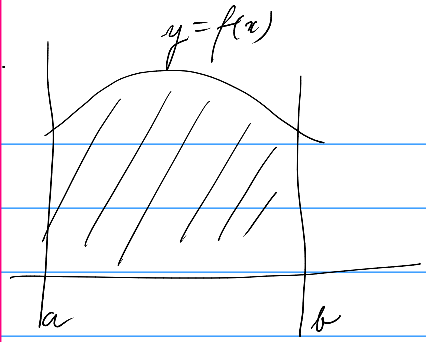
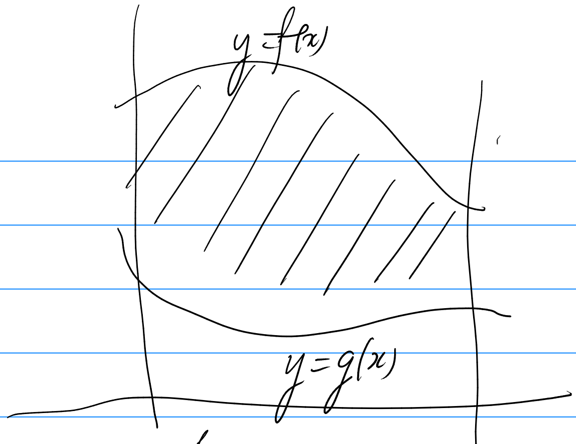
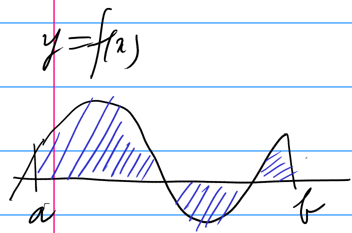
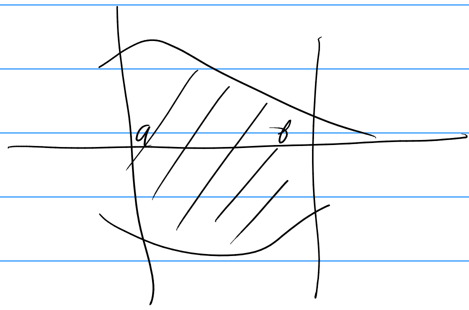


$\int_a^b f(x)\,dx$



$\int_a^b (f(x) - g(x))\,dx$





$y = -x^2 + 6x - 5, \quad y = -\frac{1}{3}x - \frac{1}{2}, \quad x = 1, \quad x = 4$

$S = ?$

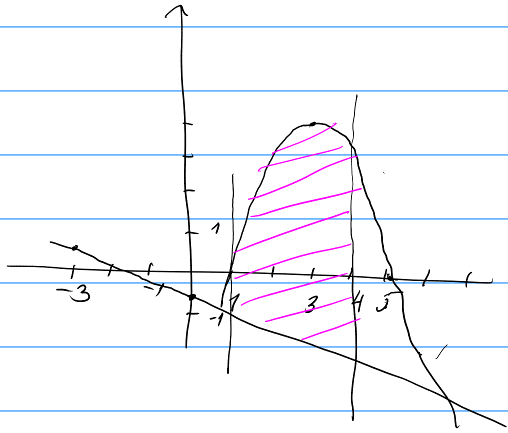
$x^2 - 6x + 5 = 0 \qquad x = \frac{6 \pm \sqrt{36-20}}{2} = \frac{6 \pm 4}{2} \begin{matrix} 5 \\ 1 \end{matrix}$

$9 - 18 + 5$

$x = 3$ lok. max.

$y = -\frac{1}{3}x - \frac{1}{2}$

$x = -3: \quad \frac{1}{2}$



$$S = \int_1^4 \left(-x^2 + 6x - 5 - \left(-\frac{1}{3}x - \frac{1}{2}\right)\right) dx = \int_1^4 \left(-x^2 + 6x - 5 + \frac{1}{3}x + \frac{1}{2}\right) dx =$$

$$= \int_1^4 \left(-x^2 + \frac{19}{3}x - \frac{9}{2}\right) dx = \left(-\frac{1}{3}x^3 + \frac{19}{6}x^2 - \frac{9}{2}x\right)\Big|_1^4 =$$

$$= -\frac{1}{3}4^3 + \frac{19}{6}4^2 - \frac{9}{2}\cdot 4 + \frac{1}{3} - \frac{19}{6} + \frac{9}{2} =$$

