Definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $a \neq b$, then $f(a) \neq f(b)$.

Alternative definition: A function f is **one-to-one (or injective)** if for all a and b in its domain, if $f(a) = f(b)$, then $a = b$.

Definition: A function f is **increasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) \leq f(b)$.

Definition: A function f is **strictly increasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) < f(b)$.

Definition: A function f is **decreasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) \geq f(b)$.

Definition: A function f is **strictly decreasing** on an interval I if for all a and b in I, if $a < b$, then $f(a) > f(b)$.

Definition: A function f is **even** if for all x in its domain , $f(-x) = f(x)$. The graph of an even function is symmetrical to the $y-$axis.

Definition: A function f is **odd** if for all x in its domain , $f(-x) = -f(x)$. The graph of an odd function is symmetrical to the origin.

Note that while an integer is either even or odd, most functions are neither even, nor odd. Even and odd functions are sort of rare but these properties are very useful for us.

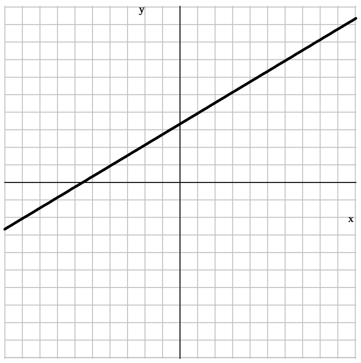
Definition: A **rational function** is a quotient of two polynomial functions.

The concept of a **continuous function** is very important. Although this term will not be precisely defined, the intuitive idea of a continuous function is thatwe can draw its graph without lifting the pencil. For example, $f(x) = x^2$ is a continuous function but $g(x) = \dfrac{1}{x}$ is not; it is not continuous at $x = 0$.

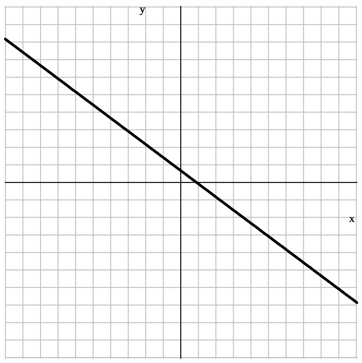
Basic Functions

1.) Linear functions $f(x) = mx + b$ where $m \neq 0$.

The graph is a straight line. One useful form of the equation can be obtained by factoring out the slope: $f(x) = mx + b = m\left(x + \frac{b}{m}\right)$



$m > 0$



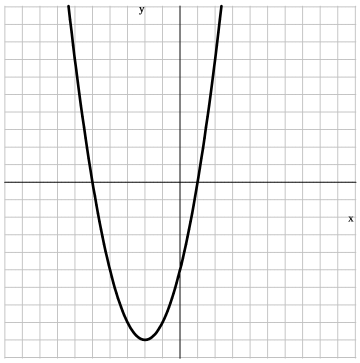
$m < 0$

Case 1. If $m > 0$
domain: $\mathbb{R}$
range: $\mathbb{R}$
y−intercept: $(0, b)$
x−intercept: $\left(-\frac{b}{m}, 0\right)$
one-to-one
no maximum or minimum
strictly increasing
continuous on $\mathbb{R}$

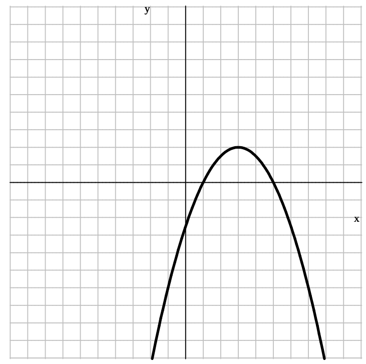
Case 2. If $m < 0$
domain: $\mathbb{R}$
range: $\mathbb{R}$
y−intercept: $(0, b)$
x−intercept: $\left(-\frac{b}{m}, 0\right)$
one-to-one
no maximum or minimum
strictly decreasing
continuous on $\mathbb{R}$

2.) Quadratic functions $f(x) = ax^2 + bx + c$ where $a \neq 0$.

The graph is a parabola. It opens upward if $a > 0$ and opens downward if $a < 0$.



