Geometrie 2

Tabule ze cvičení

Obsah

Afinní struktura 1
Afinní souřadnice 5
Afinní zobrazení 11
Vyjádření podprostorů 21
Vzájemné polohy 26
Příčky 33
Omezené podprostory, těžiště 40
Vzdálenosti 51
Obsahy a objemy 67
Odchylky 74

Poslední aktualizace 14. ledna 2021.
Mírně editované tabule, které vznikly během distanční výuky na podzim 2020.
Obsah a organizace materiálu jsou založeny na souboru `cviceni.pdf`.

CVIČENÍ (1) co je a co není AFINNÍ PROSTOR?

obecný postřeh:

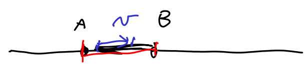
$V = \vec{a}$

- zejména musíme vidět ZAMĚŘENÍ = vekt. prostor
- teprve pak můžeme kontrolovat ostatní požadavky ...

(a) $\{A, B\} \subset a$ $A \xrightarrow{v} B$, $v = \vec{AB}$ $\rightsquigarrow$ "V" $= \{\vec{AB}\}$

aby byla množina $\{A, B\}$ af. (pod)prostor,
musí být $\{\vec{AB}\}$ vekt. (pod)prostor

... což NENÍ!

(b) 

$a = \mathbb{R}$

$\mathcal{B}$ = interval $(A, B) \subset \mathbb{R}$

aby byla $\mathcal{B}$ af. (pod)pr.,
musí být "$\vec{\mathcal{B}}$" $= \{\vec{CD} \mid C, D \in \mathcal{B}\}$ vekt. (pod)pr.

... což NENÍ!

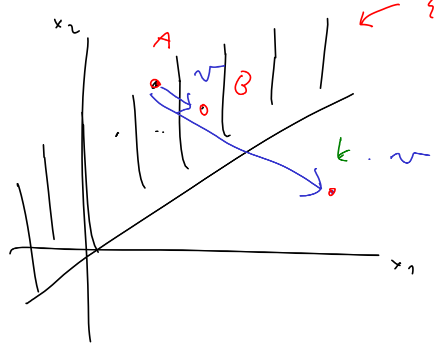
PŘIPOMENUTÍ:

U ... vekt. podpr. ve V

tj. $u, v \in U \Rightarrow u + v \in U$

$u \in U,\ k \in \mathbb{R} \Rightarrow k \cdot u \in U$

(c) $\{(x_1, x_2) \mid x_2 \geq x_1\} \subset \mathbb{R}^2$



... stejný problém

$\Rightarrow$ **NENÍ**

(d) $\{(x_1, x_2, x_3) \mid \boxed{x_3 = 1 + x_2 + x_3}\} \subset \underline{\mathbb{R}^3}$

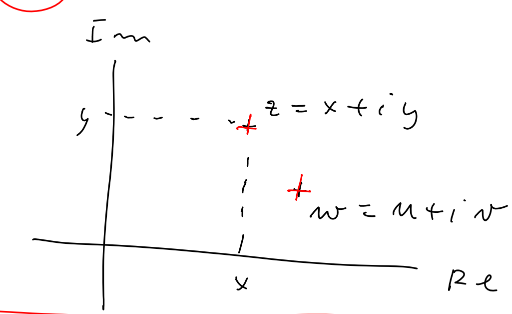
(soustava) LIN. ROVNIC **!**

$\Rightarrow$ **ANO** (dim 2)

$3 - 1$

(e) $\mathbb{C}$ chápáno nad $\underline{\underline{\mathbb{R}}}$

a =



$\mathbb{C} \cong \mathbb{R}^2$

$+$ ... $+$ ← po složkách

sčítání kompl. čísel atd.

(*)

$z + w = (x + u) + i(y + v)$

... odpovídá ✓

$\underset{\mathbb{C}}{\underset{\parallel}{a}} \times \underset{\mathbb{C}}{\underset{\parallel}{a}} \to \underset{\mathbb{R}^2}{\underset{\parallel}{V}}$ ... obyč. rozdíl

$\underbrace{(z, w)} \mapsto w - z$ (dim = 2)

ANO, splňuje vše, co má (díky ztotožnění (*)) !

(f) soust. lin. rovnic nad

$\underbrace{\mathbb{R} \supset \mathbb{Q}}_{\text{těleso (pole)}} \supset \mathbb{Z}$ jenom "lepší" $\underline{\underline{\text{okruh}}}$

- nad TĚLESEM jistě **ANO** (typický příklad)
- nad OKRUHEM **ANO** pouze v případě, že soust. má $\boxed{1}$ řešení!! (triviální případ dim 0)
- nyní x_3 ... $\underline{\underline{\text{lib}}}$

  $\Downarrow$

  $\underbrace{\text{soustava má}}$ $\begin{cases} 0 \text{ řešení} \\ \boxed{\infty} \text{ řešení} \end{cases}$

... **NENÍ** AF. PROSTOR !

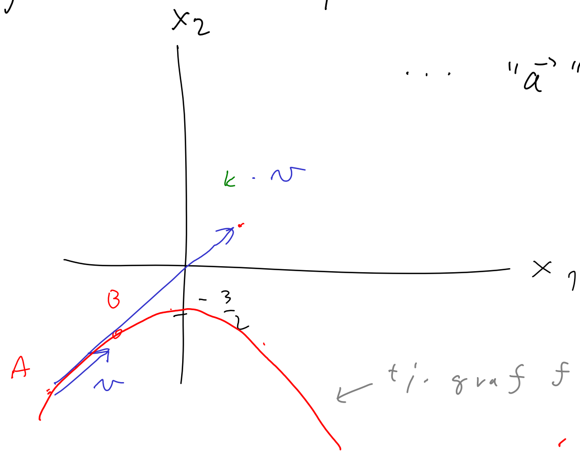
(konkrétní řešení ... $x_1 = -1 - 10\ell$, $x_2 = -1 + 5\ell$, $x_3 = k$, $x_4 = -1 + 4\ell$ | $k, \ell \in \underline{\underline{\mathbb{Z}}}$)

nad OKRUHEM více práce ... viz DIOFANTICKÉ ROVNICE

(g) (*) $\boxed{x_1^2 + 2x_2 = -3}$ pro x_1, x_2 ...

jistě NE pro stand. strukturu na $\mathbb{R}^2$

... "$\vec{a}$" není vekt. prostor



tj. graf fce $x_2 = \frac{-3 - x_1^2}{2}$

a = řešení (*)

Lze uvažovat jiné přiřazení $a \times a \to V$

v duchu "UMĚLÉHO PŘÍKLADU" (b)

.. prezentace str. 20

CVIČENÍ (3) SOUŘADNICE

(a)

$x_1, x_2, x_3, x_4 \in \mathbb{R}$

$$a = \left\{ \begin{array}{l} x_1 + 2x_2 \qquad 0 = -3 \\ 0 \quad 4x_2 - 5x_4 = 1 \end{array} \right\}$$

EKVIV. SOUSTAVA
(lib. lin. kombinace)

$$= \left\{ \begin{array}{l} 2x_1 + 4x_2 = -6 \\ x_1 + 6x_2 - 5x_4 = -2 \end{array} \right\} = \dots$$

ŘEŠENÍ

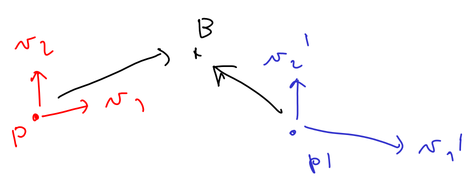
$$= \left\{ \boxed{\begin{array}{l} x_1 = -3 - 2t \\ x_2 = t \\ x_3 = s \\ x_4 = -\frac{1}{5} + \frac{4}{5}t \end{array}} \;\middle|\; t, s \in \mathbb{R} \right\} =$$

dim

$\boxed{\underbrace{4}_{\text{neznámé}} - \underbrace{2}_{\text{nezáv. rovnice}} = 2}$

$$= \left\{ \boxed{\begin{array}{l} x_1 = -7/2 - 5/2\,k \\ x_2 = 1/4 + 5/4\,k \\ x_3 = l \\ x_4 = k \end{array}} \;\middle|\; k, l \in \mathbb{R} \right\} = \dots$$

$$\left(x_1 = -3 - 2 x_2 = -3 - \frac{1 + 5k}{2} = -\frac{7}{2} - \frac{5}{2}k \right)$$



(b)

$$a \Rightarrow \begin{bmatrix} x_1 = -1 \\ x_2 = -1 \\ x_3 = -1 \\ x_4 = -1 \end{bmatrix}$$ je řešení?

$-1-2=-3$ ✓

$-4+5=1$ ✓

$t, s \overset{?}{\in} \mathbb{R}$ tak, aby

$$\boxed{\begin{aligned} -1 &= -3-2t \; ✓ \\ -1 &= t \\ -1 &= s \\ -1 &= -11/5 + 4/5\,t \; ✓ \end{aligned}}$$

$\underline{\underline{t = s = -1}}$

(soustava lin. rovnic)

$k, l \overset{?}{\in} \mathbb{R}$ tak, aby

$$\boxed{\begin{aligned} -1 &= -7/2 - 5/2\,k \; ✓ \\ -1 &= 1/4 + 5/4\,k \; ✓ \\ -1 &= l \\ -1 &= k \end{aligned}}$$

$\underline{\underline{l = k = -1}}$

(soustava lin. rovnic)

(c) OBECNÝ PŘECHOD

$$\begin{pmatrix} -3-2t \\ t \\ s \\ -\frac{1}{5}+\frac{4}{5}t \end{pmatrix} \stackrel{!}{=} \begin{pmatrix} -7/2-5/2\,k \\ 1/4+5/4\,k \\ l \\ k \end{pmatrix}$$

$$\boxed{\begin{aligned} t &= 1/4 + 5/4\,k \\ s &= l \end{aligned}}$$

[kontrola na zbylých ...] ✓

$$\Updownarrow$$

$$\boxed{\begin{aligned} l &= s \\ k &= -1/5 + \frac{4}{5}t \end{aligned}}$$

[kontrola ...] ✓

pomocí matic:

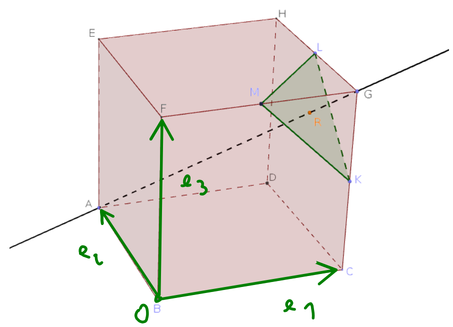
$$\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 5/4 \end{pmatrix} \cdot \begin{pmatrix} l \\ k \end{pmatrix} + \begin{pmatrix} 0 \\ 1/4 \end{pmatrix}$$

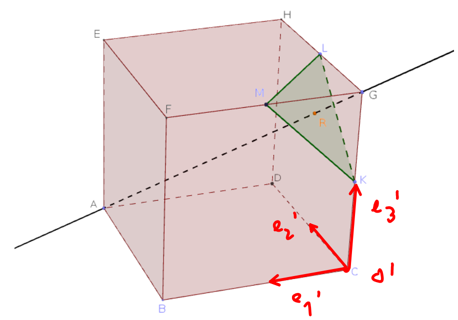
$$\Updownarrow$$

$$\begin{pmatrix} l \\ k \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{4}{5} \end{pmatrix} \cdot \begin{pmatrix} s \\ t \end{pmatrix} + \begin{pmatrix} 0 \\ -1/5 \end{pmatrix}$$

inverzní matice ...

(4) SOUŘADNICE





volba 1

počátek = B

$e_1 = \vec{BC}, \quad e_2 = \vec{BA}, \quad e_3 = \vec{BF}$

souřadnice

$B = [0, 0, 0]$

$C = [1, 0, 0]$

$\vdots$

$A = [0, 1, 0]$

$G = [1, 0, 1]$ ✓ . . .

$\vdots$

$K = [1, 0, 1/2]$ . . .

$L = [1, 1/2, 1]$ . . .

$M = [1/2, 0, 1]$ ✓ . . .

volba 2

počátek' = C

$e_1' = \frac{1}{2}\vec{CB}, \quad e_2' = \frac{1}{2}\vec{CG}, \quad e_3' = \frac{1}{2}\vec{CD}$

souřadnice

$A' = C + 2e_1' + 2e_2' + 0 \cdot e_3'$

$A' = [2, 2, 0]$

$G' = [0, 0, 2]$

$K' = [0, 0, 1]$

$L' = [0, 1, 2]$

$M' = [1, 0, 2]$ ✓

obecný přechod ?

$X = [x_1, x_2, x_3] \rightsquigarrow [x_1', x_2', x_3']$

... určeno vztahem mezi souř. repéry.

(*)

$$\boxed{\begin{array}{ll} B = & C + 2e_1' + 0e_2' + 0e_3' \\ \hline e_1 = & -2e_1' \\ e_2 = & \quad +2e_2' \\ e_3 = & \quad\quad +2e_3' \end{array}}$$

$[2,0,0]$
$(-2,0,0)$
$(0,2,0)$
$(0,0,2)$

obecný bod ...

