$(u,v) = \sum u_i v_i$

$\int uv\,dx$

$$f(x) = \frac{a_0}{2} + \sum (a_n \cos nx + b_n \sin nx)$$

$$\int_{-\pi}^{\pi} f(x) \cos mx$$

$$\int_{-\pi}^{\pi} \cos nx \cos mx \qquad \sin nx \sin \ldots$$

I délky 2π

$$f(x) \sim \underbrace{\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)}_{\text{Fourierova řada}}$$

?

f + podmínky $\Rightarrow$

$\Rightarrow$ Four. řada v bodě x = $f(x)$, je-li f v bodě x spojitá

$\dfrac{f(x+) + f(x-)}{2}$, má-li funkce f v bodě x skok

$$\int_0^{+\infty} \frac{dx}{x^\alpha} - ?$$

$$\lim_{b\to+\infty} \int_{10}^{b} x^2\,dx = \int_{10}^{+\infty} x^2\,dx = +\infty$$

$\int$ nevl. — nekonečná mez

— fce je v jedné z mezí neohraničená.

$$\int_1^5 \frac{dx}{\sqrt{x-1}} = \lim_{a\to 1+} \int_a^5 \frac{dx}{\sqrt{x-1}}$$

$$\int_{-\infty}^{2} f(x)\,dx = \lim_{a\to-\infty} \int_a^2 f(x)\,dx$$

$$\int_{-\infty}^{+\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_c^{+\infty} f(x)\,dx$$

$\int_a^b f(x)\,dx$ f je neomezená v c, $a < c < b$

$$= \underbrace{\int_a^{c} f(x)\,dx + \int_c^b f(x)\,dx}$$

$\int_0^{+\infty} \frac{dx}{x^\alpha}$ $\alpha \in \mathbb{R}$ — nekonv. pro žádné α

$$f(x) = \frac{1}{x^\alpha}$$

$\alpha < 0$ $\frac{1}{x^\alpha} = x^{-\alpha}$, kde $-\alpha > 0$

↳ integrál nevlastní I. typu

$\alpha > 0$: $\frac{1}{x^\alpha}$ má singul. v bodě 0

↳ integrál nevlastní „smíš." typu:

$$\int_0^1 \frac{dx}{x^\alpha} + \underbrace{\int_1^{+\infty} \frac{dx}{x^\alpha}}$$

II. I.

fce má singul. v bodě 0 — fce je spojitá na $(1, +\infty)$

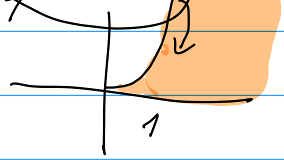
$$\int_1^{+\infty} \frac{dx}{x^\alpha}$$

$\alpha < 0$: $\displaystyle\int_1^{+\infty} \frac{dx}{x^\alpha} = \int_1^{+\infty} x^{-\alpha}\, dx =$

$-\alpha > 0$

$\alpha = -2$ $\qquad \displaystyle\int_1^{+\infty} x^2\, dx$

$$= \lim_{b\to+\infty} \int_1^b x^{-\alpha}\,dx = \lim_{b\to+\infty} \left.\frac{x^{-\alpha+1}}{-\alpha+1}\right|_1^b$$



$$= \lim_{b\to+\infty}\frac{b^{1-\alpha}-1}{1-\alpha} = +\infty.$$

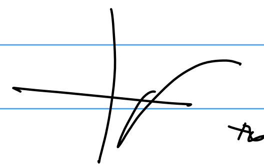
$(x^\alpha)' = \alpha x^{\alpha-1}$

$1-\alpha>0 \Rightarrow \displaystyle\int_1^{+\infty}\frac{dx}{x^\alpha} = +\infty$ (diverg.)

