$$\int R(\cos x, \sin x)\,dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{5+3\sin x}\,dx \;\stackrel{\circ}{=}$$

$$\operatorname{tg}\frac{x}{2} = t$$

$$\frac{x}{2} = \operatorname{arctg} t, \quad x = 2\operatorname{arctg} t$$

$$dx = \frac{2\,dt}{t^2+1}$$

$$x = 0 \to t = \operatorname{tg} 0 = 0$$

$$x = \frac{\pi}{2} \to t = \operatorname{tg}\frac{\pi}{4} = 1$$

$$\sin x = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{1} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \frac{2\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1+\frac{\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}}} = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\stackrel{\circ}{=} \int_0^1 \frac{\frac{2t}{1+t^2}}{5+3\cdot\frac{2t}{1+t^2}}\cdot\frac{2\,dt}{t^2+1} = 4\int_0^1 \frac{t\,dt}{[5(1+t^2)+6t](t^2+1)} =$$

$5+5t^2+6t$

$$= 4\int_0^1 \frac{t\,dt}{(5t^2+6t+5)(t^2+1)}$$

$$\frac{t}{(5t^2+6t+5)(t^2+1)}$$

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$5t^2+6t+5 \qquad D = 36-4\cdot 25 = 36-100 < 0$

$$\frac{t}{(5t^2+6t+5)(t^2+1)} = \frac{\overset{0}{A}t+\overset{1/6}{B}}{t^2+1} + \frac{\overset{0}{C}t+\overset{-5/6}{D}}{5t^2+6t+5}$$

$A, B, C, D = ?$

$$t = (At+B)(5t^2+6t+5) + (Ct+D)(t^2+1) \quad \text{(musí platit } \forall t)$$

$t=0: \quad 0 = 5B + D \qquad D = -5B = -\frac{5}{6}$

$t^3: \quad 0 = 5A + C \qquad C = 0$

$t^2: \quad 0 = 5B + 6A + D \quad \to \quad A = 0$

$t: \quad 1 = 5A + 6B + C \quad \to \quad 1 = 6B, \quad B = \frac{1}{6}$

$$\frac{t}{(5t^2+6t+5)(t^2+1)} = \frac{1}{6}\cdot\frac{1}{t^2+1} - \frac{5}{6}\cdot\frac{1}{5t^2+6t+5}$$

$$5t^2+6t+5 = 5\left(t^2+\frac{6}{5}t+1\right) = 5\left(t^2+2\cdot t\cdot\frac{3}{5}+\left(\frac{3}{5}\right)^2+1-\left(\frac{3}{5}\right)^2\right) =$$

$$= 5\left(\left(t+\frac{3}{5}\right)^2+1-\frac{9}{25}\right) = 5\left(\left(t+\frac{3}{5}\right)^2+\frac{16}{25}\right) = 5\left(\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2\right)$$

$$4\int_0^1 \frac{t\,dt}{(5t^2+6t+5)(t^2+1)} = \frac{2}{3}\int_0^1 \frac{dt}{t^2+1} - \frac{10}{3\cdot 5}\int_0^1 \frac{d\left(t+\frac{3}{5}\right)}{\left(t+\frac{3}{5}\right)^2+\left(\frac{4}{5}\right)^2} =$$

$$= \frac{2}{3}\,\mathrm{arctg}\, t\Big|_0^1 - \frac{2}{3}\cdot\frac{5}{4}\,\mathrm{arctg}\,\frac{5}{4}\left(t+\frac{3}{5}\right)\Big|_0^1$$

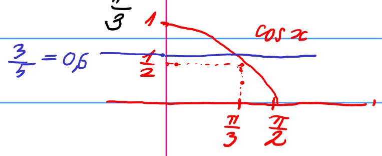
$$= \frac{2}{3}\,\mathrm{arctg}\,1 - 0 - \frac{5}{6}\,\mathrm{arctg}\left(\frac{5}{4}t+\frac{3}{4}\right)\Big|_0^1$$

$$= \frac{2}{3}\cdot\frac{\pi}{4} - \frac{5}{6}\,\mathrm{arctg}\,2 + \frac{5}{6}\,\mathrm{arctg}\,\frac{3}{4} = \frac{\pi}{6} - \frac{5}{6}\,\mathrm{arctg}\,2 + \frac{5}{6}\,\mathrm{arctg}\,\frac{3}{4}.$$

$$\int\frac{dx}{x^2+a^2} = \int\frac{dx}{a^2\left(\left(\frac{x}{a}\right)^2+1\right)} = \frac{1}{a^2}\int\frac{dx}{\left(\frac{x}{a}\right)^2+1} = \frac{1}{a}\int\frac{d\left(\frac{x}{a}\right)}{\left(\frac{x}{a}\right)^2+1} = \frac{1}{a}\,\mathrm{arctg}\,\frac{x}{a}+C$$

$$\int_{\pi/3}^{\pi/2}\frac{dx}{3-5\cos x} \;=$$

def. obor: $\cos x \neq \frac{3}{5}$ (splněno pro $\frac{\pi}{3} \le x \le \frac{\pi}{2}$)