$X = [x_1, x_2, x_3] \rightsquigarrow [x_1', x_2', x_3']$

$$X = B + x_1 e_1 + x_2 e_2 + x_3 e_3 \rightsquigarrow C + x_1' e_1' + x_2' e_2' + x_3' e_3'$$

... po dosazení (*) a úpravě:

$$X = (C + 2e_1' + 0e_2' + 0e_3') +$$
$$+ x_1(-2e_1') + x_2(+2e_2') + x_3(+2e_3')$$
$$= C + \underbrace{(2-2x_1)}_{x_1'} e_1' + \underbrace{(0+2x_2)}_{x_2'} e_2' + \underbrace{(0+2x_3)}_{x_3'} e_3'$$

$$\boxed{\begin{array}{l} x_1' = 2 - 2x_1 \\ x_2' = 0 + 2x_2 \\ x_3' = 0 + 2x_3 \end{array}}$$

kontrola pro K:

$0 \overset{?}{=} 2 - 2 \cdot 1$ ✓

$0 = 0 + 2 \cdot 0$ ✓

$1 = 0 + 2 \cdot 1/2$ ✓

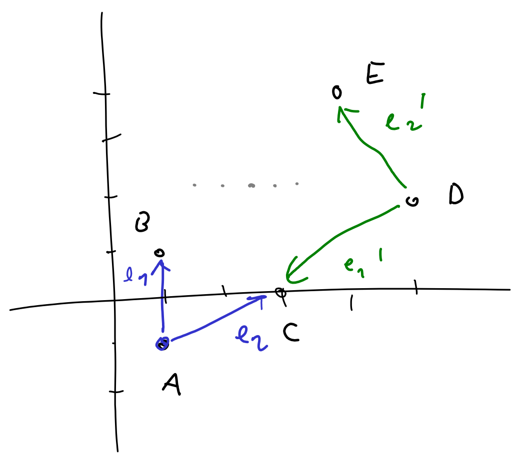
TOTÉŽ pomocí matic:

$$\boxed{\begin{pmatrix} -2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}}$$

... lze skládat přímo z (*)

---

(5)



Nic moc NOVÉHO

(6) ZOBRAZENÍ

(b) stejnolehlosti = PODOBNOSTI
$\Longrightarrow$ AFINNÍ

(c) symetrie krychle = SHODNOSTI
$\Longrightarrow$ AFINNÍ

... vzpomínáme z konstrukční geom.
(kolinearnost + poměry ~~xx~~ + rovnoběžnost)

$$f : \underset{a}{\mathbb{R}} \to \underset{a'}{\mathbb{R}}$$

(a) $k = 0 \; .. \; f(x) = 1$ ANO

$k = 1 \; .. \; f(x) = x + 1$ ANO

$k = 2 \; .. \; f(x) = x^2 + 1$ NE

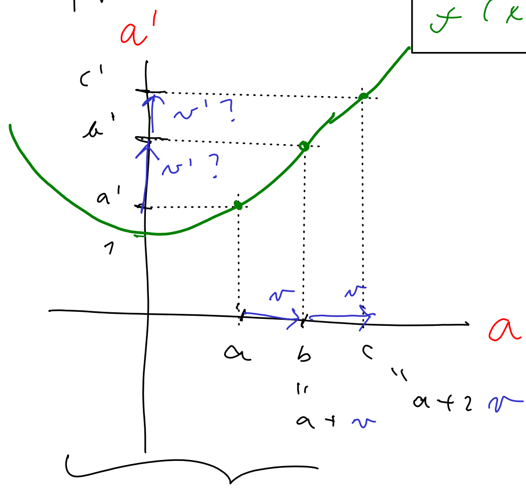
$a = \mathbb{R}^1$ ... stand. str. = obyč. rozdíl

OBECNÝ POSTŘEH:

- zejména musíme vidět indukované zobrazení mezi **VEKTORY**
- teprve potom můžeme kontrolovat zbytek ...

Např.

$\boxed{f(x) = x^2 + 1}$



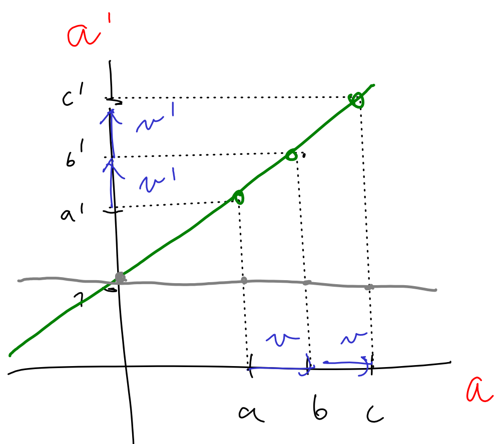
$\vec{ab} = \vec{bc}$

$\vec{a'b'} \neq \vec{b'c'}$!

nevymyslíme zobr. $\vec{f}: V \to V$

$\Rightarrow$ NENÍ AFINNÍ !

Pro $\boxed{f(x) = k x + l}$ ... lib. $k, l \in \mathbb{R}$



← spec. $f(x) = 0 \cdot x + 1$

$\vec{f}(x) = 0$

$\vec{ab} = \vec{bc}$ ... v

$\vec{a'b'} \overset{!}{=} \vec{b'c'}$ ... $v' = k v$

Tedy pro $\boxed{f(x) = kx + l}$ $\quad f : a \to a$

1. indukt. lin. zobrazení je $\boxed{\vec{f}(x) = kx}$ $\quad \vec{f} : \vec{a} \to \vec{a}$

2. a skutečně platí

- $\vec{f}(\vec{ab}) = \vec{f}(b-a) = k(b-a) = kb - ka$
- $\overrightarrow{f(a)f(b)} = (kb + \cancel{l}) - (ka + \cancel{l}) = kb - ka$

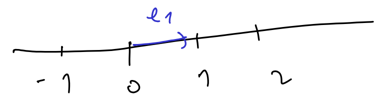
$f(a) = (ka + l)$, $f(b) = (kb + l)$

pro lib. $a, b \in \mathbb{R}^a$

LINEÁRNÍ FUNKCE $f : \mathbb{R} \to \mathbb{R}$ JE AFINNÍ ZOBR.

(7)

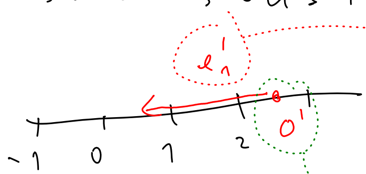
(a) O = počátek, "$e_1 = \overrightarrow{O1}$"



souř. vyjádření = původní předpis

$$\boxed{f(x) = k\,x + l}$$

(jiná souř. soustava ⇝ jiný předpis



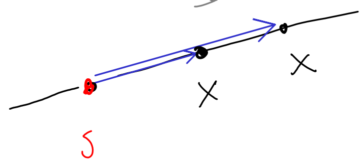
$$\boxed{f(x) = \square\, x + \square}$$
)

(b) **stejnolehlost**

... určená středem S

a koeficientem $k \in \mathbb{R}$

$(k \doteq +1{,}6)$



(pevný bod)

$$\boxed{\overrightarrow{SX'} = k \cdot \overrightarrow{SX}} \qquad (*)$$

Pro obecné vyjádření obrazu X' stačí přepsat definující rovnost $(*)$:

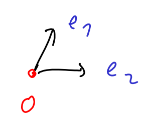
$$\overrightarrow{SX'} = k \cdot \overrightarrow{SX}$$

$$X' - S = k\,(X - S)$$

$$X' = k(X - S) + S \overset{!}{=} \underline{\underline{k}}\,X \underbrace{-\,kS + S}$$

Nezávisle na **dimenzi** a volbě **báze**

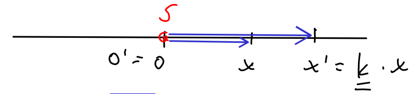
$$(**) \qquad \boxed{X' = \boxed{k}\, X + \boxed{-kS + S}}$$



lin. část $(k \cdot id)$

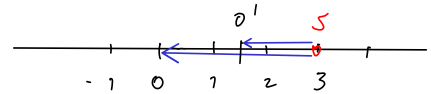
obraz počátku

Např. dim 1



- počátek = S ⇝ $f(x) = \boxed{k}\, x + \boxed{0}$

- počátek ≠ S . . .



$\left.\begin{array}{l} S \doteq 3 \\ k \doteq 1/2 \end{array}\right\} \; 0' \doteq \frac{3}{2}$

⇝ $x' \doteq \boxed{\frac{1}{2}}\, x + \boxed{\frac{3}{2}}$

obecně pro $\left.\begin{array}{l} S = s \\ \text{koef} = k \end{array}\right\} \; 0' = (1-k)\, s$

⇝ $x' = \boxed{k}\, x + \boxed{(1-k)\, s}$

. . . souhlasí s ob. vyjádřením (**) ✓

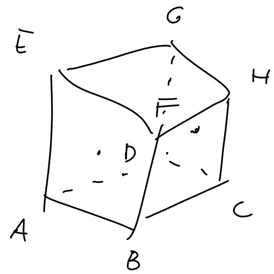
. . . viz též př. (7a)

$$\boxed{f(x) = k\, x + l}$$

⇝ STEJNOLEHLOST
s koef $= k$
a středem $= \dfrac{l}{1-k}$

$f(x) = x \iff x = \dfrac{l}{1-k}$ ✓

(c) symetrie krychle



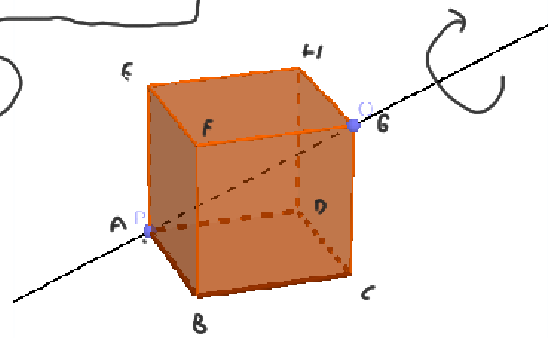
- střed. sym.
- osové sym
- obecnější rotace
  (úhly 90°, 120°)
- sym. podle rovin

Pro některé volby z př. (4)
a některé symetrie chceme:

$$\overset{?}{\rightsquigarrow} \qquad \underset{X'}{\begin{pmatrix} \vdots \end{pmatrix}} = \underset{\vec{f}}{\boxed{\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}}} \cdot \underset{X}{\begin{pmatrix} \vdots \end{pmatrix}} + \underset{O'}{\boxed{\begin{pmatrix} \vdots \end{pmatrix}}}$$

cvič. (7)

(C)

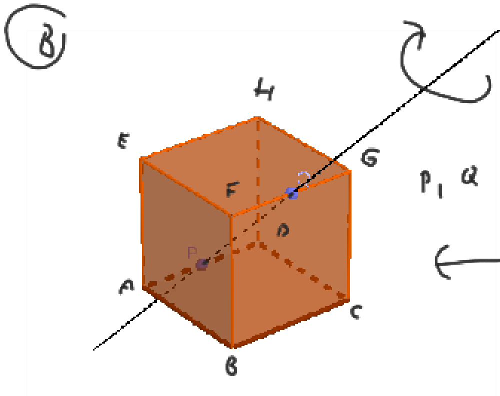


možné úhly: ~~180°~~ ±120°

možné permutace vrcholů:

A B C D E F G H
A E F B D H G C

(B)

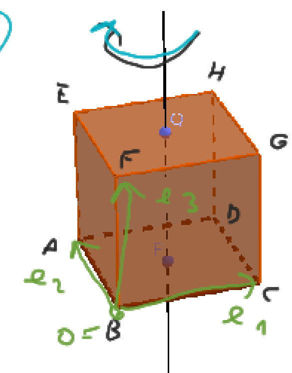


P, Q ... středy stran

možné úhly: 180°

A B C D E F G H
D H E A C G F B

(A)



P, Q středy stěn

možné úhly: 180° 90°

A B C D E F G H
A A B C H E F G

ANALYTICKY vzhledem k souř. soust ZELENÉ

$e_1 = \vec{BC} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$e_1' = \vec{B'C'} = \vec{AB} = -e_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$

$e_2' \dots = e_1$

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

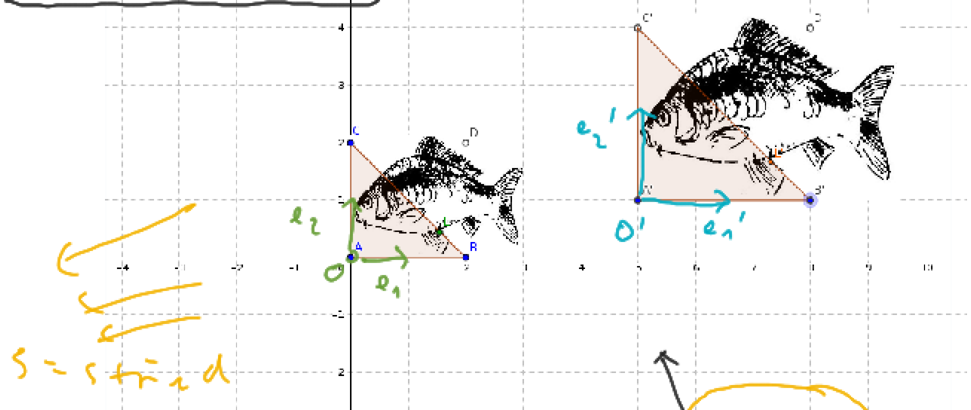
X' $\quad \vec{f} \quad$ $e_3' = \dots = e_3$ $\quad$ X $\quad$ $O' = B' = A$

OBECNĚ: e_1', e_2', e_3' obrazy bázových vektorů!

$$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

X' $\quad \vec{f} \quad$ X $\quad$ O' .. obraz počátku...!

cvič. (7)–(8)   AF. zobr. v rovině zadáno třemi body:



$k = \frac{3}{2}$

STEJNOLEHLOST

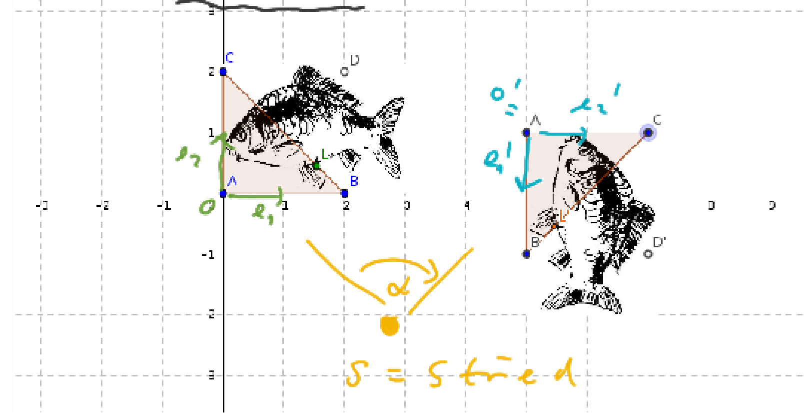
$[0,0] \mapsto [5,1]$

$[2,0] \mapsto [8,1]$

$[0,2] \mapsto [5,4]$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \left(\begin{array}{c|c} 3/2 & 0 \\ 0 & 3/2 \end{array}\right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$e_1' = k e_1$ $e_2' = k e_2$ o'



OTÁČENÍ ...