$$= -\frac{64}{3} + \frac{19\cdot 16 - 19}{6} - \frac{36}{2} + \frac{1}{3} + \frac{9}{2} =$$

$$= -\frac{63}{3} + \frac{19\cdot 15}{6} - \frac{27}{2} = -\frac{126 + \overset{285}{19\cdot 15} - 81}{6} = \frac{78}{6} = \frac{39}{3} = 13.$$

$$\begin{array}{r} 19 \\ \times 15 \\ \hline 95 \\ 19 \\ \hline 285 \\ -126 \\ \hline 159 \\ -81 \\ \hline 78 \end{array}$$

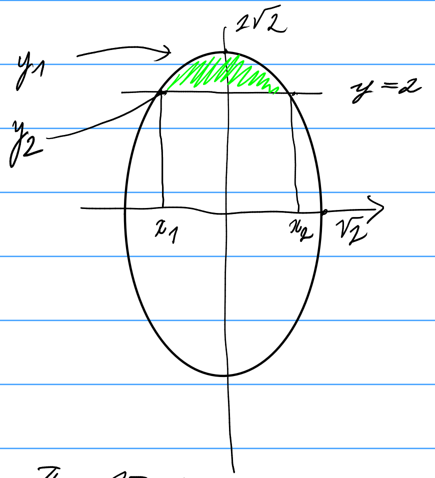
$S = 13.$

!

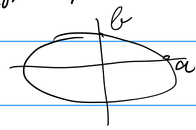
$$\begin{cases} x = \sqrt{2}\cdot\cos t \\ y = 2\sqrt{2}\cdot\sin t \\ y = 2 \end{cases}$$ (parametrické zadání)

$S = ?$

$y \geq 2$



$x = a\cos t$
$y = b\sin t$
$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

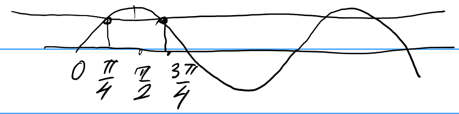


$\cancel{2}\sqrt{2}\cdot\sin t = \cancel{2}$, $\quad \sqrt{2}\sin t = 1$

$\sin t = \frac{1}{\sqrt{2}}$

$t = \frac{\pi}{4} + 2\pi n$

$t = \frac{3\pi}{4} + 2\pi n$



$x_1 \leftarrow t_1 = \frac{3\pi}{4}, \quad x_2 \leftarrow t_2 = \frac{\pi}{4}$

$$S = \int_{x_1}^{x_2} (y_1(x) - y_2(x))\,dx = \int_{x_1}^{x_2} y_1(x)\,dx - \int_{x_1}^{x_2} y_2(x)\,dx$$

$$\int_{x_1}^{x_2} y_1(x)\,dx = \int_{\frac{3\pi}{4}}^{\frac{\pi}{4}} \overset{y}{2\sqrt{2}\sin t}\cdot(-\sqrt{2}\sin t)\,dt = -4\int_{\frac{3\pi}{4}}^{\frac{\pi}{4}} \sin^2 t\,dt =$$

$[\, x = \sqrt{2}\cos t \rightarrow dx = -\sqrt{2}\sin t\,dt \,]$

$$= -4\int_{\frac{3\pi}{4}}^{\frac{\pi}{4}} \frac{1-\cos 2t}{2}\,dt = 2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (1-\cos 2t)\,dt =$$

$\cos^2 x - \sin^2 x = \cos 2x$
$1 - 2\sin^2 x = \cos 2x$
$\sin^2 x = \frac{1-\cos 2x}{2}$

$$= 2\cdot\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) - 2\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos 2t\,dt = \pi - \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d(\sin 2t) =$$

$$= \pi - \sin 2t\Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \pi - \sin\frac{3\pi}{2} + \sin\frac{\pi}{2} = \pi + 1 + 1 = \pi + 2$$

