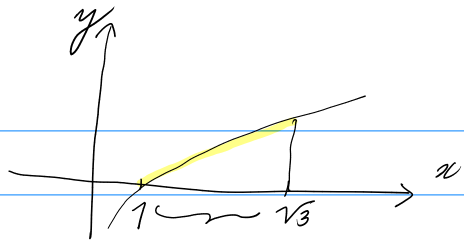


$$y = \underbrace{\ln x}_{f(x)}, \qquad 1 \le x \le \sqrt{3}$$

$$f'(x) = \frac{1}{x}$$

$$y = f(x)$$

$$l = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

$$l = \int_1^{\sqrt{3}} \sqrt{1 + \left(\frac{1}{x}\right)^2}\, dx =$$

$$= \int_1^{\sqrt{3}} \sqrt{\frac{x^2+1}{x^2}}\, dx = \int_1^{\sqrt{3}} \frac{\sqrt{x^2+1}}{x}\, dx =$$

$$x^2 + 1 = t^2$$

$$\boxed{2x\,dx = 2t\,dt}$$

$$x\,dx = t\,dt$$

$$x^2 = t^2 - 1$$

$$x = \sqrt{t^2-1}$$

$$t = \sqrt{x^2+1}$$

$$x = 1 \to t = \sqrt{2}$$

$$x = \sqrt{3} \to t = \sqrt{3+1} = 2.$$

$$= \int_1^{\sqrt{3}} \frac{\overbrace{\sqrt{x^2+1}}^{t}}{\underbrace{x^2}_{t^2-1}}\, \overbrace{x\,dx}^{t\,dt} =$$

$$= \int_{\sqrt{2}}^{2} \frac{t}{t^2-1} \cdot t\,dt = \int_{\sqrt{2}}^{2} \frac{t^2}{t^2-1}\, dt =$$

$$= \int_{\sqrt{2}}^{2} \frac{t^2-1+1}{t^2-1}\, dt = \int_{\sqrt{2}}^{2} \left(1 + \frac{1}{t^2-1}\right) dt =$$

$$= 2 - \sqrt{2} + \int_{\sqrt{2}}^{2} \frac{dt}{t^2-1}$$

$$\frac{1}{t^2-1} = \frac{1}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} = \frac{A(t+1) + B(t-1)}{(t-1)(t+1)}$$

$$t = 1: \quad 1 = 2A, \qquad A = \frac{1}{2}$$

$$t = -1: \quad 1 = -2B, \qquad B = -\frac{1}{2}.$$

$$= 2 - \sqrt{2} + \frac{1}{2}\int_{\sqrt{2}}^{2} \frac{dt}{t-1} - \frac{1}{2}\int_{\sqrt{2}}^{2} \frac{dt}{t+1} = 2 - \sqrt{2} + \frac{1}{2}\int_{\sqrt{2}}^{2} \frac{d(t-1)}{t-1} -$$

$$- \frac{1}{2}\int_{\sqrt{2}}^{2} \frac{d(t+1)}{t+1} = 2 - \sqrt{2} + \frac{1}{2} \ln|t-1| \Big|_{\sqrt{2}}^{2} - \frac{1}{2} \ln|t+1| \Big|_{\sqrt{2}}^{2} =$$

$$= 2 - \sqrt{2} + \frac{1}{2}\left(\underbrace{\ln 1}_{0} - \ln(\sqrt{2}-1)\right) - \frac{1}{2}\left(\ln 3 - \ln(\sqrt{2}+1)\right) =$$

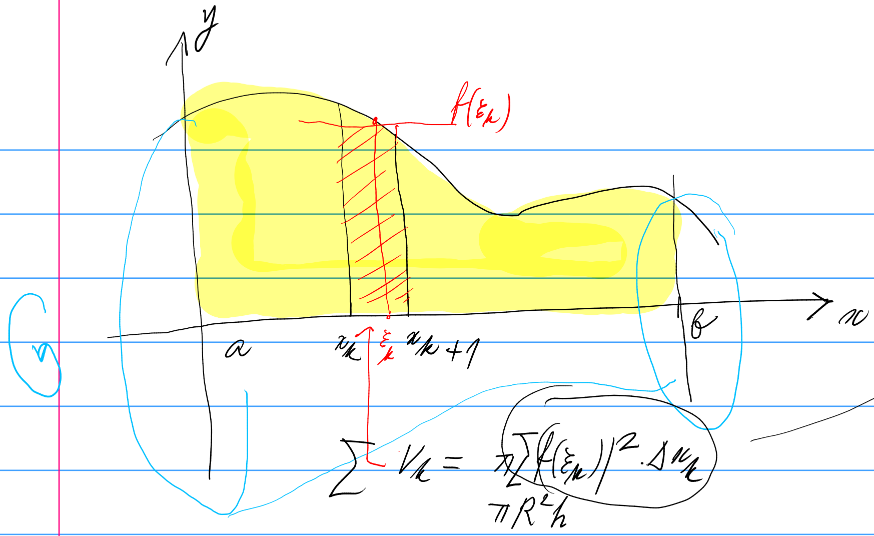
$$= 2 - \sqrt{2} - \frac{1}{2}\ln(\sqrt{2}-1) - \frac{1}{2}\ln\frac{3}{\sqrt{2}+1} =$$