$[0,0] \mapsto [5,1]$

$[2,0] \mapsto [5,-1]$

$[0,2] \mapsto [7,1]$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \left(\begin{array}{c|c} 0 & 1 \\ -1 & 0 \end{array}\right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$e_1' = -e_2$ $e_2' = e_1$ o'

S = STŘED $\Rightarrow$ $S' \overset{!}{=} S$ $\Leftrightarrow$ $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \boxed{\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \cdot \\ \cdot \end{pmatrix} \overset{!}{=} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}$ SOUSTAVA LIN. ROVNIC

POZN... STEJNOLEHLOST OBECNĚ (minule)

$$\boxed{\vec{SX'} = k \cdot \vec{SX}}$$

$k = \frac{3}{2}$ $\quad S = ?$

$$X' - S = k(X - S)$$

$$X' = k(X - S) + S = \underline{\underline{k}}X - kS + S$$

$$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$(1-k)S = -\frac{1}{2}S = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

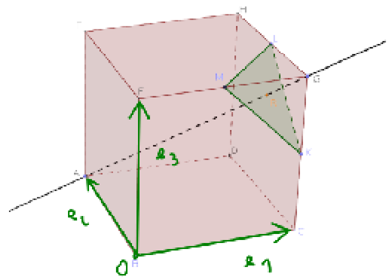
$$\iff S = \underline{\underline{\begin{bmatrix} -10 \\ -2 \end{bmatrix}}}$$

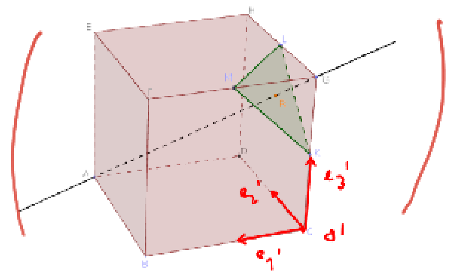
kontrola

$$S' = \begin{pmatrix} 3/2 & 0 \\ 0 & 3/2 \end{pmatrix} \cdot \begin{pmatrix} -10 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} -10 \\ -2 \end{pmatrix} = S \ \checkmark$$

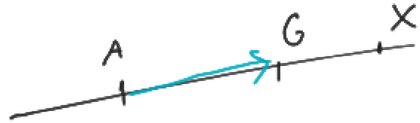
(A)

$A = [0, 1, 0]$
$G = [1, 0, 1]$
$\vdots$
$K = [1, 0, 1/2]$
$L = [1, 1/2, 1]$
$M = [1/2, 0, 1]$





$p = AG$
$\alpha = KLM$



$X = A + t\,\overrightarrow{AG}$

$\overrightarrow{AG} = (1, -1, 1)$

a) parametricky

$$p = \{A + t\,\overrightarrow{AG} \mid t \in \mathbb{R}\} = \left\{ \begin{array}{l} x_1 = 0 + t \cdot 1 \\ x_2 = 1 + t\,(-1) \\ x_3 = 0 + t \cdot 1 \end{array} \;\middle|\; t \in \mathbb{R} \right\}$$

$$\alpha = \{K + r\,\overrightarrow{KL} + s\,\overrightarrow{KM} \mid r, s \in \mathbb{R}\} = \left\{ \begin{array}{l} x_1 = 1 + r \cdot 0 + s \cdot (-1/2) \\ x_2 = 0 + r \cdot 1/2 + s \cdot 0 \\ x_3 = 1/2 + r \cdot 1/2 + s \cdot 1/2 \end{array} \;\middle|\; \dots \right\}$$

$\overrightarrow{KL}$ $\overrightarrow{KM}$

b) rovnicemi

$$p = \left\{ \boxed{\begin{array}{l} 1x_1 + 1x_2 + 0x_3 = 1 \\ 0x_1 + 1 \cdot x_2 + 1 \cdot x_3 = 1 \end{array}} \right\}$$

← 2 rovnice (takové, abych eliminoval t) resp. r, s

$$\alpha = \left\{ (+1)x_1 + (-1)x_2 + 1x_3 = \frac{3}{2} \right\}$$

Pozn. ... $(1, -1, 1)$ = "normála α"

CO KDYŽ

$$\alpha = \left\{ \begin{aligned} x_1 &= 1 + r \begin{pmatrix} 1 \\ 3 \\ 7 \end{pmatrix}_{1} + s \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}_{1} \\ x_2 &= 0 + r \cdot 3 + s \cdot 1 \\ x_3 &= 1/2 + r \cdot 7 + s \cdot 3 \end{aligned} \;\middle|\; \ldots \right\}$$

$$= \left\{ \begin{aligned} x_1 - 1 &= r - 2s \\ x_2 &= \boxed{3r} + s \\ x_3 - 1/2 &= \boxed{7r} + 3s \end{aligned} \right\} \quad \cdot(-3),\ \cdot(-7)$$

$$= \left\{ \begin{aligned} x_1 - 1 &= r - 2s \\ -3x_1 + x_2 + 3 &= 0 + 7s \\ -7x_1 + x_3 + \tfrac{13}{2} &= 0 + \boxed{17s} \end{aligned} \right\} \quad \cdot(-17),\ \cdot 7$$

$$= \left\{ \begin{aligned} x_1 - 1 &= r - 2s \\ -3x_1 + x_2 + 3 &= 0 + 7s \\ 2x_1 - 17x_2 + 7x_3 - \tfrac{11}{2} &= 0 \end{aligned} \right\}$$

toto nás nezajímá

toto je hledaná rovnice

GAUSSOVA ELIMINACE

NIKDY NEŘÍKÁME!

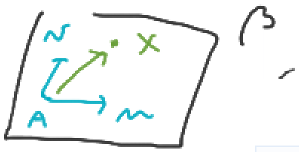
$$\alpha = \left\{ 2x_1 - 17x_2 + 7x_3 = \frac{11}{2} \right\}$$

MINULE ... rovnicové vyjádření podpr. (cv. 110)

$$\alpha = \left\{ \begin{array}{l} x_1 = 1 + r\cdot 0 + s\cdot(-1/2) \\ x_2 = 0 + r\cdot 1/2 + s\cdot 0 \\ x_3 = 1/2 + r\cdot 1/2 + s\cdot 1/2 \end{array} \;\middle|\; r, s \in \mathbb{R} \right\} = \{ \dots \} = \{ x_1 - x_2 + x_3 = -\tfrac{1}{2} \}$$

K $\quad \vec{KL}$ $\quad \vec{KM}$

z hlavy



$$\beta = \left\{ \begin{array}{l} x_1 = 1 + r\cdot 1 + s\cdot(-2) \\ x_2 = 0 + r\cdot 3 + s\cdot 1 \\ x_3 = 1/2 + r\cdot 7 + s\cdot 3 \end{array} \;\middle|\; \dots \right\} = \{ \dots \} = \{ 2x_1 - 17x_2 + 7x_3 = \tfrac{11}{2} \}$$

X $\quad A$ $\quad u$ $\quad v$

systematická (GAUSSOVA) eliminace

NAJDĚTE

- jiné řešení ← např. pomocí DETERMINANTŮ

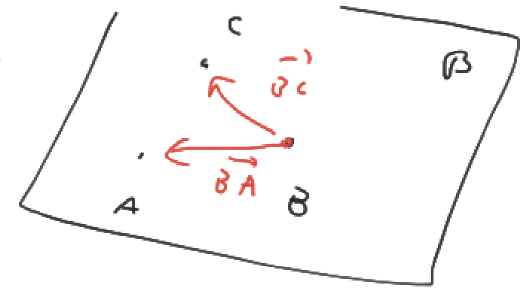
$$\vec{AX} \quad \begin{vmatrix} x_1 - 1 & 1 & -2 \\ x_2 & 3 & 1 \\ x_3 - 1/2 & 7 & 3 \end{vmatrix} = 2x_1 - 17x_2 + 7x_3 - \frac{11}{2} = 0 \; !$$

- jiné vyjádření ← "AF. KOMBINACE" bodů K L M

$$\alpha = \{ \text{"}X = t_0K + t_1L + t_2M\text{"} \mid \boxed{t_0 + t_1 + t_2 = 1} \} = \left\{ \begin{array}{l} x_1 = t_0\cdot 1 + t_1 \dots + t_2 \dots \\ x_2 = t_0\cdot 0 + t_1 \dots + t_2 \dots \\ x_3 = t_0\cdot 1/2 + t_1 \dots + t_2 \dots \end{array} \;\middle|\; t_0 + t_1 + t_2 = 1 \right\}$$

TOTÉŽ pro af. podpr. $\mathcal{B}$ určený body   cv. (11)

$A = [1, -1, 0, 2]$ $B = [4, 1, 0, 2]$ $C = [2, -1, 1, 1]$



- af. kombinace

$$\mathcal{B} = \left\{ \begin{array}{l} x_1 = t_0 + 4t_1 + 2t_2 \\ x_2 = -t_0 + t_1 - t_2 \\ x_3 = t_2 \\ x_4 = 2t_0 + 2t_1 + t_2 \end{array} \;\middle|\; \underbrace{t_0 + t_1 + t_2 = 1}_{} \right\}$$

$t_1 = 1 - t_0 - t_2$

- Parametricky

$$\mathcal{B} = \left\{ \begin{array}{l} x_1 = 4 + t_0 \cdot (-3) + t_2 \cdot (-2) \\ x_2 = 1 + t_0 \cdot (-2) + t_2 \cdot (-2) \\ x_3 = 0 + t_0 \cdot 0 + t_2 \cdot 1 \\ x_4 = 2 + t_0 \cdot 0 + t_2 \cdot (-1) \end{array} \;\middle|\; t_0, t_2 \in \mathbb{R} \right\}$$

B $\vec{BA}$ $\vec{BC}$

$\leftarrow$ $\vec{BA}, \vec{BC}$ LIN. NEZÁVISLÉ tj. $\dim \mathcal{B} = 2$

- Rovnicově

$$\mathcal{B} = \left\{ \begin{array}{l} x_3 + x_4 = 2 \\ \underbrace{2x_1 - 3x_2}_{5 + 0 \cdot t_0 + 2t_2} \underbrace{- x_3 + x_4}_{2 - 2t_2} = 7 \end{array} \right\}$$

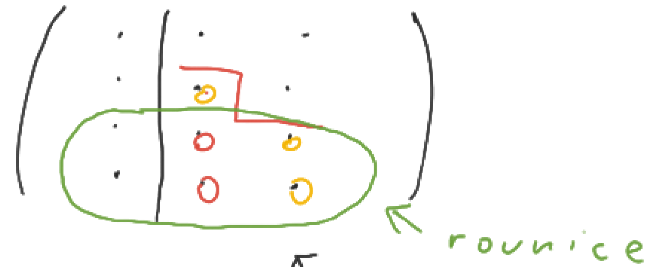
$\leftarrow$ HLEDÁME 2 (NEZÁVISLÉ) ROVNICE

$\nwarrow$ $4 - 2$

$\nwarrow$ 2 HLAVY

$$\mathcal{B} = \left\{ \begin{array}{l} x_1 = 4 + t_0 \cdot (-3) + t_2 \cdot (-2) \\ x_2 = 1 + t_0 \cdot (-2) + t_2 \cdot (-2) \\ x_3 = 0 + t_0 \cdot 0 + t_2 \cdot 1 \\ x_4 = 2 + t_0 \cdot 0 + t_2 \cdot (-1) \end{array} \;\middle|\; t_0, t_2 \in \mathbb{R} \right\}$$

$$B \qquad \vec{BA} \qquad \vec{BC}$$



ELIMINACE

$$\left(\begin{array}{rcl} x_1 - 4 & = & -3t_0 - 2t_2 \\ x_2 - 1 & = & -2t_0 - 2t_2 \\ x_3 & = & 0 \quad t_2 \\ x_4 - 2 & = & 0 \quad -t_2 \end{array} \right) \sim \left(\begin{array}{rcl} x_1 - 4 & = & -3t_0 - 2t_2 \\ 2x_1 - 3x_2 - 5 & = & 0 \quad + 2t_2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array} \right) \quad \text{ATD.}$$

SUBDETERMINANTY

$$\left(\begin{array}{c|c|c} x_1 - 4 & -3 & -2 \\ x_2 - 1 & -2 & -2 \\ x_3 & 0 & 1 \\ x_4 - 2 & 0 & -1 \end{array} \right)$$

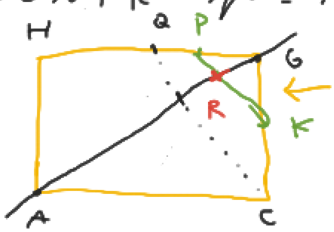
$\xrightarrow{\det}$ $-2(x_1 - 4) + 6x_3 - 4x_3 + 3(x_2 - 1) = 0$

$-2x_1 + 3x_2 + 2x_3 + 5 = 0$ ✓

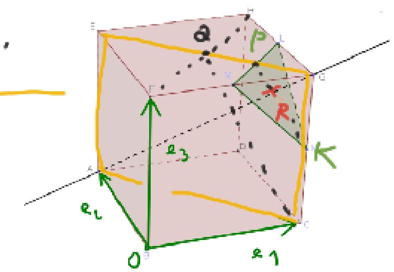
$\xrightarrow{\det}$ $-2(x_4 - 2) - 2x_3 = 0$

$-2x_3 - 2x_4 + 4 = 0$ ✓

cv. (13) Určete PRŮNIK $p: AG$ a $\alpha = KLM$ ...