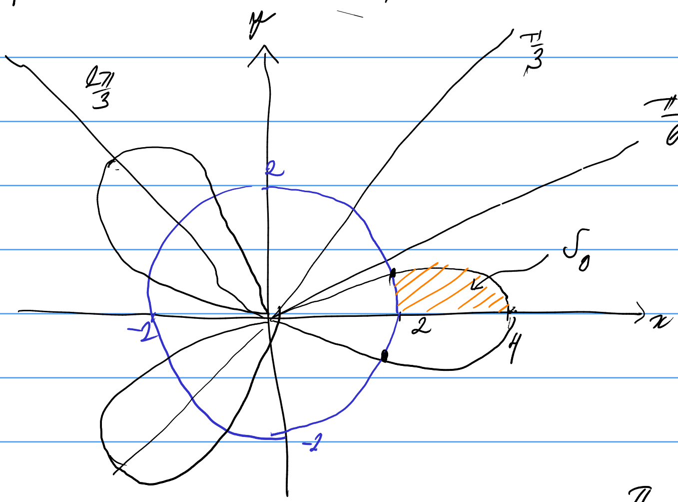
$$\int_{x_1}^{x_2} \underbrace{y_2(x)}_{=2}\,dx = \int_{x_1}^{x_2} 2\,dx = \int_{-1}^{1} 2\,dx = 2\cdot 2 = 4$$

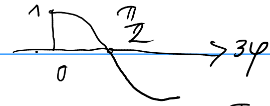
$x_1 \leftarrow t_1 = \frac{3\pi}{4}, \quad x_2 \leftarrow t_2 = \frac{\pi}{4}$
$x = \sqrt{2}\cos t;$
$x_1 = \sqrt{2}\cdot\cos\frac{3\pi}{4} = -1, \quad x_2 = \sqrt{2}\cos\frac{\pi}{4} = 1$

$S = \pi + 2 - 4 = \pi - 2.$

$\rho = 4\cos 3\varphi$, $\rho = 2$ vnější $(\rho \geq 2)$

$S = ?$





$3\varphi = \frac{\pi}{2}$

$\varphi = \frac{\pi}{6}$

$S = 6 \cdot S_0$

$4\cos 3\varphi = 2$

$\cos 3\varphi = \frac{1}{2}$, $3\varphi = \pm\frac{\pi}{3} + 2\pi n$, $\varphi = \pm\frac{\pi}{9} + \frac{2\pi}{3}n$

$\underbrace{\cos(3\varphi)}_{\frac{\rho(\varphi)}{4}} = \cos(3\varphi + 2\pi) = \underbrace{\cos 3\left(\varphi + \frac{2\pi}{3}\right)}_{\frac{\rho(\varphi + \frac{2\pi}{3})}{4}}$ $T = \frac{2\pi}{3}$ perioda

$$S_0 = \frac{1}{2}\int_0^{\frac{\pi}{9}} \left(\rho_1^2(\varphi) - \rho_2^2(\varphi)\right) d\varphi$$

$$S = 6 \cdot \frac{1}{2}\int_0^{\frac{\pi}{9}} \left((4\cos 3\varphi)^2 - 2^2\right) d\varphi =$$

$$= 3\int_0^{\frac{\pi}{9}} (16\cos^2 3\varphi - 4)\, d\varphi =$$

$$= 3\int_0^{\frac{\pi}{9}} \left(16 \cdot \frac{1 + \cos 6\varphi}{2} - 4\right) d\varphi =$$

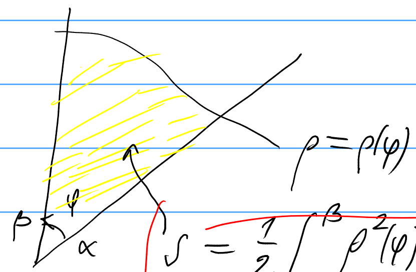
$$= 3\int_0^{\frac{\pi}{9}} (8 + 8\cos 6\varphi - 4)\, d\varphi =$$

$$= 3\int_0^{\frac{\pi}{9}} (8\cos 6\varphi + 4)\, d\varphi =$$

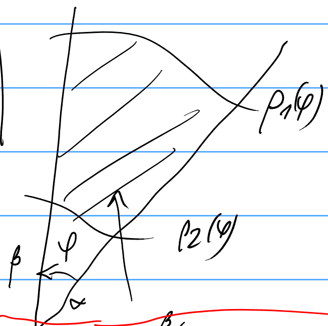
$$= 3 \cdot 8 \int_0^{\frac{\pi}{9}} \cos 6\varphi + 12 \cdot \frac{\pi}{9} =$$