$\alpha<1$

$\alpha=1$: $\displaystyle\int_1^{+\infty}\frac{dx}{x^\alpha} = \int_1^{+\infty}\frac{dx}{x} = \lim_{b\to+\infty}\int_1^b\frac{dx}{x} =$

$$= \lim_{b\to+\infty}(\ln b - \underbrace{\ln 1}_{0}) = \lim_{b\to+\infty}\ln b = +\infty$$



$\alpha>1$: $\to 1-\alpha<0$

$$\int_1^{+\infty}\frac{dx}{x^\alpha} = \lim_{b\to+\infty}\frac{\boxed{b^{1-\alpha}}\to 0 \;-1}{1-\alpha} = \frac{-1}{1-\alpha} = \frac{1}{\alpha-1}\ (>0)$$

$\displaystyle\int_1^{+\infty}\frac{dx}{x^\alpha}$ — konverguje pro $\alpha>1$; div. v ost. pří.

$$\boxed{\int_0^1 \frac{dx}{x^\alpha}} = \lim_{a\to0+}\int_a^1 \frac{dx}{x^\alpha} = \lim_{a\to 0+}\int_a^1 x^{-\alpha}\,dx = \lim_{a\to0+} \left.\frac{x^{-\alpha+1}}{-\alpha+1}\right|_a^1 =$$

$\alpha\ne1$

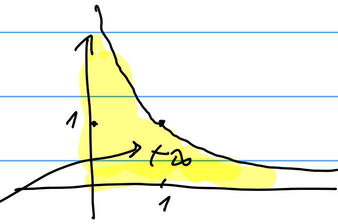
$= \displaystyle\lim_{a\to0+}\frac{1-a^{1-\alpha}}{1-\alpha}$
- $\alpha<1$: $1-\alpha>0 \to \exists$ limita konečná
- $1-\alpha<0$, tj. $\alpha>1 \to$ div.

$\alpha=1$: $\displaystyle\int_0^1 \frac{dx}{x} = \lim_{a\to0+}\int_a^1\frac{dx}{x} = \lim_{a\to0+}(\ln 1 - \ln a)$ — div.

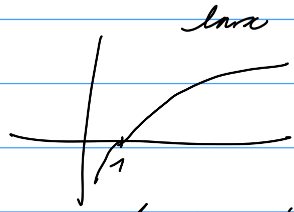
$$\int_1^{+\infty} \frac{\ln x}{x}\,dx \quad \xrightarrow{?} \text{ diverguje.}$$

integrál nevl. I. typu

$\hookrightarrow \dots$ $\displaystyle\int_1^{+\infty} \frac{dx}{x}$ diverguje



$$\frac{\ln x}{x} = \frac{1}{x}\cdot \ln x$$



$$\int_a^{+\infty} f(x) \qquad \int_a^{+\infty} g(x)\,dx$$

pro dostatečně velká x platí $\ln x > 1$

$\Downarrow$

$$\frac{\ln x}{x} > \frac{1}{x}$$

$$0 \le f(x) \le g(x)$$

$\int f$ diverguje. $\rightarrow$ $\int g$ taky div.

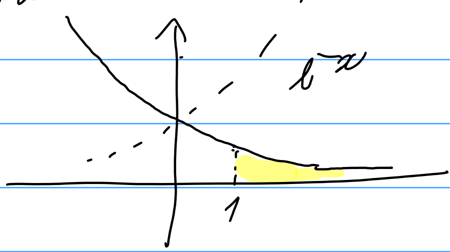
$$\int_0^{+\infty} e^{-x}\,dx \quad \text{— I. typu}$$

$$\int_0^{+\infty} e^{-x}\,dx = \lim_{b\to+\infty} \int_0^b e^{-x}\,dx = \lim_{b\to+\infty} -\int_0^b d\left(e^{-x}\right) =$$

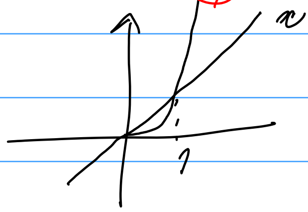
$$d\left(e^{-x}\right) = e^{-x}\cdot(-1)\,dx$$

$$= -\lim_{b\to+\infty} e^{-x}\Big|_0^b = -\lim_{b\to+\infty}\left(e^{-b} - e^0\right) =$$

$$= \lim_{b\to+\infty}\left(1 - e^{-b}\right) = 1 - \underbrace{\lim_{b\to+\infty} e^{-b}}_{=0} = 1$$



$\int_0^{+\infty} e^{-x^2}dx$ — ? I. typu



$x > 1$: $x^2 > x$

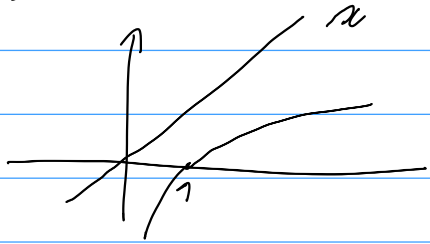
$0 < e^{-x^2} < e^{-x}$ $\qquad \int_1^{+\infty} e^{-x}dx < +\infty$

$\Downarrow$

$\int_1^{+\infty} e^{-x^2}dx < +\infty$ konwerguje.