R ... $1/2 \cdot 1/3 = 1/6$ medzi AG



– Bez počítání

a) parametricky

$p = \{A + t\vec{AG} \mid t \in \mathbb{R}\} = \left\{ \begin{array}{l} x_1 = 0 + t \cdot 1 \\ x_2 = 1 + t \cdot (-1) \\ x_3 = 0 + t \cdot 1 \end{array} \middle| \; t \in \mathbb{R} \right\}$

$\vec{AG} = (1, \dots)$

$\alpha = \{K + r\vec{KL} + s\vec{KM} \mid r, s \in \mathbb{R}\} = \left\{ \begin{array}{l} x_1 = 1 + r \cdot 0 + s \cdot (-1/2) \\ x_2 = 0 + r \cdot 1/2 + s \cdot 0 \\ x_3 = 1/2 + r \cdot 1/2 + s \cdot 1/2 \end{array} \middle| \; \dots \right\}$

$\vec{KL}$, $\vec{KM}$

b) rovnicemi

$p = \left\{ \boxed{\begin{array}{l} 1x_1 + 1x_2 + 0x_3 = 1 \\ 0x_1 + 1 \cdot x_2 + 1 \cdot x_3 = 1 \end{array}} \right\}$

$\alpha = \{(+1)x_1 + (-1)x_2 + 1x_3 = 3/2\}$

← z úlohy (tak, aby sme eliminovali t) resp. r, s

pozn ... $(1, -1, 1)$ = "normála α"

– Početně

1) p ... param, α ... param:

$$\begin{aligned} 0 + t &= 1 - 1/2\, s \\ 1 - t &= 1/2\, r \\ 0 + t &= 1/2 + 1/2\, r + 1/2\, s \end{aligned}$$

3 rovnice
3 neznámé

2) p ... rov, α ... rov:

$$\begin{aligned} x_1 + x_2 &= 1 \\ x_2 + x_3 &= 1 \\ x_1 - x_2 + x_3 &= -1/2 \end{aligned}$$

3 rovnice
3 neznámé

3) p ... param, α ... rov

1 rov.
1 neznámá

$$\underbrace{(t)}_{x_1} - \underbrace{(1 - t)}_{x_2} + \underbrace{(t)}_{x_3} = -1/2$$

$$3t - 1 = 3/2$$
$$3t = 5/2, \quad t = 5/6 \; ✓$$

4) p ... rov, α ... param

2 rovnice
2 neznámé

$R = p \cap \alpha = [5/6, 1 - 5/6, 5/6] = [5/6, 1/6, 5/6]$ ✓

5) $p = \{ X = t_0 A + t_1 G \mid \boxed{t_0 + t_1 = 1} \}$ ← af. kombinace

$\alpha = \{ X = s_0 K + s_1 L + s_2 M \mid \boxed{s_0 + s_1 + s_2 = 1} \}$ ←

$p \cap \alpha \ \ldots$

$$\begin{array}{l} x_1 = \\ x_2 = \\ x_3 = \end{array} \left[\begin{array}{lcl} t_1 & = & s_0 + s_1 + \tfrac{1}{2} s_2 \\ t_0 & = & 1/2\, s_1 \\ t_1 & = & 1/2\, s_0 + s_1 + s_2 \end{array} \right]$$

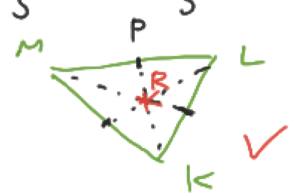
$$\left[\begin{array}{l} t_0 + t_1 = 1 \\ 1 = s_0 + s_1 + s_2 \end{array} \right]$$

$3 + 2 = \underline{5}$ rovnic

$\underline{5}$ neznámých

musí vyjít totéž ....

... tedy $R = \frac{1}{6} A + \frac{5}{6} G = \frac{1}{3} K + \frac{1}{3} L + \frac{1}{3} M$



cv. (14) VZÁJEMNOU POLOHU podpr. . . . v závislosti na $a \in \mathbb{R}$

$$\mathcal{B} = \left\{ \begin{bmatrix} -4 \\ 4 \\ 8 \end{bmatrix} + s \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \,\middle|\, s \in \mathbb{R} \right\}, \quad \mathcal{C} = \left\{ \begin{bmatrix} a \\ 6 \\ -5 \end{bmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$$

$\mathcal{B} \cap \mathcal{C}$ . . . . .

$$\begin{aligned} -4 + 2s &= a + t \\ 4 + s &= 6 - 3t \\ 8 - 4s &= -5 + 3t \end{aligned}$$

3 rovnice, 2 neznámé . . . vzhledem k a

např. $a = 0$ . . . nemá řeš., $\mathcal{B} \cap \mathcal{C} = \emptyset$, tj. // nebo

vektory lin NEZÁVISLÉ

obecně

$$\begin{aligned} 2s - t &= a + 4 \\ s + 3t &= 2 \\ -4s - 3t &= -13 \end{aligned} \sim \begin{aligned} \text{I. } 2s - t &= a + 4 \\ \text{II. } s + 3t &= 2 \\ \text{III. } -3s \ \boxed{0} &= -11 \end{aligned} \sim \ . . . .$$

$\text{III. } s = \frac{11}{3}$ $\quad$ $\text{II. } 3t = 2 - \frac{11}{3} = -\frac{5}{3}$

$t = -\frac{5}{9}$

$\text{I. } \frac{22}{3} + \frac{5}{9} = a + 4 \iff \boxed{a = \frac{22}{3} + \frac{5}{9} - 4 = \frac{35}{9}}$

MEZISHRNUTÍ

$\mathcal{B} \cap \mathcal{C} \neq \emptyset$

$\iff a = 35/9$

SHRNUTÍ

a) vektory $u = \begin{pmatrix} 2 \\ 1 \\ -4 \end{pmatrix} \in \vec{\mathcal{B}}$ a $v = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} \in \vec{\mathcal{C}}$ NEZÁVISLÉ

$\Rightarrow \quad \vec{\mathcal{B}} \cap \vec{\mathcal{C}} = \{o\} \quad \Rightarrow$ NIKDY $\mathcal{B} = \mathcal{C}$ ani $\mathcal{B} \parallel \mathcal{C}$

b) $\mathcal{B} \cap \mathcal{C}$ neprázdný $\Leftrightarrow a = \frac{35}{9}$

Pro $a = \frac{35}{9}$ .... $\mathcal{B} \times \mathcal{C}$ ... RŮZNOBĚŽNÉ ($\mathcal{B} \cap \mathcal{C} =$ bod)

Pro $a \neq \frac{35}{9}$ .... $\mathcal{B} \mathbin{-\!\!\backslash\!-} \mathcal{C}$ ... MIMOBĚŽNÉ

POZN. K SOUSTAVĚ:

$$\boxed{\begin{aligned} 2s - t &= a + 4 \\ s + 3t &= 2 \\ -4s - 3t &= -13 \end{aligned}}$$ má řešení $\Leftrightarrow \det \begin{pmatrix} 2 & -1 & a+4 \\ 1 & 3 & 2 \\ -4 & -3 & -13 \end{pmatrix} = \dots = -35 + 9a = 0$

$\Leftrightarrow a = \frac{35}{9}$

cv. (15) | VZÁJEMNÁ POLOHA . . .

=	X
//	X/

?

$$\mathcal{B} = \left\{ \begin{array}{|l} x_1 + 4x_2 - x_3 = 10 \\ 2x_2 + x_4 = 11 \end{array} \right\}_{\text{dim } 2}, \qquad \mathcal{C} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 1 \\ 4 \end{pmatrix} + t_1 \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix} + t_2 \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix} \middle| \, t_1, t_2 \in \mathbb{R} \right\}_{\text{dim } 2}$$

$\mathcal{B} \cap \mathcal{C}$ . . .
$$\begin{array}{|l} (\cancel{2t_2}) + 4(1+2t_1) - (1+t_1+\cancel{2t_2}) = 10 \\ 2(1+2t_1) + (4+t_1) = 11 \end{array}$$
← 2 rov. / 2 neznámé

(a) $\left(\begin{array}{cc|c} * & * & * \\ \hline 0 & 0 & 0 \end{array} \right)$

$$\sim \ \ldots\ldots \ \sim \ \boxed{\begin{array}{l} 7t_1 = 7 \\ 5t_1 = 5 \end{array}} \ \sim \ \begin{array}{l} t_1 = 1 \\ t_2 = \text{lib} \end{array}$$

závěry:

. . . má řeš. $\Rightarrow \mathcal{B} \cap \mathcal{C} \neq \emptyset$

. . . $\dim \mathcal{B} \cap \mathcal{C} = 1$

$\Big\}$ $\mathcal{B} \times \mathcal{C}$

CO KDYBY

(b) $\left(\begin{array}{cc|c} * & * & * \\ 0 & * & * \end{array} \right) \Rightarrow$ t_1, t_2 .. jednoznačně
tj. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$
dim 0

$\Big\}$ $\mathcal{B} \times \mathcal{C}$

$\mathcal{B} \cap \mathcal{C}$ = bod

"  "

(c) $\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \Rightarrow$ t_1, t_2 .. lib
tj. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$
dim 2

$\mathcal{B} \cap \mathcal{C} = \mathcal{B} = \mathcal{C}$

$\mathcal{B}\cap\mathcal{C}$ (d) $\left(\begin{array}{cc|c} x & x & x \\ 0 & 0 & x \end{array}\right) \Rightarrow$ nemá řeš. $\Rightarrow$ $\mathcal{B}\cap\mathcal{C}=\emptyset$ $\begin{cases} \parallel \Leftrightarrow \vec{\mathcal{B}}=\vec{\mathcal{C}} \\ \times \Leftrightarrow \vec{\mathcal{B}}\neq\vec{\mathcal{C}} \end{cases}$?

$\vec{\mathcal{B}}\cap\vec{\mathcal{C}}$ $\left(\begin{array}{cc|c} x & x & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow$ $\dim \vec{\mathcal{B}}\cap\vec{\mathcal{C}}=1$ $\Rightarrow$ $\underbrace{\vec{\mathcal{B}}\neq\vec{\mathcal{C}}}_{\mathcal{B}\ \times\ \mathcal{C}}$

$\mathcal{B}$ X $\mathcal{C}$

se společným směrem

(„částečná //")

$\left(\vec{\mathcal{B}}=\vec{\mathcal{C}} \Leftrightarrow \dim \vec{\mathcal{B}}\cap\vec{\mathcal{C}}=2 \right)$

tj. $\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$

Pozměňte zadání tak, abyste vyčerpali všechny možnosti ...

$\mathcal{B}\cap\mathcal{E}$

$$\overset{(e)}{\left(\begin{array}{cc|c} 0 & 0 & * \\ 0 & 0 & 0 \end{array}\right)} \Rightarrow \text{není řeš.} \Rightarrow \mathcal{B}\cap\mathcal{E}=\emptyset$$

$\vec{\mathcal{B}}\cap\vec{\mathcal{E}}$

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow \dim \vec{\mathcal{B}}\cap\vec{\mathcal{E}} = \underline{\underline{2}} \Rightarrow \vec{\mathcal{B}} = \vec{\mathcal{E}}$$

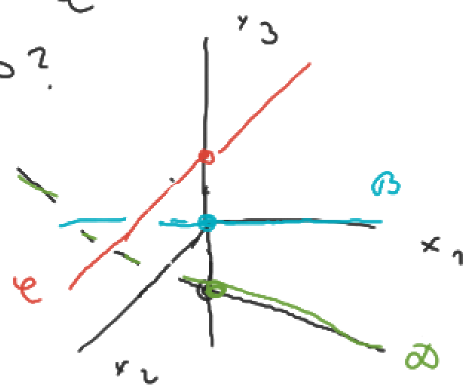
$\Bigg\}$ $\underline{\underline{\mathcal{B} \parallel \mathcal{E}}}$

---

cv (16) | Př. tři navzájem mimob. podpr. v $\underline{\underline{\dim 3}}$

přímky?