$$= \frac{3 \cdot 8}{6} \sin 6\varphi \Big|_0^{\pi/9} - \frac{4}{3}\pi = 4\sin 6 \cdot \frac{\pi}{9} - 4\sin 0 - \frac{4}{3}\pi = 4\sin\frac{2\pi}{3} - \frac{4\pi}{3} =$$

$$= 4 \cdot \frac{\sqrt{3}}{2} - \frac{4\pi}{3} = 2\sqrt{3} - \frac{4\pi}{3}.$$



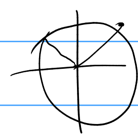
$$\boxed{S = \frac{1}{2}\int_\alpha^\beta \rho^2(\varphi)\, d\varphi}$$



$$\boxed{S = \frac{1}{2}\int_\alpha^\beta \left(\rho_1^2(\varphi) - \rho_2^2(\varphi)\right) d\varphi}$$

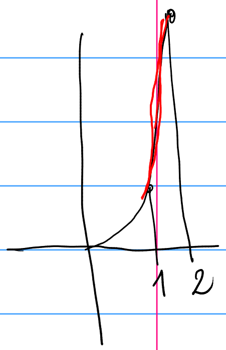
$\cos^2 x - \sin^2 x = \cos 2x$

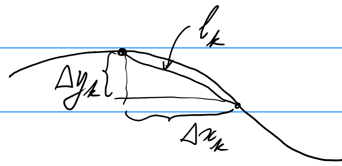
$\cos^2 x = \frac{1 + \cos 2x}{2}$



$y = x^2 \qquad l = ? \qquad A(1;1)\ ,\ B(2;4)$

mezi A, B





$x_k \ \ldots$

$x = \varphi(t)$

$y = \psi(t)$

$l_k = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2}$

$(\varphi(t_{k+1}) - \varphi(t_k))^2 \ \ldots$

$$l_k = \sqrt{\underbrace{(\varphi(t_{k+1}) - \varphi(t_k))^2}_{\varphi'(c_k)\cdot(t_{k+1}-t_k) = \varphi'(c_k)\cdot\Delta t_k} + \underbrace{(\psi(t_{k+1}) - \psi(t_k))^2}_{\psi'(\tilde c_k)\Delta t_k}} = \sqrt{(\varphi'(c_k))^2(\Delta t_k)^2 + (\psi'(\tilde c_k))^2(\Delta t_k)^2}$$

$$= \sqrt{(\varphi'(c_k))^2 + (\psi'(\tilde c_k))^2}\cdot \Delta t_k$$

$$\boxed{l = \int_{t_1}^{t_2} \sqrt{(\varphi'(t))^2 + (\psi'(t))^2}\, dt}$$

$y = f(x) \qquad t = x \qquad \begin{cases} x = x \\ y = f(x) \end{cases}$

$$\boxed{l = \int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2}\, dx}$$

$f(x) = x^2, \ x_1 = 1, \ x_2 = 2.$

$$l = \int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2}\, dx = \int_1^2 \sqrt{1 + (2x)^2}\, dx =$$

$$= \int_1^2 \sqrt{1 + 4x^2}\, dx \qquad \left| \begin{array}{l} u = \sqrt{1+4x^2}, \quad du = \dfrac{8x\,dx}{2\sqrt{1+4x^2}} = \dfrac{4x\,dx}{\sqrt{1+4x^2}} \\ dv = dx, \quad v = x \end{array} \right|$$

$$l = x\sqrt{1+4x^2}\Big|_1^2 - \int_1^2 x \frac{4x}{\sqrt{1+4x^2}}\, dx = 2\sqrt{1+16} - \sqrt{5} - \int_1^2 \frac{4x^2\, dx}{\sqrt{1+4x^2}} =$$

$$= 2\sqrt{17} - \sqrt{5} - \int_1^2 \frac{4x^2+1-1}{\sqrt{4x^2+1}}\, dx = 2\sqrt{17} - \sqrt{5} - \underbrace{\int_1^2 \sqrt{1+4x^2}\, dx}_{= l} - \int_1^2 \frac{dx}{\sqrt{4x^2+1}}$$

$$2l = 2\sqrt{17} - \sqrt{5} - \frac{1}{2}\int_1^2 \frac{d(2x)}{\sqrt{(2x)^2+1}} = 2\sqrt{17} - \sqrt{5} -$$