$a > 0$



$a < 0$

Case 1. If $a > 0$

Example: $f(x) = x^2 + 4x - 5$

standard form: $f(x) = (x+2)^2 - 9$

factored form: $f(x) = (x+5)(x-1)$

domain: $\mathbb{R}$ range: $[-9, \infty)$

$y-$intercept: $(0, -5)$

$x-$intercepts: $(-5, 0)$ and $(1, 0)$

not one-to-one
no maximum
minimum: $(-2, -9)$
strictly decreasing on $(-\infty, -2)$
strictly increasing on $(-2, \infty)$
continuous on $\mathbb{R}$

Case 2. If $a < 0$

Example: $f(x) = -\dfrac{1}{2}x^2 + 3x - \dfrac{5}{2}$

standard form: $f(x) = -\dfrac{1}{2}(x-3)^2 + 2$

factored form: $f(x) = -\dfrac{1}{2}(x-1)(x-5)$

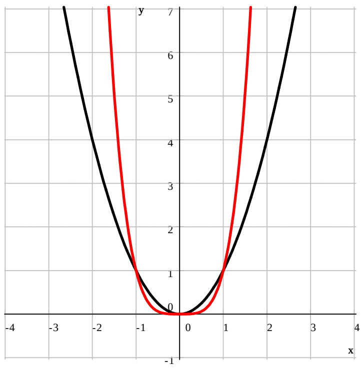
domain: $\mathbb{R}$ range: $(-\infty, 2]$

$y-$intercept: $\left(0, -\dfrac{5}{2}\right)$

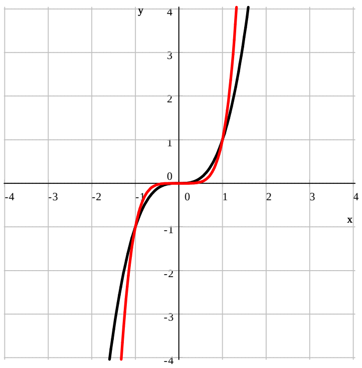
$x-$intercepts: $(1, 0)$ and $(5, 0)$

not one-to-one
no minimum
maximum: $(3, 2)$
strictly increasing on $(-\infty, 3)$
strictly decreasing on $(3, \infty)$
continuous on $\mathbb{R}$

3.) Monomials $f(x) = x^n$



n is even



n is odd

Case 1. If n is even

domain: $\mathbb{R}$ range: $[0, \infty)$

$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

not one-to-one
no maximum
minimum: $(0, 0)$
strictly decreasing on $(-\infty, 0)$
strictly increasing on $(0, \infty)$
continuous on $\mathbb{R}$

black graph: $f(x) = x^2$
red graph: $f(x) = x^4$

Case 2. If n is odd

domain: $\mathbb{R}$ range: $\mathbb{R}$

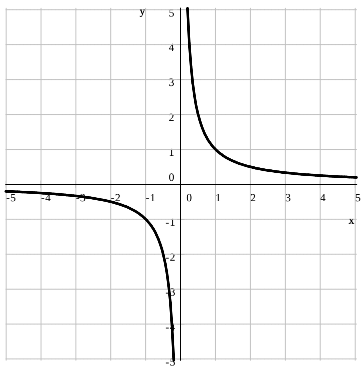
$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

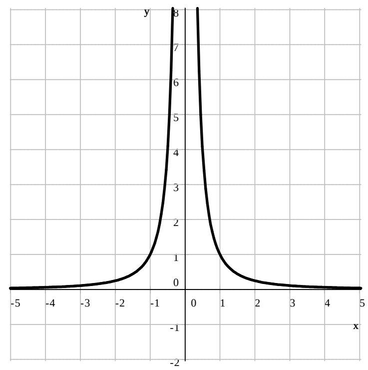
one-to-one
no minimum or maximum
strictly increasing on $\mathbb{R}$
continuous on $\mathbb{R}$

black graph: $f(x) = x^3$
red graph: $f(x) = x^5$

4.) The rational functions $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{1}{x^2}$



$f(x) = \dfrac{1}{x}$



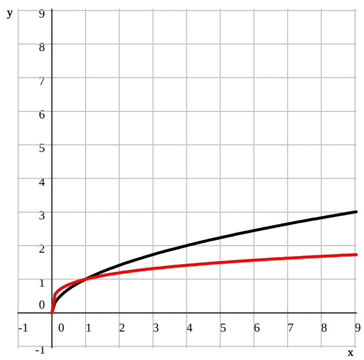
$g(x) = \dfrac{1}{x^2}$

domain: $\mathbb{R} \setminus \{0\}$ range: $\mathbb{R} \setminus \{0\}$
no $y-$intercept
no $x-$intercept
one-to-one
no maximum or minimum
strictly decreasing on $(-\infty, 0)$ and on $(0, \infty)$

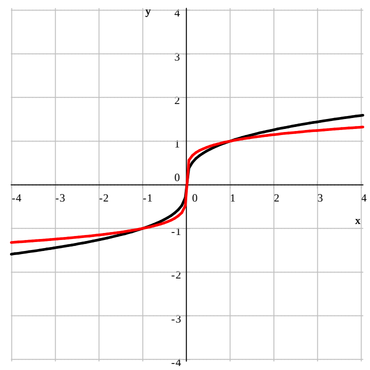
not continuous at $x = 0$
vertical asymptote: the line $x = 0$
horizontal asymptote: the line $y = 0$

domain: $\mathbb{R} \setminus \{0\}$ range: $(0, \infty)$
no $y-$intercept
no $x-$intercept
not one-to-one
no minimum or maximum
strictly increasing on $(-\infty, 0)$ and
strictly decreasing on $(0, \infty)$
not continuous at $x = 0$
vertical asymptote: the line $x = 0$
horizontal asymptote: the line $y = 0$