Např: $\rightarrow$



BODY NESTAČÍ

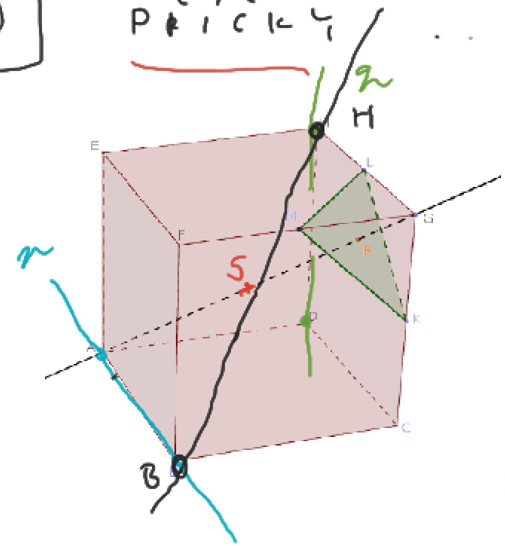
$\mathcal{B}\,\cdot$

$\cdot\,\mathcal{E}$

$\overrightarrow{\text{různé}} \Rightarrow \mathcal{B}\cap\mathcal{E}=\emptyset$

$\underline{\underline{\text{rovnob.}}} \Leftarrow \underset{\{0\}}{\vec{\mathcal{B}}} \overset{!}{=} \underset{\{0\}}{\vec{\mathcal{E}}}$

cv (17) PŘÍČKY .....

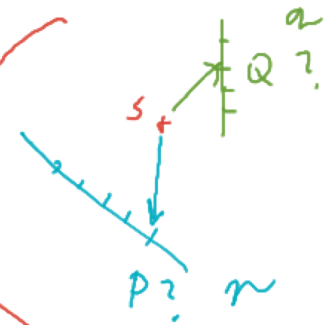


$p = AB$, $q = DH$

+ a) STŘED KRYCHLE

b) resp. SMĚR LM

MYŠLENKA

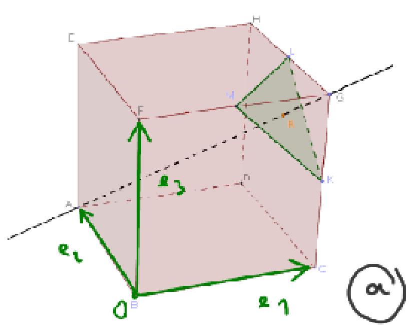


PQ ... příčka proch. S

$\Updownarrow$

$\vec{SP}$ a $\vec{SQ}$ LIN. ZÁVISLÉ

POČÍTÁNÍ:



$$\left.\begin{array}{l} A = [0,1,0] \\ B = [0,0,0] \end{array}\right\} p \Rightarrow P = [0, t, 0] \leftarrow \{A + t\vec{AB}\}$$

$$\left.\begin{array}{l} D = [1,1,0] \\ H = [1,1,1] \end{array}\right\} q \Rightarrow Q = [1, 1, s] \leftarrow \{D + s\vec{DH}\}$$

a) $S = [\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}]$

$$\vec{SP} = \left(-\tfrac{1}{2}, t - \tfrac{1}{2}, -\tfrac{1}{2}\right)$$

$$\vec{SQ} = \left(\tfrac{1}{2}, \tfrac{1}{2}, s - \tfrac{1}{2}\right)$$

$$\left.\begin{array}{l}\vec{SP} = \left(-\frac{1}{2},\ t-\frac{1}{2},\ -\frac{1}{2}\right) \\ \vec{SQ} = \left(\frac{1}{2},\ \frac{1}{2},\ s-\frac{1}{2}\right)\end{array}\right\}$$ LIN. ZÁVISLÉ $\Leftrightarrow$??

(a) 2 HLAVY .... $\vec{SP} = -1 \cdot \vec{SQ} \rightsquigarrow$ $t-\frac{1}{2} = -\frac{1}{2}$ $\rightsquigarrow$ $t = 0$

$+\frac{1}{2} = s - \frac{1}{2}$ $\qquad \underline{\underline{s = 1}}$

$\left(\begin{array}{l} t = 0 \rightsquigarrow P = B \\ s = 1 \rightsquigarrow \underline{\underline{Q = H}} \end{array}\right.$ ... souhlasí s obrázkem ✓ $\left.\right)$

(b) VÝPOČET ... $\underbrace{\vec{SP} = k \cdot \vec{SQ}}$ $\leftarrow$ $\underline{3}$ neznámé

$$\left(\begin{array}{l} -\frac{1}{2} = k \cdot \frac{1}{2} \\ t-\frac{1}{2} = k \cdot \frac{1}{2} \\ -\frac{1}{2} = k \cdot \left(s - \frac{1}{2}\right) \end{array}\right)$$

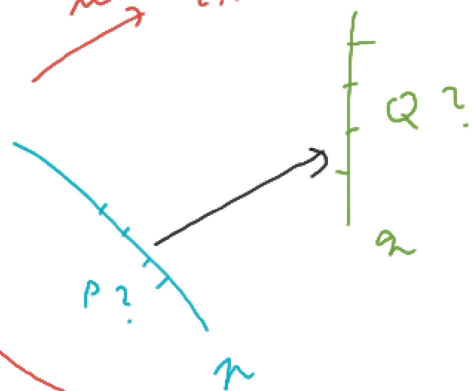
NELINEÁRNÍ vzhledem t, s, k,

ale LINEÁRNÍ vzhledem $t, s, k \cdot s$

alt. $\left(\begin{array}{c|c|c} -\frac{1}{2} & t-\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & s-\frac{1}{2} \end{array}\right)$ $\xrightarrow{\text{det}}$ $\boxed{\begin{array}{l} \ldots\ldots = 0 \\ \ldots\ldots = 0 \end{array}}$ $\leftarrow$ $\underline{2}$ neznámé ...

h)

$\vec{w} = \vec{l_m}$



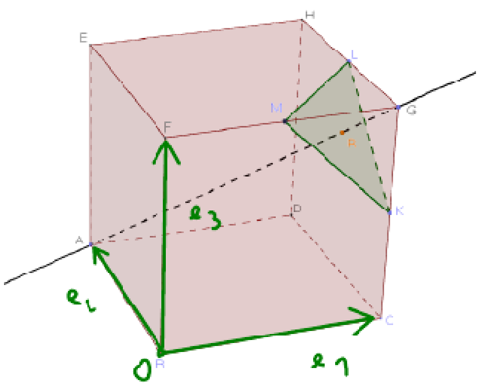
PQ = příčka // w

$\Uparrow$

$\overrightarrow{PQ}$, w lin. závislé

MYŠLENKA OBDOBNÁ

VÝPOČTY TAKY ...



A = —''—
B = —''— } P —''— t

D = —''—
M = —''— } Q —''— s

$w = (1, 1, 0)$ ←

$$\boxed{\overrightarrow{PQ} = k \cdot w} \; ?$$

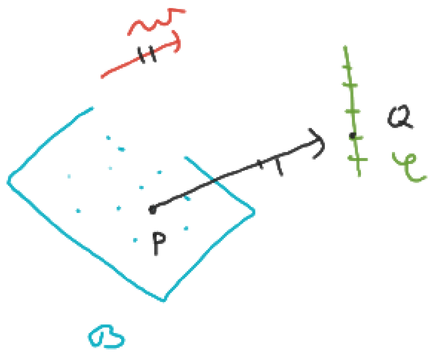
LINEÁRNÍ vzhledem t, s, k !

cv (19) | PŘÍČKY ...

dim 2 — dim 1

$\mathcal{B} = \left\{ \begin{matrix} x_2 - x_4 = 2 \\ x_3 = 1 \end{matrix} \right\}$, $\quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \middle| \ t \in \mathbb{R} \right\}$ $\quad$ + SMĚR $\quad w = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

(a)



PŘEDCHOZÍ MYŠLENKA:

- param. vyjádření $\mathcal{B} = \left\{ \left[\vdots \right] + s_1 \left(\vdots \right) + s_2 \left(\vdots \right) \right\}$
- soustava $\boxed{\overrightarrow{PQ} = k \cdot w}$ ← 4 rovnice, 4 neznámé

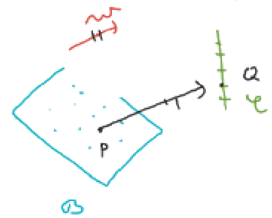
(b) JINÝ NÁPAD ... viz prezentace / příště ...

cv (19) | PŘÍČKY ...

dim 2 $\qquad\qquad\qquad\qquad$ dim 1

$\mathcal{B} = \left\{ \begin{matrix} x_2 - x_4 = 2 \\ x_3 = 1 \end{matrix} \right\}, \quad \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \,\middle|\, t \in \mathbb{R} \right\}$ + SMĚR $\underline{w = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}}$

(a)



PŘEDCHOZÍ MYŠLENKA:

- param. vyjádření $\mathcal{B} = \{ [\vdots] + r_1 (\vdots) + r_2 (\vdots) \}$
- soustava $\boxed{\overrightarrow{PQ} = k \cdot w}$ ← 4 rovnice, 4 neznámé

(b) JINÝ NÁPAD ... viz prezentace ~~| příště ...~~ → další str...

- "$r = \mathcal{C} + w$" .... $\boxed{\begin{matrix} x_1 = 1 + t & x_3 = 3 + s \\ x_2 = t & x_4 = 3 \end{matrix}}$
- $P = \mathcal{B} \cap r$ .... $\boxed{\begin{matrix} t - 3 = 2 \\ 3 + s = 1 \end{matrix}} \Rightarrow \begin{matrix} t = 5 \\ s = -2 \end{matrix} \Rightarrow \underline{\underline{P = \begin{bmatrix} 6 \\ 5 \\ 1 \\ 3 \end{bmatrix}}}$
- příčka "$p = P + w$" $= \underline{\underline{\left\{ \begin{bmatrix} 6 \\ 5 \\ 1 \\ 3 \end{bmatrix} + s \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \,\middle|\, s \in \mathbb{R} \right\}}}$

(• druhý bod $Q \in \mathcal{C}$ ... $Q = p \cap \mathcal{C}$)

⇝ vyjde: $Q = \begin{bmatrix} 6 \\ 5 \\ 3 \\ 3 \end{bmatrix}$

OBECNÉ NÁPADY

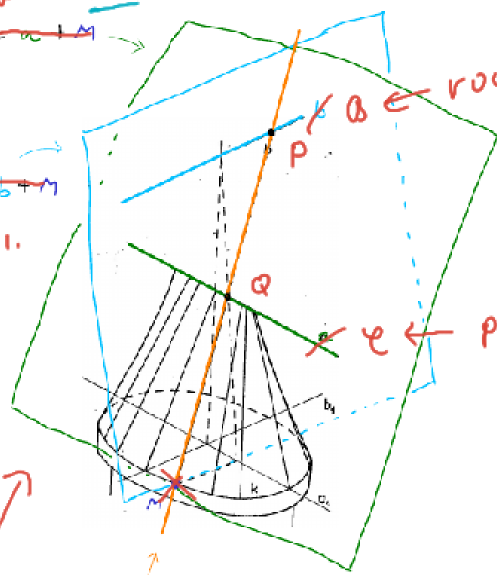
(b) ~~(a)~~ průnik NADPROSTORŮ:

"$\beta = \mathcal{B} + w$"

~~$\alpha = a + M$~~

~~$\beta = b + M$~~

"$\pi = e + w$"



$\mathcal{B}$ ← rovnice dim 2

e ← param dim 1

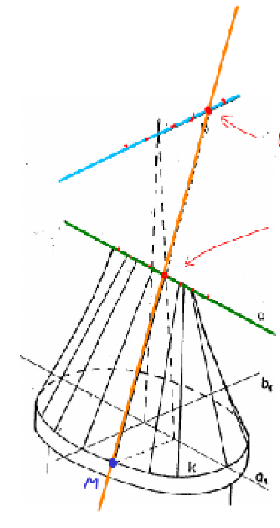
w

~~$p = \alpha \cap \beta$~~ $\boxed{P = \beta \cap \pi}$

resp. $\boxed{P = \mathcal{B} \cap \pi}$ ← 2 rovnice / 2 neznámé

$Q = p \cap e$

~~(b)~~ (a) spojnice KONCOVÝCH BODŮ:

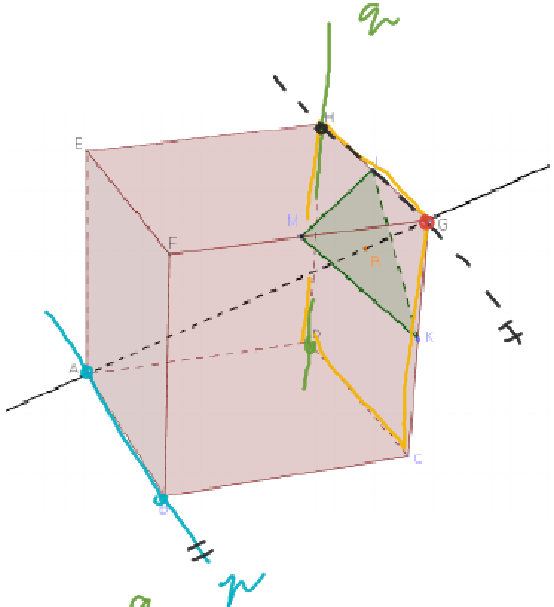


$B \in b$, $A \in a$ } tak, aby A, B, M kolin.

tj. $\boxed{\overrightarrow{MA} = k \cdot \overrightarrow{MB}}$

$p = A + B$

cv (17)



p = AB

q = DH

Genericky 1 riešení, ale MŮŽE se stát cokoli...

