^2 \frac{d\left(\frac{u}{2}\right)}{\left(\frac{u}{2}\right)^2+1} = 4 - 4\cdot\int_0^2 \frac{d\left(\frac{u}{2}\right)}{\left(\frac{u}{2}\right)^2+1} =$$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$

$$= 4 - 4\cdot\operatorname{arctg}\frac{u}{2}\Big|_0^2 = 4 - 4(\operatorname{arctg} 1 - \operatorname{arctg} 0) =$$

$$= 4 - 4\left(\frac{\pi}{4} - 0\right) = 4 - \pi.$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{5+3\sin x}\,dx \qquad \operatorname{tg}\frac{x}{2} = t$$

$$2\cdot\operatorname{arctg} t = x$$

$$\sin x = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$x=0: \ t=\operatorname{tg}0=0$

$x=\frac{\pi}{2}: \ t=\operatorname{tg}\frac{\pi}{4}=1$

$$dx = 2\,d(\operatorname{arctg} t) = 2\frac{dt}{1+t^2} = \frac{2}{1+t^2}dt$$

$$= \int_0^1 \frac{\frac{2t}{1+t^2}}{5+3\cdot\frac{2t}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt = \int_0^1 \frac{2t}{5(1+t^2)+6t}\cdot\frac{2}{1+t^2}\,dt =$$

$5+5t^2+6t$

$$= 4\int_0^1 \frac{t\,dt}{(5t^2+6t+5)(t^2+1)}$$

$5t^2+6t+5$

$D = 36-4\cdot5\cdot5 < 0$

$$\frac{t}{(5t^2+6t+5)(t^2+1)} = \frac{\overset{0}{A}t+\overset{\frac{1}{6}}{B}}{t^2+1} + \frac{\overset{0}{C}t+\overset{-\frac{5}{6}}{D}}{5t^2+6t+5} \qquad A,B,C,D = ?$$

$$t = (At+B)(5t^2+6t+5)+(Ct+D)(t^2+1)$$

$t=0: \quad 0 = B\cdot5+D \qquad D=-5B \qquad D=-\frac{5}{6}$

$t^3: \ 5A+C=0 \qquad C=0$

$t^2: \ 5B+6A+D=0 \qquad \to 6A=0, \qquad A=0$

$t: \ 5A+6B+C=1 \qquad 6B=1, \qquad B=\frac{1}{6}$

$$\frac{t}{(5t^2+6t+5)(t^2+1)} = \frac{1}{6}\cdot\frac{1}{t^2+1} - \frac{5}{6}\frac{1}{5t^2+6t+5}$$

$$\text{integral} = 4\int_0^1 (\ \downarrow\ )\,dt = \frac{2}{3}\int_0^1\frac{dt}{t^2+1} - \frac{10}{3}\int_0^1\frac{dt}{5t^2+6t+5} =$$

$$\underset{\neq 0}{5}t^2+6t+5 = 5\left(t^2+\frac{6}{5}t+1\right) = 5\left(\boxed{t^2+2\cdot\frac{3}{5}\cdot t+\left(\frac{3}{5}\right)^2}+1-\left(\frac{3}{5}\right)^2\right) =$$

$\left(t+\frac{3}{5}\right)^2 \qquad \frac{25-9}{25}=\frac{16}{25}$

$$= 5\left(\left(t+\frac{3}{5}\right)^2+\frac{16}{25}\right) = \boxed{5\left(\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2\right)}$$

$$= \frac{2}{3}\operatorname{arctg} t\Big|_0^1 - \frac{10}{3\cdot5}\int_0^1\frac{dt}{\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} = \frac{2}{3}\cdot\frac{\pi}{4} -$$

$$-\frac{2}{3}\int_0^1\frac{d\left(t+\frac{3}{5}\right)}{\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} = \frac{\pi}{6}-\frac{2}{3}\int_0^1\frac{d\left(t+\frac{3}{5}\right)}{\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} =$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a^2}\int \frac{dx}{\left(\frac{x}{a}\right)^2+1} = \frac{1}{a}\int \frac{d\left(\frac{x}{a}\right)}{\left(\frac{x}{a}\right)^2+1} =$$

$$= \frac{1}{a}\operatorname{arctg}\frac{x}{a} + C$$

$$= \frac{\pi}{6} - \frac{2}{3}\cdot\frac{5}{4}\cdot \operatorname{artg}\frac{t+\frac{3}{5}}{\frac{4}{5}}\Bigg|_0^1 = \frac{\pi}{6} - \frac{5}{6}\operatorname{artg}\underbrace{\frac{5}{4}\left(t+\frac{3}{5}\right)}_{\left(\frac{5}{4}t+\frac{3}{4}\right)}\Bigg|_0^1$$

$$= \frac{\pi}{6} - \frac{5}{6}\operatorname{artg}\left(\frac{5}{4}t+\frac{3}{4}\right)\Bigg|_0^1 = \frac{\pi}{6} - \frac{5}{6}\operatorname{artg}\left(\frac{5}{4}+\frac{3}{4}\right) +$$

$$+ \frac{5}{6}\operatorname{arctg}\left(\frac{3}{4}\right) = \boxed{\frac{\pi}{6} - \frac{5}{6}\operatorname{arctg}2 + \frac{5}{6}\operatorname{arctg}\frac{3}{4}}.$$

$$J = \int_0^{\frac{\pi}{4}} e^x \sin x\,dx = \left| \begin{matrix} u = \sin x, & u' = \cos x \\ v' = e^x, & v = e^x \end{matrix} \right| =$$

$$= e^x \sin x\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} e^x \cos x\,dx = e^{\frac{\pi}{4}}\sin\frac{\pi}{4} - \int_0^{\frac{\pi}{4}} e^x \cos x\,dx =$$

$$\left| \begin{matrix} u = \cos x, & u' = -\sin x \\ v' = e^x, & v = e^x \end{matrix} \right| =$$

$$= \frac{e^{\frac{\pi}{4}}}{\sqrt{2}} - e^x\cos x\Big|_0^{\frac{\pi}{4}} + \int_0^{\frac{\pi}{4}} e^x(-\sin x)\,dx = \frac{e^{\frac{\pi}{4}}}{\sqrt{2}} - e^{\frac{\pi}{4}}\cos\frac{\pi}{4} +$$

$$+ \cos 0 - \underbrace{\int_0^{\frac{\pi}{4}} e^x \sin x\,dx}_{J}$$

$$J = \cancel{\frac{e^{\frac{\pi}{4}}}{\sqrt{2}}} - \cancel{\frac{e^{\frac{\pi}{4}}}{\sqrt{2}}} + 1 - J, \qquad 2J = 1, \qquad J = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{4}} e^x \sin x\,dx = \frac{1}{2}.$$