$\mathrm{tg}\,\frac{x}{2} = t$, $\quad x = 2\,\mathrm{arctg}\,t$, $\quad dx = \frac{2dt}{1+t^2}$

$$\cos x = \frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}} = \frac{1-\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

$x = \frac{\pi}{2}$: $t = \mathrm{tg}\,\frac{\pi}{4} = 1$

$x = \frac{\pi}{3}$: $t = \mathrm{tg}\,\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$$= \int_{\frac{1}{\sqrt{3}}}^{1}\frac{1}{3-5\cdot\frac{1-t^2}{1+t^2}}\cdot\frac{2\,dt}{t^2+1} = 2\int_{\frac{1}{\sqrt{3}}}^{1}\frac{dt}{3(t^2+1)-5(1-t^2)} =$$

$3t^2+3-5+5t^2 = 8t^2-2 = 2(4t^2-1)$

$$= 2\int_{\frac{1}{\sqrt{3}}}^{1}\frac{dt}{2(4t^2-1)} = \int_{\frac{1}{\sqrt{3}}}^{1}\frac{dt}{4t^2-1} = \frac{1}{2}\int_{\frac{1}{\sqrt{3}}}^{1}\frac{dt}{2t-1} - \frac{1}{2}\int_{\frac{1}{\sqrt{3}}}^{1}\frac{dt}{2t+1} =$$

$$\boxed{\begin{aligned} &\frac{1}{4t^2-1} = \frac{1}{(2t-1)(2t+1)} = \frac{\overset{\frac{1}{2}}{A}}{2t-1} + \frac{\overset{-\frac{1}{2}}{B}}{2t+1} \\ &1 = A(2t+1) + B(2t-1) \\ &t = \tfrac{1}{2}:\quad 1 = 2A,\quad A = \tfrac{1}{2} \\ &t = -\tfrac{1}{2}:\quad 1 = -2B,\quad B = -\tfrac{1}{2} \end{aligned}}$$

$$= \frac{1}{2\cdot 2}\int_{\frac{1}{\sqrt{3}}}^{1}\frac{d(2t-1)}{2t-1} - \frac{1}{2\cdot 2}\int_{\frac{1}{\sqrt{3}}}^{1}\frac{d(2t+1)}{2t+1} =$$

$$= \frac{1}{4}\ln|2t-1|\Big|_{\frac{1}{\sqrt{3}}}^{1} - \frac{1}{4}\ln|2t+1|\Big|_{\frac{1}{\sqrt{3}}}^{1} = \frac{1}{4}\left(\ln 1 - \ln\left(\frac{2}{\sqrt{3}}-1\right)\right) -$$

$$- \frac{1}{4}\left(\ln 3 - \ln\left(\frac{2}{\sqrt{3}}+1\right)\right) = -\frac{1}{4}\ln 3 -$$

$$- \frac{1}{4}\ln\left(\frac{2}{\sqrt{3}}-1\right) + \frac{1}{4}\ln\left(\frac{2}{\sqrt{3}}+1\right) = \frac{1}{4}\ln\frac{\frac{2}{\sqrt{3}}+1}{\frac{2}{\sqrt{3}}-1} - \frac{1}{4}\ln 3$$

$$= \frac{1}{4}\ln\frac{2+\sqrt{3}}{2-\sqrt{3}} - \frac{1}{4}\ln 3.$$

$$\int_0^{\frac{\pi}{4}} x\,tg^2x\,dx = \left| \begin{array}{l} u = x, \quad u' = 1 \\ v' = tg^2 x, \end{array} \right. \quad \text{per partes}$$

$$1 + tg^2 x = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = (tg\,x)'$$

$$v = \int tg^2 x\,dx = \int \left(\frac{1}{\cos^2 x} - 1\right) dx = \int d(tg\,x) - \int dx = tg\,x - x$$

$$= x\cdot(tg\,x - x)\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} 1\cdot(tg\,x - x)\,dx = \frac{\pi}{4}\left(tg\frac{\pi}{4} - \frac{\pi}{4}\right) - \int_0^{\frac{\pi}{4}} tg\,x\,dx +$$

$$+ \int_0^{\frac{\pi}{4}} x\,dx = \frac{\pi}{4}\left(1 - \frac{\pi}{4}\right) - \int_0^{\frac{\pi}{4}} \frac{\sin x\,dx}{\cos x} + \frac{x^2}{2}\Big|_0^{\frac{\pi}{4}} =$$

$$d(\cos x) = -\sin x\,dx$$

$$= \frac{\pi}{4} - \frac{\pi^2}{16} + \int_0^{\frac{\pi}{4}} \frac{d(\cos x)}{\cos x} + \frac{1}{2}\cdot\left(\frac{\pi}{4}\right)^2 =$$