M = G

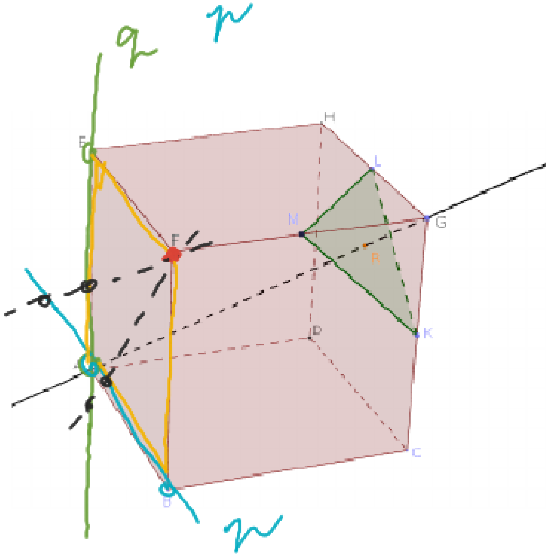
resp. $w = \vec{GH}$

tuki, aby

– neex. příčka

(rovina $\beta = q + G$ je $\parallel p$)

resp. GH je "příčka" s nevl. průsečíkem s p



p = AB

q = AE

M = F

resp. $w = \vec{BE}$

– ∞ příček

(F ∈ rovině p + q)

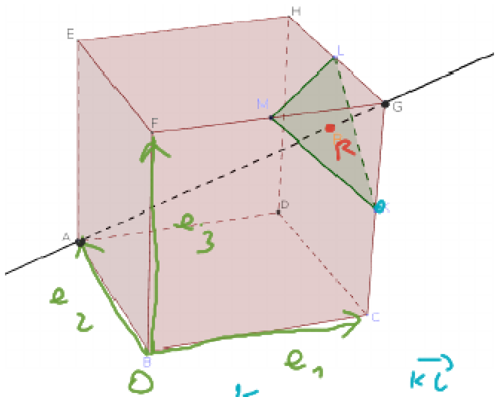
cv. (20)

$\alpha = KLM$, $p = AG$

(a) A, G v opač. poloprostorech vzhledem k α?

(b) $R = p \cap \alpha$ = těžiště ΔKLM

⋮



$$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix},\quad L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix},\quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$\alpha = \left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}}_{K} + r \underbrace{\begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}}_{\vec{KL}} + s \underbrace{\begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix}}_{\vec{KM}} \right\} = \left\{ \boxed{x_1 - x_2 + x_3 = \frac{3}{2}} \right\} \quad (*)$$

$$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\quad G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\quad R = \begin{bmatrix} 5/6 \\ 1/6 \\ 5/6 \end{bmatrix}$$

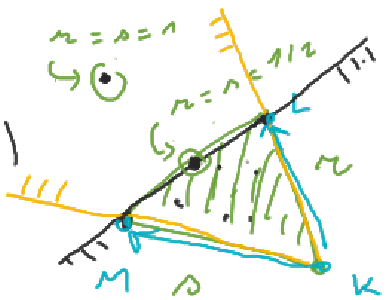
(a) "dosaď A, G do rov. (*)" ⇝

A: $0 - 1 + 0 = -1 < \frac{3}{2}$

G: $1 - 0 + 1 = 2 > \frac{3}{2}$

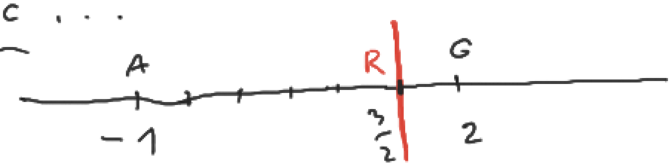
⇒ A, G v OPAČ. POLOPR.

(b)



$$\boxed{\begin{aligned} &0 \le r, s \le 1 \\ &r + s \le 1 \end{aligned}}$$

NAVÍC ...



... POMĚRY souhlasí ✓

(b) $\left\{ \underbrace{\overset{K}{\begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}} + r \overset{\overrightarrow{KL}}{\begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix}} + s \overset{\overrightarrow{KM}}{\begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix}}} \;\middle|\; \boxed{\begin{matrix} 0 \le r, s \le 1 \\ 0 \le r+s \le 1 \end{matrix}} \right\} = $ trojúhelník KLM

$$R = \begin{bmatrix} 5/6 \\ 1/6 \\ 5/6 \end{bmatrix}$$

$R \in \triangle KLM \iff \boxed{\begin{aligned} 5/6 &= 1 \qquad - 1/2\, s \\ 1/6 &= \quad 1/2\, r \\ 5/6 &= 1/2 + 1/2\, r + 1/2\, s \end{aligned}}$ a $\boxed{\begin{matrix} 0 \le r, s \le 1 \\ 0 \le r+s \le 1 \end{matrix}}$

I. $\underbrace{5/6 - 1}_{-1/6} = -1/2\, s \rightsquigarrow s = 1/3$

II. $r = 1/3$

III. $5/6 \overset{?}{=} 1/2 + 1/6 + 1/6$ ✓

má řešení

(tj. $R \in$ rovině KLM)

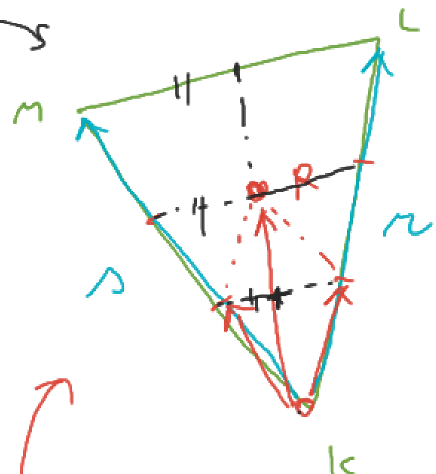
a $\left[\begin{matrix} 0 \le 1/3 \le 1 \\ 0 \le 1/3 + 1/3 \le 1 \end{matrix}\right.$ ✓

$\Downarrow$

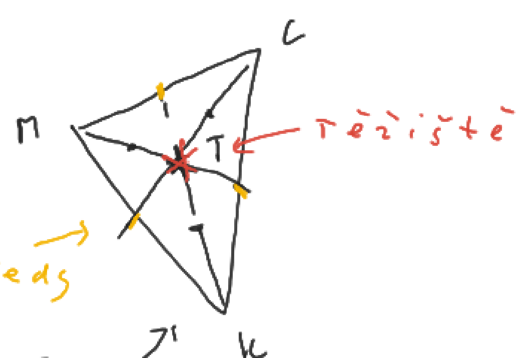
$R \in \triangle KLM$

(b) TĚŽIŠTĚ △ KLM ?

vyšlo nám →



TOTÉŽ ✓



těžiště def

pomocí param. vyjádř...

$$R = K + \underbrace{\tfrac{1}{3}}_{\mu}\overrightarrow{KL} + \underbrace{\tfrac{1}{3}}_{\lambda}\overrightarrow{KM}$$

$$= L + \tfrac{1}{3}\overrightarrow{LK} + \tfrac{1}{3}\overrightarrow{LM}$$

$$= M + \tfrac{1}{3}\overrightarrow{ML} + \tfrac{1}{3}\overrightarrow{MK}$$

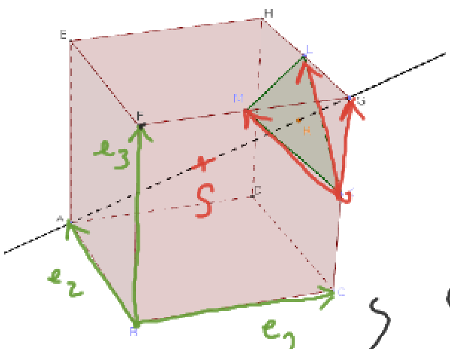
pomocí afinních souř.

$$\text{"}\ R = \underbrace{\tfrac{1}{3}}_{1-\mu-\lambda}K + \underbrace{\tfrac{1}{3}}_{\mu}L + \underbrace{\tfrac{1}{3}}_{\lambda}M\ \text{"}$$

$$\text{"}\ R = K + \underbrace{\tfrac{1}{3}}_{\mu}(L-K) + \underbrace{\tfrac{1}{3}}_{\lambda}(M-K)\ \text{"}$$

cv. (20)

$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \quad L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad S = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$



(c) $S \in$ KONVEXNÍ OBAL bodů K, L, M, G ?

σ

① param. vyjádř. ... $\sigma = \{ K + r\vec{KL} + s\vec{KM} + t\vec{KG} \mid \boxed{0 \le r, s, t \le 1,\ 0 \le r+s+t \le 1} \}$

② rovnicové vyjádř. ... $\sigma = \{ \boxed{x_3 \le 1}\ \boxed{x_2 \ge 0}\ \boxed{x_1 - x_2 + x_3 \ge \tfrac{3}{2}}\ \boxed{x_1 \le 1} \}$

③ af. kombinace ... $\sigma = \{ t_K K + t_L L + t_M M + t_G G \mid t_K + t_L + t_M + t_G = 1,\ \boxed{t_K, t_L, t_M, t_G \ge 0} \}$

$\{x_3 = 1\}$ = rovina MLG

$\{x_3 \le 1\}$ = poloprostor ... obs. bod K

$\{x_1 - x_2 + x_3 = \tfrac{3}{2}\}$ = rovina KLM

$\{ -\,\|\,- \ge \tfrac{3}{2} \}$ = poloprostor ... ... obs. G

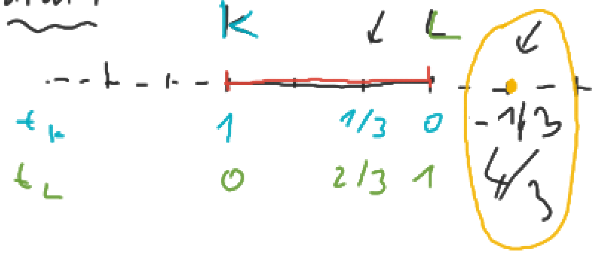
např. ② ... $x_3 = \tfrac{1}{2} \le 1$ ✓ $\quad x_1 - x_2 + x_3 = 1/2 \cancel{\ge} 3/2 \quad \Longrightarrow S \cancel{\in} \sigma$

$x_2 = \tfrac{1}{2} \ge 0$ ✓ $\quad x_1 = \tfrac{1}{2} \le 1$ ✓

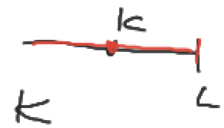
ODBOČKA

$X = -1/3\,K + 4/3\,L \iff X = (1-\frac{4}{3})K + \frac{4}{3}L = K + \frac{4}{3}(L-K) = K + \frac{4}{3}\vec{KL}$ ✓

dim 1

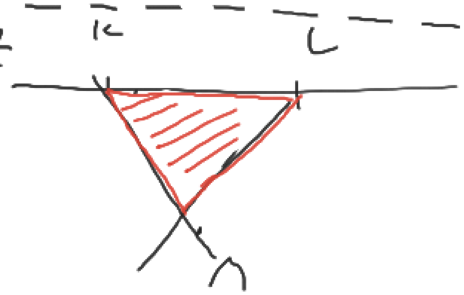


- $\{ t_K K + t_L L \mid t_K + t_L = 1 \}$ = přímka KL
- $\{$ — " — $\mid$ — " — , $\boxed{t_K \ge 0}$ $\}$ = polopř. LK



- $\{$ — " — $\mid$ — " — , $\boxed{t_K, t_L \ge 0}$ $\}$ = úsečka KL

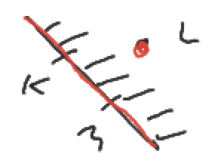
dim 2



- $\{ t_K K + t_L L + t_M M \mid t_K + t_L + t_M = 1 \}$ = rovina KLM

⋮

- $\{$ — " — $\mid$ — " — , $\boxed{t_L \ge 0}$ $\}$ = polorovina



- $\{$ — " — $\mid$ — " — , $\boxed{t_K, t_L, t_M \ge 0}$ $\}$ = $\triangle$ KLM

dim 3

obdobně ⟿ čtyřstěn KLMG . . . . . . $\boxed{t_K, t_L, t_M, t_G \ge 0}$

Jak by se řešilo pomocí vyjádření ③ ?