$$2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$$

$$= 2 - \sqrt{2} - \frac{1}{2}\ln\frac{(\sqrt{2}-1)\cdot 3}{\sqrt{2}+1} = 2 - \sqrt{2} - \frac{1}{2}\ln\frac{(\sqrt{2}-1)^2 \cdot 3}{2-1} =$$

$$= 2 - \sqrt{2} - \frac{1}{2}\ln 3(3 - 2\sqrt{2}).$$

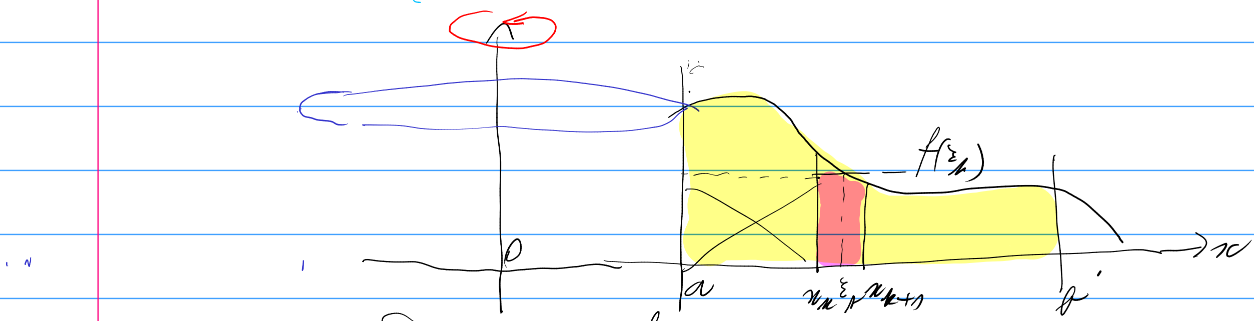
$f(x) \geq 0$, $a \leq x \leq b$
spoj.



$\sum V_k = \pi \sum (f(\xi_k))^2 \cdot \Delta x_k$

$\pi R^2 h$

$$V = \pi \int_a^b f^2(x)\,dx$$

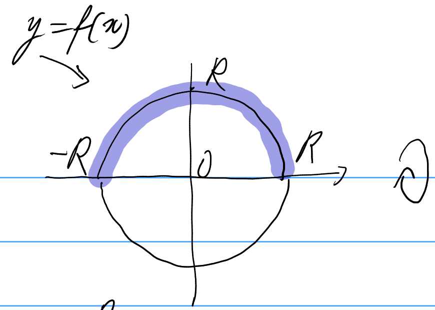


$$\boxed{V = 2\pi \int_a^b x f(x)\,dx}$$

$\pi x_{k+1}^2 f(\xi_k) - \pi x_k^2 f(\xi_k) =$

$= \pi (x_{k+1}^2 - x_k^2) f(\xi_k) = \pi (\underbrace{x_{k+1} + x_k}_{\approx 2\xi_k}) f(\xi_k) \underbrace{\Delta x_k}_{x_{k+1} - x_k}$

Koule R



$$x^2+y^2=R^2$$

$$y=\sqrt{R^2-x^2}$$

$$V=\pi\int_{-R}^{R}\underbrace{\left(\sqrt{R^2-x^2}\right)^2}_{\text{sudá, } (-R,R)}dx = 2\cdot\pi\int_0^R\left(\sqrt{R^2-x^2}\right)^2dx = 2\pi\int_0^R\left(R^2-x^2\right)dx$$