$$= \frac{\pi}{4} - \frac{\pi^2}{16} + \ln|\cos x|\Big|_0^{\pi/4} + \frac{\pi^2}{32} =$$

$$= \frac{\pi}{4} - \frac{\pi^2}{32} + \ln\left|\cos\frac{\pi}{4}\right| - \ln|\cos 0| = \frac{\pi}{4} - \frac{\pi^2}{32} + \ln\frac{1}{\sqrt{2}} - \ln 1 =$$

$$= \frac{\pi}{4} - \frac{\pi^2}{32} + \ln 1 - \ln\sqrt{2} = \frac{\pi}{4} - \frac{\pi^2}{32} - \frac{1}{2}\ln 2$$

$$\int_{\frac{\pi}{4}}^{\pi} \frac{\sin^3 x}{2 - \cos x}\,dx = \int_{\frac{\pi}{4}}^{\pi} \frac{\sin^2 x}{2 - \cos x}\,\sin x\,dx = \left| \begin{array}{l} \cos x = t \\ \sin^2 x = 1 - t^2 \\ dt = -\sin x\,dx \end{array} \right.$$

$$x = \frac{\pi}{4} \to t = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$x = \pi \to t = \cos\pi = -1$$

$$= -\int_{\frac{1}{\sqrt{2}}}^{-1} \frac{1-t^2}{2-t}\,dt = \int_{-1}^{\frac{1}{\sqrt{2}}} \frac{1-t^2}{2-t}\,dt = \int_{-1}^{\frac{1}{\sqrt{2}}} \frac{t^2-1}{t-2}\,dt$$

$$\frac{t^2-1}{t-2}$$

$$\begin{array}{r|l} t^2 - 1 & t-2 \\ \hline -(t^2 - 2t) & t+2 \\ 2t - 1 & \\ -(2t - 4) & \\ 3 & \end{array}$$

$$\frac{t^2-1}{t-2} = t + 2 + \frac{3}{t-2}$$

$$= \int_{-1}^{\frac{1}{\sqrt{2}}} (t+2)\,dt + 3\cdot\int_{-1}^{\frac{1}{\sqrt{2}}} \frac{dt}{t-2} = \left(\frac{t^2}{2} + 2t\right)\Big|_{-1}^{\frac{1}{\sqrt{2}}} + 3\int_{-1}^{\frac{1}{\sqrt{2}}} \frac{d(t-2)}{t-2} =$$

$$= \left(\frac{1}{2}\cdot\frac{1}{2} + \frac{2}{\sqrt{2}}\right) - \left(\frac{1}{2} - 2\right) + 3\ln|t-2|\Big|_{-1}^{\frac{1}{\sqrt{2}}} = \frac{1}{4} + \sqrt{2} + \frac{3}{2} + 3\ln\left|\frac{1}{\sqrt{2}} - 2\right| -$$

$$- 3\ln|-1-2| = \frac{7}{4} + \sqrt{2} + 3\ln\left|\frac{1-2\sqrt{2}}{\sqrt{2}}\right| - 3\ln 3 =$$

$$= \frac{7}{4} + \sqrt{2} - \frac{3}{2}\ln 2 + 3\ln(2\sqrt{2} - 1) - 3\ln 3.$$

per partes

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{x}{\cos^2 x}\,dx = \left| \begin{array}{l} u = x, \quad u' = 1 \\ v' = \frac{1}{\cos^2 x}, \; v = \int \frac{dx}{\cos^2 x} = \operatorname{tg} x \end{array} \right| =$$

$$= x \operatorname{tg} x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} - \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \operatorname{tg} x\,dx = \frac{\pi}{3}\operatorname{tg}\frac{\pi}{3} - \frac{\pi}{6}\operatorname{tg}\frac{\pi}{6} + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} d(\ln \cos x)$$

$$= \frac{\pi}{3}\cdot\sqrt{3} - \frac{\pi}{6}\cdot\frac{1}{\sqrt{3}} + \ln|\cos x| \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$$

$$= \frac{\pi}{\sqrt{3}} - \frac{\pi}{6\sqrt{3}} + \ln\cos\frac{\pi}{3} - \ln\cos\frac{\pi}{6} =$$

$$\operatorname{tg} x\,dx = \frac{\sin x\,dx}{\cos x} = -\frac{d(\cos x)}{\cos x} = -d(\ln \cos x)$$

$$= \frac{5\pi}{6\sqrt{3}} + \ln\frac{1}{2} - \ln\frac{\sqrt{3}}{2}$$

$$= \frac{5\pi}{6\sqrt{3}} - \cancel{\ln 2} - \ln\sqrt{3} + \cancel{\ln 2}$$

$$= \frac{5\pi}{6\sqrt{3}} - \frac{\ln 3}{2} = \frac{5\pi\sqrt{3}}{18} - \frac{\ln 3}{2}.$$