⇝ SOUSTAVA pro $S \in \sigma$:

$$\left.\begin{aligned}
\tfrac{1}{2} &= t_K + t_L + \tfrac{1}{2} t_M + t_G \\
\tfrac{1}{2} &= + \tfrac{1}{2} t_L \\
\tfrac{1}{2} &= \tfrac{1}{2} t_K + t_L + t_M + t_G \\
\boxed{1} &\boxed{= t_K + t_L + t_M + t_G}
\end{aligned}\right\} \; \ldots \;
\begin{aligned}
t_K &= 1 \geq 0 \;✓ \\
t_L &= 1 \geq 0 \;✓ \\
t_M &= 1 \geq 0 \;✓ \\
t_G &= -2 \not\geq 0
\end{aligned}$$

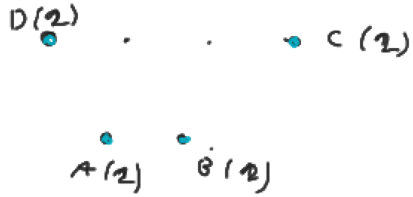
$$\Downarrow$$

$$S \notin \sigma$$

$$\sigma = \left\{ t_K K + t_L L + t_M M + t_G G \;\middle|\; \begin{array}{l} \boxed{t_K + t_L + t_M + t_G = 1} \\ \boxed{t_K, t_L, t_M, t_G \geq 0} \end{array} \right\}$$

$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$, $L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}$, $M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\qquad S = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}$

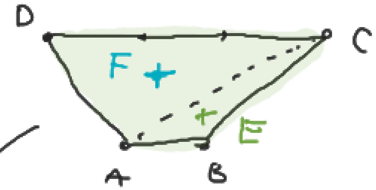
cv (21) (a) těžiště



$$T_1 = \frac{1}{4}A + \frac{1}{4}B + \frac{1}{4}C + \frac{1}{4}D$$

$$\frac{2}{2+2+2+2}$$

(b) těžiště



těžiště ACD

těžiště ABC

poměr VAH
||
poměr OBSAHŮ

T = těžiště E(1), F(3)

(1) $\left[\begin{array}{l} F = \frac{1}{3}A + \frac{1}{3}C + \frac{1}{3}D \\ E = \frac{1}{3}A + \frac{1}{3}B + \frac{1}{3}C \end{array}\right]$ "redukce"

(2) $\left[T = \frac{1}{4}E + \frac{3}{4}F \right]$ "bodová soustava"

$4 = 1 + 3$

(3) DOSAZENÍ ... $T_2 = \frac{1}{4}\cdot\frac{1}{3}(A+B+C) + \frac{3}{4}\cdot\frac{1}{3}(A+C+D)$

$$T_2 = \frac{1}{3}A + \frac{1}{12}B + \frac{1}{3}C + \frac{1}{4}D = \ldots\ldots = \ldots\ldots$$

$T_1 \neq T_2$

A, B, C, D NEJSOU V OB. POLOZE

$\Rightarrow$ "SOUŘ." NEJSOU JEDNOZNAČNÉ !!

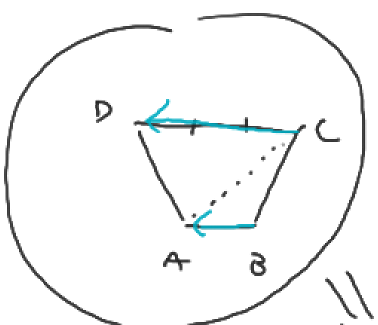
$$T_1 = \frac{1}{4}A + \frac{1}{4}B + \frac{1}{4}C + \frac{1}{4}D = 1 \cdot A - \frac{1}{2}B + \frac{1}{2}C$$

$$T_2 = \frac{1}{3}A + \frac{1}{12}B + \frac{1}{3}C + \frac{1}{4}D = \frac{13}{12}A - \frac{2}{3}B + \frac{7}{12}C$$

$$\Rightarrow \quad T_1 \neq T_2$$

VSKUTKU

JEDNOZNAČNÁ vyjádření!
(A, B, C jsou v obecné poloze)

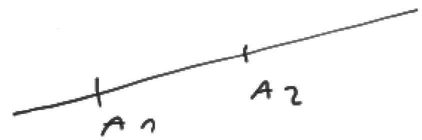


$$\boxed{D = 3A - 3B + 1C}$$

$$\overrightarrow{CD} = 3\,\overrightarrow{BA} \Rightarrow D - C = 3(A - B)$$

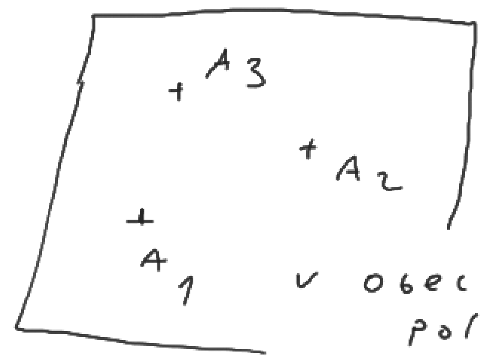
cv (22)

$A_1 = [1,-1,0,2]$, $A_2 = [3,-1,2,4]$, $A_3 = [3,1,0,0]$,
$A_4 = [5,1,2,2]$, $A_5 = [3,0,1,2]$, $A_6 = [4,-1,0,2]$

- $A_2 \overset{?}{=} A_1$ ⇝ JISTĚ $\underline{\underline{\text{NE}}}$
- $\boxed{A_3 \overset{?}{=} t_1 A_1 + t_2 A_2, \quad t_1 + t_2 = 1}$ $\iff$ $A_3 \in$ 

$\Updownarrow$

$$\boxed{\begin{aligned} 3 &= t_1 + 3t_2 \\ 1 &= -t_1 - t_2 \\ 0 &= + 2t_2 \\ 0 &= 2t_1 + 4t_2 \\ 1 &= t_1 + t_2 \end{aligned}}$$

⇝ NEMÁ řešení, tj. 

v obecné poloze

- $\boxed{A_4 \overset{?}{=} t_1 A_1 + t_2 A_2 + t_3 A_3, \quad t_1 + t_2 + t_3 = 1}$

⇝ má řešení . . . . . . PŘÍŠTĚ DOPLNÍME

cv (22)

$A_1 = [1,-1,0,2]$, $A_2 = [3,-1,2,4]$, $A_3 = [3,1,0,0]$,
$A_4 = [5,1,2,2]$, $A_5 = [3,0,1,2]$, $A_6 = [4,-1,0,2]$

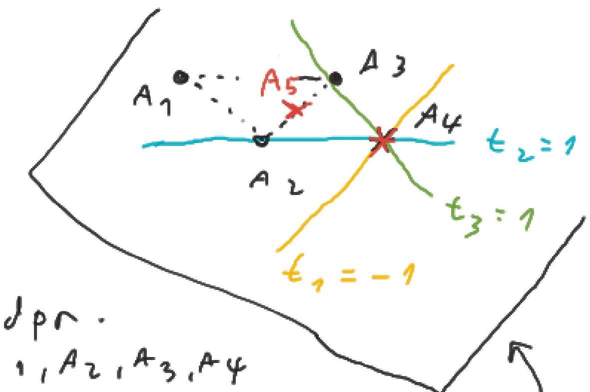
MINULE (a) $A_2 \neq A_1$
(b) $A_3 \neq t_1 A_1 + t_2 A_2$

(c) $A_4 = t_1 A_1 + t_2 A_2 + t_3 A_3$, $t_1 + t_2 + t_3 = 1$

$$\left[\begin{array}{c|ccc} 5 & 1 & 3 & 3 \\ 1 & -1 & -1 & 1 \\ 2 & 0 & 2 & 0 \\ 2 & 2 & 4 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \Longleftrightarrow \boxed{\begin{array}{l} t_1 = -1 \\ t_2 = 1 \\ t_3 = 1 \end{array}}$$

, tj. $\boxed{A_4 = -1 A_1 + 1 A_2 + 1 A_3}$

zejména $A_4 \in$ rovině A_1, A_2, A_3 ✓



(d) $A_5 = \sum_{i=1}^{4} t_i A_i$, $\sum_{i=1}^{4} t_i = 1$

$$\left[\begin{array}{c|cccc} 3 & \cdot & \cdot & \cdot & 5 \\ 0 & \cdot & \cdot & \cdot & 1 \\ 1 & \cdot & \cdot & \cdot & 2 \\ 2 & \cdot & \cdot & \cdot & 2 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right] \Longleftrightarrow \dots \Longleftrightarrow \boxed{\begin{array}{l} t_1 = \text{lib} \\ t_2 = 1/2 - t_1 \\ t_3 = 1/2 - t_1 \\ t_4 = t_1 \end{array}}$$

, tj. $A_5 \in$ podpr. A_1, A_2, A_3, A_4

z předch. $\Rightarrow$ A_1, A_2, A_3, A_4 v rovině

$\Rightarrow$ nejednoznačnost!

$\boxed{t_1 = 0} \rightsquigarrow \underbrace{t_2 = \tfrac{1}{2} = t_3,\ t_4 = 0}$

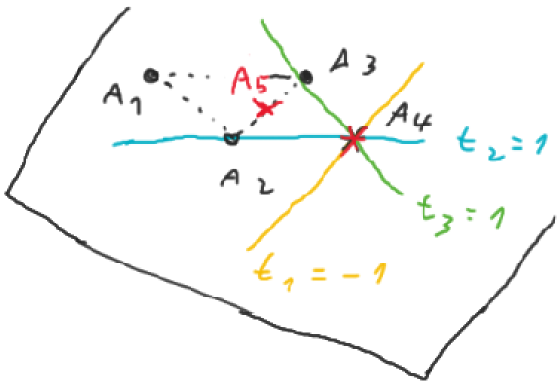
$\boxed{A_5 = \tfrac{1}{2} A_2 + \tfrac{1}{2} A_3}$

(e) $A_6 = \sum_{i=1}^{5} \dots\dots\dots$,

$$\left[\begin{array}{c|ccccc} 4 & \cdot & \cdot & \cdot & \cdot & 3 \\ -1 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & \cdot & \cdot & \cdot & \cdot & 1 \\ 2 & \cdot & \cdot & \cdot & \cdot & 2 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right] \rightsquigarrow \boxed{\text{NEMÁ ŘEŠENÍ}}$$

tj. .... $A_6 \notin$ rovině $A_1, \dots, A_5$

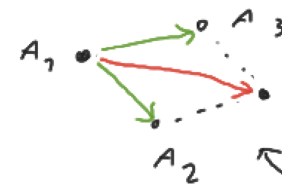
DOPLNĚNÍ



(c) $A_4 = -A_1 + A_2 + A_3 = A_1 + \underline{(A_2 - A_1)} + \underline{(A_3 - A_1)}$

$\underbrace{t_1, t_2, t_3 \text{ JEDNOZN. a } t_1 \cancel{\geq} 0}$

$A_4 \notin$ konvex. obalu A_1, A_2, A_3 ✓



rovnoběžník

(d) $A_5 = t_1 A_1 + (1/2 - t_1) A_2 + (1/2 - t_1) A_3 + t_1 A_4$ & $A_4 = -A_1 + A_2 + A_3$

koef. NEJEDNOZN.
a znaménka
mohou být LIB!

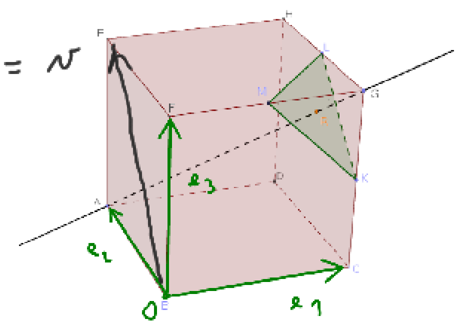
$\rightsquigarrow A_5 = \frac{1}{2} A_2 + \frac{1}{2} A_3$ ... střed $A_2 A_3$
(totéž co subs. $t_1 = 0$)
(subs. $t_1 = 1/2 \rightsquigarrow A_5 = \frac{1}{2} A_1 + \frac{1}{2} A_4$ ... střed $A_1 A_4$) ✓

Ale EX. vyjádření:
$t_1, t_2, t_3, t_4 \geq 0$
TEDY $A_5 \in$ konvex. obalu A_1, A_2, A_3, A_4 ✓

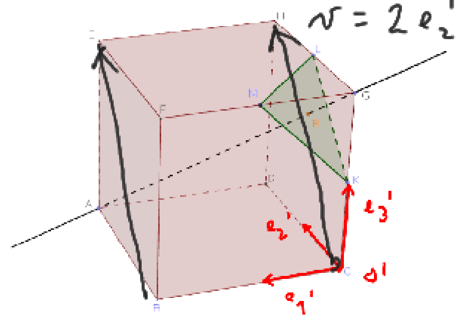
CV (23) — KARTÉZSKÁ SOUŘ. SOUSTAVA

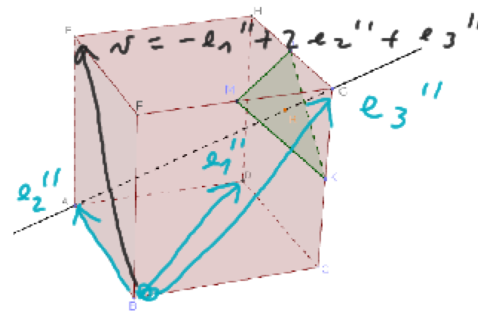
KRYCHLE S HRANOU 1

$e_2 + e_3 = v$



$v = 2e_2' + 2e_3'$





ANO

$\|e_1\| = \|e_2\| = \|e_3\| = 1$

$e_1 \perp e_2,\ e_2 \perp e_3,\ e_1 \perp e_3$

tj.

$$\boxed{\begin{aligned} &e_1 \cdot e_1 = e_2 \cdot e_2 = e_3 \cdot e_3 = 1 \\ &e_1 \cdot e_2 = e_2 \cdot e_3 = e_1 \cdot e_3 = 0 \end{aligned}}$$

$$\left.\begin{aligned} u &= (1, 2, -1) \\ v &= (0, 1, 1) \end{aligned}\right\} \quad \begin{aligned} u \cdot u &= 1 + 4 + 1 = 6 \\ u \cdot v &= 0 + 2 - 1 = 1 \\ v \cdot v &= 0 + 1 + 1 = 2 \end{aligned}$$

NE

$\|e_1'\| = \|e_2'\| = \|e_3'\| = \frac{1}{2} \neq 1$ $\quad k = 1/2$

$e_1 \perp e_2,\ e_2 \perp e_3,\ e_3 \perp e_4$

$u = (-2, 4, -2)$

$v = (0, 2, 2)$

$k^2 = \frac{1}{4}$

$u \cdot u = \frac{1}{4}(4 + 16 + 4) = \frac{1}{4} 24 = 6$

$u \cdot v = \frac{1}{4}(0 + 8 - 4) = \frac{1}{4} 4 = 1$

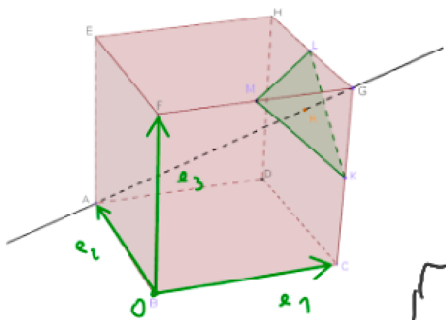
$v \cdot v = \frac{1}{4}(0 + 4 + 4) = \frac{1}{4} \cdot 8 = 2$

NE $\leftarrow$ $e_1'' \not\perp e_2''$ ...

skal. součin obecně ...