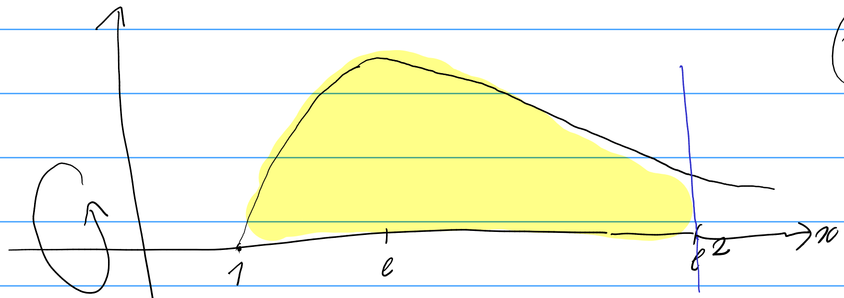
$$= 2\pi\left(R^2\cdot R - \frac{x^3}{3}\Big|_0^R\right) = 2\pi\underbrace{\left(R^3-\frac{R^3}{3}\right)}_{\frac{2}{3}R^3} = \frac{4\pi}{3}R^3.$$

$y=\dfrac{\ln x}{x}$, $\quad y=0$, $\quad x=e^2$ $\qquad$ оху x

$$\left(\frac{\ln x}{x}\right)' = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$$

кр. т. $x=e$

лок. max.



$\int e^{ax} p(x)\,dx$

$$V=\pi\int_1^{e^2}\left(\frac{\ln x}{x}\right)^2 dx = \left|\begin{array}{l} \ln x = t \quad x=e^t \\ dx = e^t dt \\ x=1 \to t=\ln 1 = 0 \\ x=e^2 \to t=\ln e^2 = 2 \end{array}\right| \qquad \left|\frac{1}{x}\right. \ln x$$

$$= \pi\int_0^2 \left(\frac{t}{e^t}\right)^2 e^t\,dt = \pi\int_0^2 \frac{t^2}{(e^t)^2} e^t\,dt = \pi\int_0^2 t^2 e^{-t}\,dt =$$

р.р.

$$= \left|\begin{array}{l} u=t^2, \; du=2t\,dt \\ dv=e^{-t}dt, \; v=\int e^{-t}dt=-e^{-t} \end{array}\right|$$

$$= \pi\left(-t^2e^{-t}\Big|_0^2 - \int_0^2(-e^{-t})\,2t\,dt\right) = \pi\left(-4e^{-2}+2\int_0^2 t e^{-t}\,dt\right) =$$

$$= -4\pi e^{-2} - 2\pi\int_0^2 t\,d(e^{-t}) = -4\pi e^{-2} - 2\pi\left(te^{-t}\Big|_0^2 - \int_0^2 e^{-t}\,dt\right) =$$

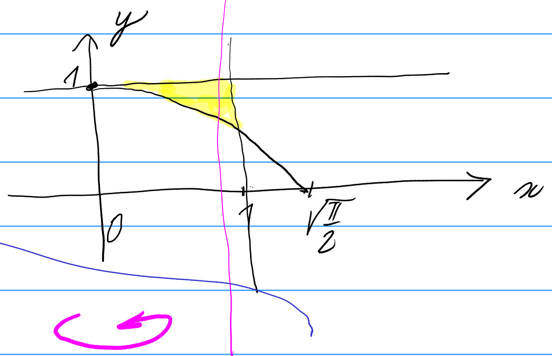
$\int u\,dv$

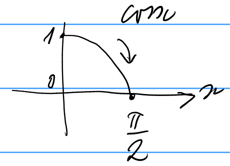
$$= -4\pi e^{-2} - 2\pi\left(2e^{-2} + \int_0^2 d(e^{-t})\right) = -4\pi e^{-2} - 4\pi e^{-2} -$$

$$-2\pi\, e^{-t}\Big|_0^2 = -8\pi e^{-2} - 2\pi e^{-2} + 2\pi = 2\pi - 10\pi e^{-2} = 2\pi(1-5e^{-2}).$$

$y = \cos(x^2)$, $y = 1$, $x = 1$, осу y

$V = ?$





$x^2 = \frac{\pi}{2}$

$x = \sqrt{\frac{\pi}{2}} > 1$

$$V = 2\pi \cdot \int_0^1 x \cdot (1 - \cos(x^2))\,dx = 2\pi \int_0^1 x\,dx - 2\pi \int_0^1 x \cos(x^2)\,dx =$$