5.) Radical functions $f(x) = \sqrt[n]{x}$



n is even



n is odd

Case 1. If n is even

domain: $[0, \infty)$ range: $[0, \infty)$

$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

one-to-one
no maximum
minimum: $(0, 0)$
strictly increasing
continuous on $(0, \infty)$

black graph: $f(x) = \sqrt{x}$
red graph: $f(x) = \sqrt[4]{x}$

Case 2. If n is odd

domain: $\mathbb{R}$ range: $\mathbb{R}$

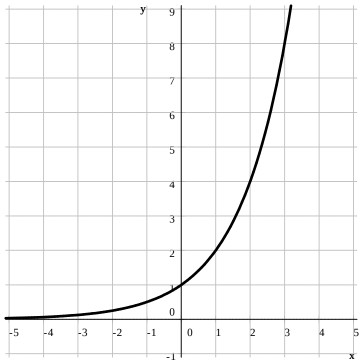
$y-$intercept: $(0, 0)$

$x-$intercept: $(0, 0)$

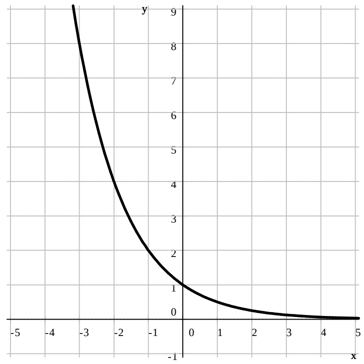
one-to-one
no minimum or maximum
strictly increasing
continuous on $\mathbb{R}$

black graph: $f(x) = \sqrt[3]{x}$
red graph: $f(x) = \sqrt[5]{x}$

6.) Exponential functions $f(x) = a^x$ where $a > 0$.



$a > 1$



$0 < a < 1$

Case 1. If $a > 1$

domain: $\mathbb{R}$ range: $(0, \infty)$

no $x-$intercepts

$y-$intercept: $(0, 1)$

one-to-one

no maximum or minimum

strictly increasing

horizontal asymptote: $y = 0$

continuous on $\mathbb{R}$

Case 2. If $0 < a < 1$

domain: $\mathbb{R}$ range: $(0, \infty)$

no $x-$intercepts

$y-$intercept: $(0, 1)$

one-to-one

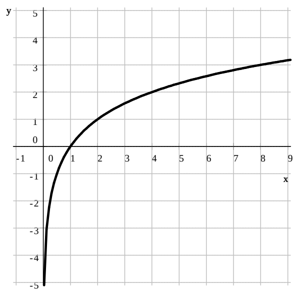
no maximum or minimum

strictly decreasing

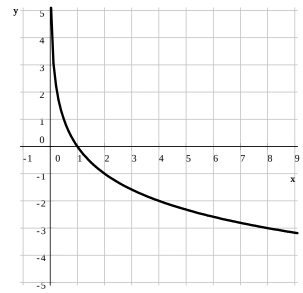
horizontal asymptote: $y = 0$

continuous on $\mathbb{R}$

7.) Logarithmic functions $f(x) = \log_a x$ where $a > 0$ and $a \neq 1$.



$a > 1$



$0 < a < 1$

Case 1. If $a > 1$

domain: $(0, \infty)$ range: $\mathbb{R}$

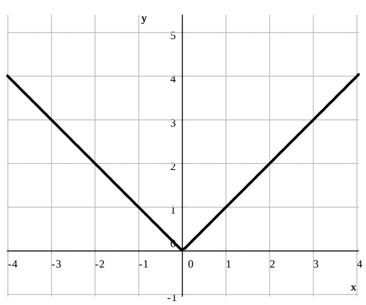
no $y-$intercepts
$x-$intercept: $(1, 0)$
one-to-one
no maximum or minimum
strictly increasing
vertical asymptote: $x = 0$
continuous on $(0, \infty)$

Case 2. If $0 < a < 1$

domain: $(0, \infty)$ range: $\mathbb{R}$

no $y-$intercepts
$x-$intercept: $(1, 0)$
one-to-one
no maximum or minimum
strictly decreasing
vertical asymptote: $x = 0$
continuous on $(0, \infty)$

8.) Absolute value function, $f(x) = |x|$



domain: $\mathbb{R}$ range: $[0, \infty)$
$x-$intercept: $(0, 0)$, $y-$intercept: $(0, 0)$
not one-to-one
no maximum
minimum: $(0, 0)$
strictly decreasing on $(\infty, 0)$ and strictly increasing on $(0, \infty)$
continuous on $\mathbb{R}$

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