$$\left| \int \frac{dx}{\sqrt{x^2+B}} = \ln\left|x + \sqrt{x^2+B}\right| \right.$$

$$- \frac{1}{2}\ln\left|2x + \sqrt{4x^2+1}\right|\Big|_1^2 = 2\sqrt{17} - \sqrt{5} -$$

$$- \frac{1}{2}\left(\ln(4 + \sqrt{17}) - \ln(2 + \sqrt{5})\right)$$

$$2l = 2\sqrt{17} - \sqrt{5} - \frac{1}{2}\ln\frac{4+\sqrt{17}}{2+\sqrt{5}}$$

$$l = \sqrt{17} - \frac{\sqrt{5}}{2} - \frac{1}{4}\ln\frac{4+\sqrt{17}}{2+\sqrt{5}}$$

$$\begin{cases} x = 7\cos^3 t \\ y = 7\sin^3 t \end{cases} \qquad \frac{\pi}{2} \le t \le \pi \qquad l = ?$$

$$l = \int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$

$x = x(t)$, $y = y(t)$

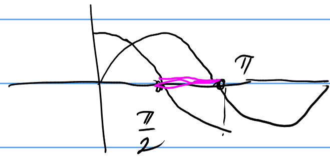
$$x'(t) = 7\cdot 3\cos^2 t\cdot(-\sin t) = -21\cos^2 t\sin t$$

$$y'(t) = 21\sin^2 t\cos t$$

$$(x'(t))^2 + (y'(t))^2 = 21^2\cos^4 t\sin^2 t + 21^2\sin^4\cos^2 t =$$

$$= 21^2\cos^2 t\sin^2 t\,(\underbrace{\cos^2 t + \sin^2 t}_{1}) = 21^2\cos^2 t\sin^2 t$$

$$l = \int_{\frac{\pi}{2}}^{\pi}\sqrt{21^2\sin^2 t\cos^2 t}\,dt =$$



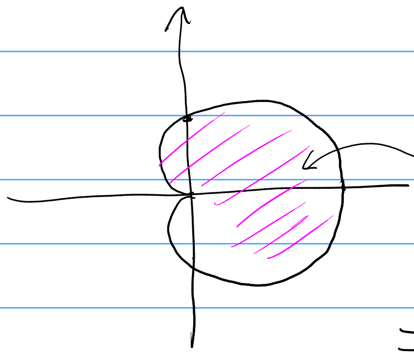
$$\sqrt{21^2\sin^2 t\cos^2 t} = -21\sin t\cos t$$

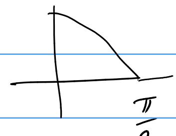
$$= -21\int_{\frac{\pi}{2}}^{\pi}\sin t\cos t\,dt = -21\cdot\int_{\frac{\pi}{2}}^{\pi}\sin t\,d(\sin t) = -21\cdot\frac{(\sin t)^2}{2}\Bigg|_{\frac{\pi}{2}}^{\pi} =$$

$$= -\frac{21}{2}\Big(\underbrace{(\sin\pi)^2}_{0} - \underbrace{(\sin\tfrac{\pi}{2})^2}_{1}\Big) = \frac{21}{2}.$$

$\rho = 2a(1+\cos\varphi)$ $\qquad a>0 \qquad 0\le\varphi\le 2\pi$

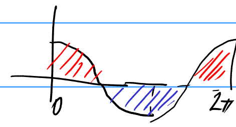
$S = ?$





$$S = \frac{1}{2}\int_0^{2\pi} R(\varphi)^2\, d\varphi =$$

$$= \frac{1}{2}\int_0^{2\pi} (2a(1+\cos\varphi))^2\, d\varphi =$$



$$= 2a^2\int_0^{2\pi}(1+\cos^2\varphi+2\cos\varphi)\,d\varphi = 2a^2\cdot 2\pi + 4a^2\int_0^{2\pi}\cos\varphi\, d\varphi +$$

$$+ 2a^2\int_0^{2\pi}\cos^2\varphi\, d\varphi = 4\pi a^2 + 4a^2\cdot 0 + a^2\int_0^{2\pi}(\cos 2\varphi+1)\,d\varphi =$$

$$= 4\pi a^2 + a^2\cdot 2\pi + a^2\cdot\underbrace{\int_0^{2\pi}\cos 2\varphi\, d\varphi}_{0} = 6\pi a^2.$$