$$= 2\pi \frac{x^2}{2}\Big|_0^1 - \pi \cdot \int_0^1 \cos(x^2)\,d(x^2)$$

$d(x^2) = 2x\,dx$

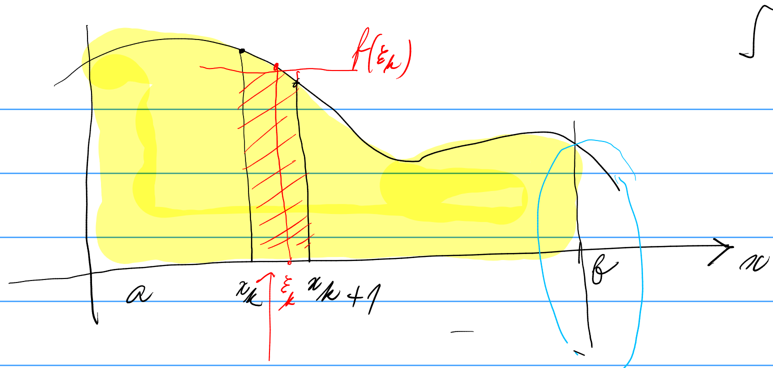
$$= \pi - \pi \sin(x^2)\Big|_0^1 = \pi - \pi \sin 1 =$$

$x^2 = s$, $2x\,dx = ds$

$\int \cos s\,ds$

$$= \pi \cdot (1 - \sin 1).$$

$S = ?$



$$S = 2\pi \int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$$

$y = f(x)$

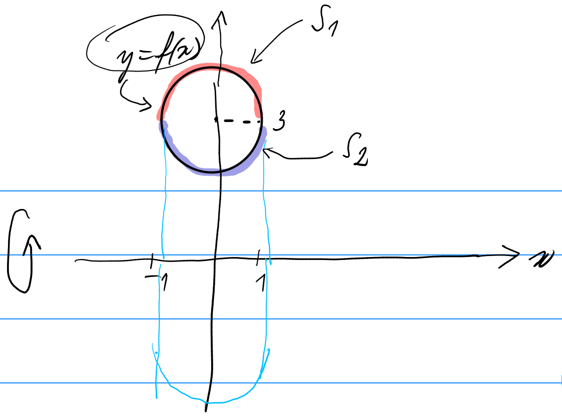
$x = x(t)$
$y = y(t)$

$$S = 2\pi \int_\alpha^\beta y(t)\sqrt{(x'(t))^2+(y'(t))^2}\,dt$$

$$x^2+(y-3)^2=1$$

$$S=?$$

$$S=S_1+S_2$$



$(0;3)$

$$y-3=\pm\sqrt{1-x^2}$$

$$y=3\pm\sqrt{1-x^2},\quad -1\le x\le 1$$

$$S_1:\quad y=3+\sqrt{1-x^2}=f(x)$$

$$(f'(x))=\left(3+\sqrt{1-x^2}\right)'=$$
$$=\frac{-2x}{2\sqrt{1-x^2}}=-\frac{x}{\sqrt{1-x^2}}$$

$$S_1=2\pi\int_{-1}^{1}\left(3+\sqrt{1-x^2}\right)\cdot\sqrt{1+\left(\frac{-x}{\sqrt{1-x^2}}\right)^2}\,dx=$$

$$=2\pi\int_{-1}^{1}\left(3+\sqrt{1-x^2}\right)\sqrt{\underbrace{1+\frac{x^2}{1-x^2}}_{\frac{1-\cancel{x^2}+\cancel{x^2}}{1-x^2}}}\,dx=2\pi\int_{-1}^{1}\underbrace{\left(3+\sqrt{1-x^2}\right)\cdot\frac{1}{\sqrt{1-x^2}}}_{\text{чёт}, (-1,1)}dx=$$

$$=2\cdot 2\pi\int_{0}^{1}\left(3+\sqrt{1-x^2}\right)\frac{1}{\sqrt{1-x^2}}\,dx=4\pi\int_0^1\left(\frac{3}{\sqrt{1-x^2}}+1\right)dx=$$

$$=12\pi\underbrace{\int_0^1\frac{dx}{\sqrt{1-x^2}}}_{\arcsin x}+4\pi=12\pi\cdot\arcsin x\Big|_0^1+4\pi=$$

$$=12\pi(\underbrace{\arcsin 1}_{\frac{\pi}{2}}-\underbrace{\arcsin 0}_{0})+4\pi=12\pi\cdot\frac{\pi}{2}+4\pi=\underbrace{6\pi^2+4\pi}_{S_1}$$

$$S_2=2\pi\int_{-1}^{1}\left(3-\sqrt{1-x^2}\right)\cdot\underbrace{\sqrt{1+\left(\frac{x}{\sqrt{1-x^2}}\right)^2}}_{\frac{1}{\sqrt{1-x^2}}}\,dx=2\pi\int_{-1}^{1}\left(3-\sqrt{1-x^2}\right)\cdot\frac{1}{\sqrt{1-x^2}}\,dx$$

$$=4\pi\cdot\int_0^1\left(\frac{3}{\sqrt{1-x^2}}-1\right)dx=\underbrace{12\pi\int_0^1\frac{dx}{\sqrt{1-x^2}}}_{6\pi^2}-4\pi=\underbrace{6\pi^2-4\pi}_{S_2}$$

$$S=\underbrace{6\pi^2+\cancel{4\pi}}_{S_1}+\underbrace{6\pi^2-\cancel{4\pi}}_{S_2}=12\pi^2.$$