$\int_1^{+\infty} \frac{\ln x}{x^{5/2}}dx$ — ? I. typu (f-a ograniczona, krzm' nie jest +∞)

$\int_1^{+\infty} \frac{dx}{x^{5/2}}$ — konwerguje $(\alpha = \frac{5}{2} > 1)$



$\ln x < x$ $(x > 1)$

int. kom.

$0 < \frac{\ln x}{x^{5/2}} < \frac{x}{x^{5/2}} = \frac{1}{x^{3/2}}$

$x > 1$ $\qquad \int_1^{+\infty} \frac{dx}{x^{3/2}} < +\infty$ $\quad (\alpha = \frac{3}{2} > 1)$.

$\Downarrow$

konwerguje i $\int_1^{+\infty} \frac{\ln x}{x^{5/2}}dx$

Obsah plochy: $y = xe^{-x}$, $y=0$, $x \geq 0$

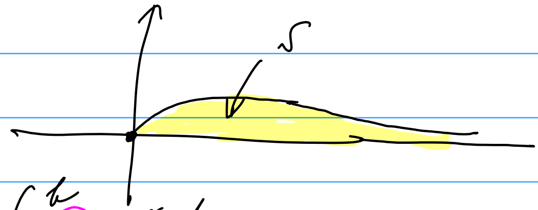
$f(x) = xe^{-x}$ má vodor. asympt. $y=0$

$\lim\limits_{x\to+\infty} xe^{-x} = 0$ $\quad (f(x) \geq 0)$

$\infty \cdot 0$ – ?

nevlastní int. I. typu

$\frac{\infty}{\infty}$ $\quad \frac{x}{e^x}$ $\cdot$ (LH) $\quad \frac{1}{e^x} \to 0$, $x \to +\infty$



$$S = \int_0^{+\infty} xe^{-x}\,dx = \lim_{b\to+\infty} \underbrace{\int_0^b xe^{-x}\,dx}_{} =$$

nevl. int. I. typu

$$\left| \begin{array}{l} u = x,\ u' = 1 \\ v' = e^{-x},\ v = -e^{-x} \end{array} \right|$$

$$= \lim_{b\to+\infty} \left(-xe^{-x}\Big|_0^b + \int_0^b e^{-x}\,dx \right) =$$

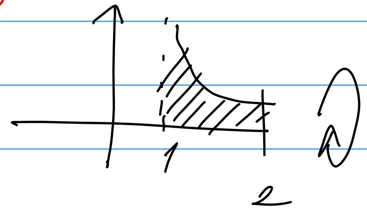
$$= \lim_{b\to+\infty} \left(-be^{-b} - \int_0^b d(e^{-x}) \right) = \lim_{b\to+\infty} \left(-be^{-b} - e^{-x}\Big|_0^b \right) =$$

$$= \lim_{b\to+\infty} \left(\underbrace{-be^{-b}}_{\to 0} - \underbrace{e^{-b}}_{\to 0} + 1 \right) = 1.$$

$$\boxed{S = 1}$$

$\frac{b}{e^b}$, kde $b \to +\infty$ $(\to 0)$

Objem rotačného telesa: $\quad f(x) = \dfrac{1}{\sqrt[4]{x-1}}, \; x=1, \; x=2$



$\lim\limits_{x\to 1+} \dfrac{1}{\sqrt{x-1}} = +\infty$

nevlastný II. typu

$$V = \pi \int_a^b f^2(x)\,dx$$

$$V = \pi \int_1^2 \frac{dx}{\left(\sqrt[4]{x-1}\right)^2} = \pi \int_1^2 \frac{dx}{\sqrt{x-1}} =$$

$$= \pi \int_1^2 (x-1)^{-\frac{1}{2}}\,dx = \pi \left.\frac{(x-1)^{1-\frac{1}{2}}}{1-\frac{1}{2}}\right|_1^2 =$$

$$= 2\pi \left.(x-1)^{\frac{1}{2}}\right|_1^2 = 2\pi(1-0) = 2\pi$$

$\to 0$

Zde vskutku počítame limitu pro $x \to 1+$