$a>0$ $\qquad r = a(1+\cos\varphi)$, $\qquad 0 \le \varphi \le 2\pi$

$r = r(\varphi)$ $\qquad x = x(t),\ y = y(t)$

$$\begin{cases} x(\varphi) = r(\varphi)\cos\varphi \\ y(\varphi) = r(\varphi)\sin\varphi \end{cases} \longrightarrow S = 2\pi \int_\alpha^\beta y(t)\sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

$$\sqrt{(x'(\varphi))^2 + (y'(\varphi))^2} = \sqrt{(r'(\varphi)\cos\varphi - r(\varphi)\sin\varphi)^2 + (r'(\varphi)\sin\varphi + r(\varphi)\cos\varphi)^2} =$$

$$= \sqrt{r'^2\cos^2\varphi + r^2\sin^2\varphi - \cancel{2rr'\cos\varphi\sin\varphi} + r'^2\sin^2\varphi + r^2\cos^2\varphi + \cancel{2r'r\sin\varphi\cos\varphi}}$$

$$= \sqrt{r'^2 + r^2}$$

$$S = 2\pi \int_\alpha^\beta r(\varphi)\sin\varphi \cdot \sqrt{(r'(\varphi))^2 + (r(\varphi))^2}\, d\varphi.$$

$r(\varphi) = a(1+\cos\varphi)$

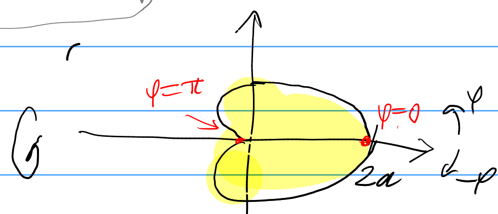
$r'(\varphi) = -a\sin\varphi$

$$S = 2\pi \int_0^\pi a(1+\cos\varphi)\cdot\sin\varphi\sqrt{a^2\sin^2\varphi + a^2(1+\cos\varphi)^2}\, d\varphi =$$

$a^2\sin^2\varphi + a^2 + a^2\cos^2\varphi + 2a^2\cos\varphi$

$$= 2\pi a \int_0^\pi (1+\cos\varphi)\sin\varphi\sqrt{2a^2 + 2a^2\cos\varphi}\, d\varphi$$

$\sqrt{2a^2(1+\cos\varphi)}$

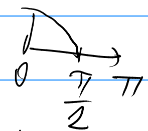


$$= 2\pi a \int_0^\pi 2\cos^2\frac{\varphi}{2}\cdot 2\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\cdot\sqrt{2a^2\cdot 2\cos^2\frac{\varphi}{2}}\, d\varphi$$

$1 + \cos\varphi = 2\cos^2\frac{\varphi}{2}$

$4a^2\cos^2\frac{\varphi}{2}$

$$= 16a^2\pi \int_0^\pi \cos^3\frac{\varphi}{2}\sin\frac{\varphi}{2}\cdot\cos\frac{\varphi}{2}\, d\varphi$$



$$= 16a^2\pi \int_0^\pi \cos^4\frac{\varphi}{2}\sin\frac{\varphi}{2}\, d\varphi = -32a^2\pi \int_0^\pi \cos^4\frac{\varphi}{2}\, d\left(\cos\frac{\varphi}{2}\right)$$

$d\left(\cos\frac{\varphi}{2}\right) = -\frac{1}{2}\sin\frac{\varphi}{2}\, d\varphi$

$$= -32a^2\pi \left.\frac{\cos^5\frac{\varphi}{2}}{5}\right|_0^\pi = -\frac{32a^2\pi}{5}\left(\underbrace{\cos^5\frac{\pi}{2}}_{0} - \underbrace{\cos^5 0}_{1}\right) =$$

$\cos\frac{\varphi}{2} = s$

$\to \int s^4\, ds$

$$= \frac{32}{5}a^2\pi.$$