... potřeba víc sčítanců $\Rightarrow$

KARTÉZSKÁ



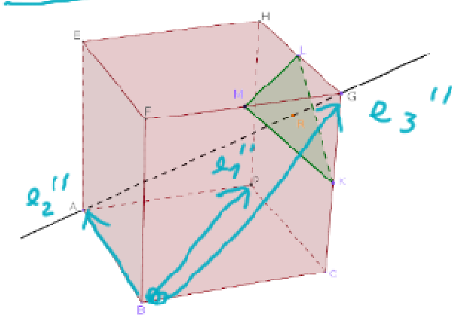
POZN. $\underbrace{(1,2,-1)}_{u} \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{skal. souč.}} \cdot \underbrace{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}_{v} = 2 - 1 = 1$

TOTÉŽ

$$\left.\begin{aligned} u &= e_1 + 2e_2 - e_3 \\ v &= e_2 + e_3 \end{aligned}\right\} \quad u \cdot v = 1\, \underline{e_1 \cdot e_2}^{\,0} + 1\, \underline{e_1 \cdot e_3}^{\,0} + 2\, \underline{e_2 \cdot e_2}^{\,1} + 2\, \underline{e_2 \cdot e_3}^{\,0} - \underline{e_3 \cdot e_2}^{\,0} - \underline{e_3 \cdot e_3}^{\,1} = 2 - 1 = 1$$

$$\boxed{\begin{aligned} e_1'' &= e_1 + e_2 \\ e_2'' &= e_2 \\ e_3'' &= e_1 + e_3 \end{aligned}} \rightleftarrows \boxed{\begin{aligned} e_1 &= e_1'' - e_2'' \\ e_2 &= e_2'' \\ e_3 &= -e_1'' + e_2'' + e_3'' \end{aligned}}$$

OBECNÁ



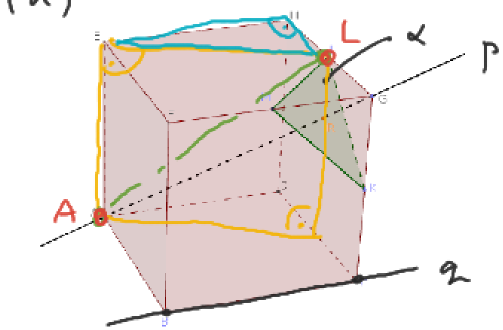
$$\left.\begin{aligned} u &= 2e_1'' - e_3'' \\ v &= -e_1'' + 2e_2'' + e_3'' \end{aligned}\right\} \quad u \cdot v = -2\, \underline{e_1'' \cdot e_1''}^{\,2} + 4\, \underline{e_1'' \cdot e_2''}^{\,1} + 2\, \underline{e_1'' \cdot e_3''}^{\,1} + \underline{e_3'' \cdot e_1''}^{\,1} - 2\, \underline{e_3'' \cdot e_2''}^{\,0} - \underline{e_3'' \cdot e_3''}^{\,2}$$

$$= -4 + 4 + 2 + 1 - 2 = 1$$

$\underbrace{(2,0,-1)}_{u} \cdot \underbrace{\begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}}_{\text{skal. souč.}} \cdot \underbrace{\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}}_{v} = \dots = 1$

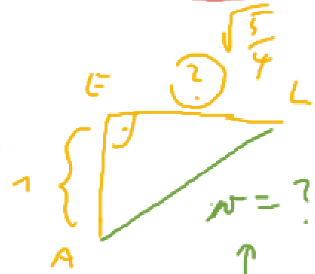
cv (24) VZDÁLENOST $v(A,L)$, $v(L,\alpha)$, $v(L,p)$, $v(p,q) = ?$

(a)

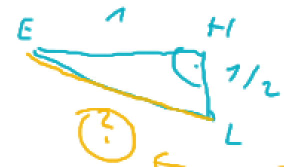


KRYCHLE

s hranou 1



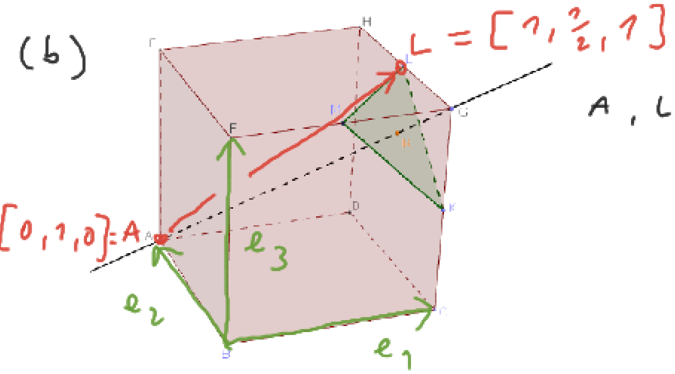
$$\sqrt{1 + \frac{5}{4}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$



$$\sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}}$$

2 x Pythagorova věta

(b)



Analyticky (totéž)

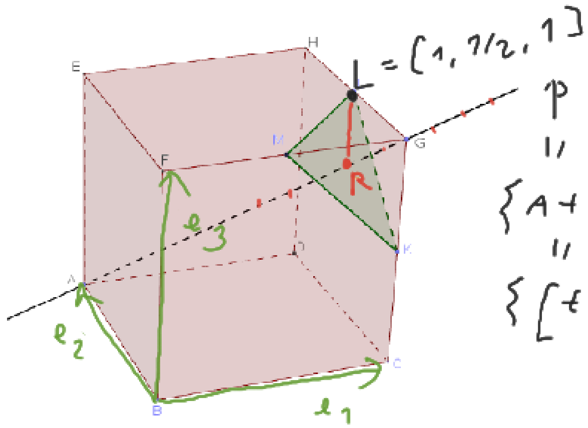
$A, L \rightsquigarrow \vec{AL} = (1, -\frac{1}{2}, 1) = u$

$\rightsquigarrow |AL| = \|u\| = \sqrt{u \bullet u}$

$$= \sqrt{\underbrace{1 + \frac{1}{4}}_{\frac{5}{4}} + 1} = \sqrt{\frac{9}{4}} = \frac{3}{2} \checkmark$$

• $\underline{v(L, \alpha) = 0} \iff L \in \alpha$

$\alpha = KLM$



$p = \{A + t\,\vec{AG} \mid t \in \mathbb{R}\} = \{[t, 1-t, t]\}$

• $\underline{v(L, p) \neq 0}$?

Definice ("minimum")

(a) $\underline{v(L, p) = \min\{|LP|,\ P \in p\}}$

$\vec{LP} = (t-1,\ 1/2 - t,\ t-1)$

$|LP| = \|\vec{LP}\| = \sqrt{(t-1)^2 + (\tfrac{1}{2} - t)^2 + (t-1)^2}$

$= \sqrt{3t^2 - 5t + 9/4}$

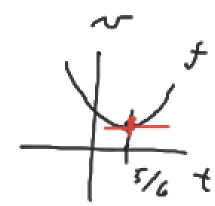
$$\boxed{v = \min\left\{ f(t) = \sqrt{3t^2 - 5t + \tfrac{9}{4}} \;\middle|\; t \in \mathbb{R} \right\}}$$

derivace $\longrightarrow f'(t) = \frac{1}{2}\,\frac{6t-5}{\sqrt{\ldots}} = 0$

$\Updownarrow$

$\boxed{6t - 5 = 0}$

$t = 5/6$



$P = R = [5/6,\ 1/6,\ 5/6] \longleftarrow$ – – – – – – – – –

(NUTNĚ $f''(5/6) > 0$ tj. minimum)

$\vec{LR} = \left(-\frac{1}{6}, -\frac{1}{3}, -\frac{1}{6}\right) = -\frac{1}{6}(1, 2, 1) \rightsquigarrow v = \frac{1}{6}\sqrt{6}$

(b) $v(L,p) = |LP| \iff LP \perp p$, tj. $\boxed{\vec{LP} \cdot \vec{AG} = 0}$

Geom. charakterizace ("pata kolmice")

$P = [t, 1-t, t]$, $\vec{AG} = (1,-1,1)$

$\vec{LP} = (t-1, 1/2 - t, t-1)$

$$\boxed{(t-1) - (\tfrac{1}{2} - t) + (t-1) = 0}$$

$$3t - \frac{5}{2} = 0$$

$$t = 5/6$$

totéž ... $P = R = \dots$

$v = \dots$

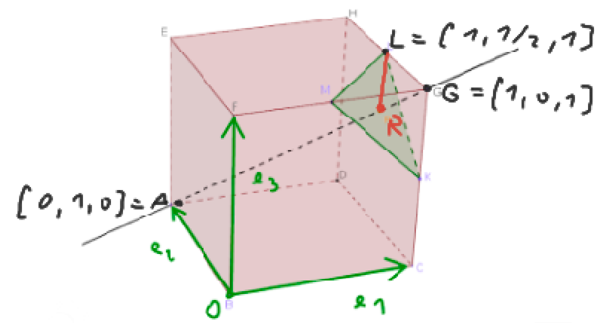
(a) $\boxed{6t - 5 = 0}$ ✓

POZN

(a)–(b) ... velmi OBECNÉ postupy!

Další (spec.) nápady příště...

cv (24) MINULE $v(L, p)$ . . .

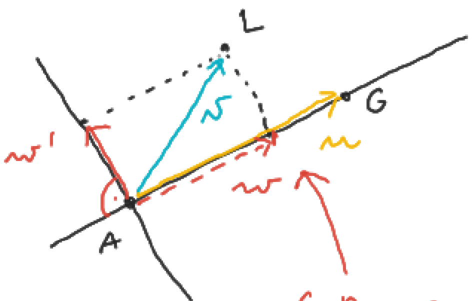


(a) MINIMUM

(b) PATA KOLMICE $\Big\}$ $R = A + \frac{5}{6}\overrightarrow{AG}$

$\rightsquigarrow$ vzdál $= |LR| = \frac{\sqrt{6}}{6}$

(c) KOLMÝ PRŮMĚT



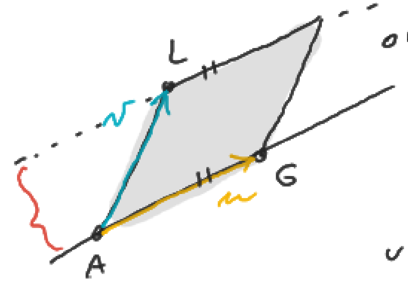
$(R = A + w)$

$w = \perp$ průmět v do u

$w' = v - w$

vzdál $= \|w'\|$

(d) VÝŠKA ROVNOBĚŽNÍKU

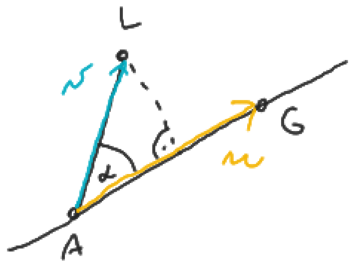


$\underbrace{\text{obsah } ▱ = \text{zákl} \cdot \overleftarrow{AG} \times \text{výška}}$

výška $= \dfrac{\text{obsah } ▱}{\|u\|}$

vzdál = výška

POZN



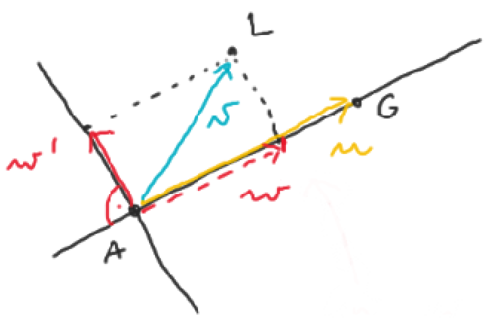
$\cos\alpha = \dfrac{u \cdot v}{\|u\| \cdot \|v\|}$

$\sin\alpha = \sqrt{1 - \cos^2\alpha}$ $\leftarrow$ $\sin^2\alpha + \cos^2\alpha = 1$

vzdál $= \|v\| \cdot \sin\alpha$

(c) KOLMÝ PRŮMĚT OBECNĚ

$w = \perp$ průmět v do $\langle u \rangle$, resp. $\langle u \rangle^{\perp}$



$$\boxed{\begin{aligned} w &= a \cdot u \qquad a = ? \\ w' &= v - w \perp u \end{aligned}}$$

$$(v - a \cdot u) \cdot u = 0$$

$$v \cdot u - a\,(u \cdot u) = 0$$

$$a = \frac{v \cdot u}{u \cdot u}$$

$$\boxed{\begin{aligned} w &= \left(\frac{v \cdot u}{u \cdot u}\right) u \\ w' &= v - w \end{aligned}}$$

$$w \cdot w = \left(\frac{v \cdot u}{u \cdot u}\right)^2 (u \cdot u) = \frac{(v \cdot u)^2}{u \cdot u}$$

$$w' \cdot w' = (v - w) \cdot (v - w) = v \cdot v - 2\, v \cdot w + w \cdot w = \dots = v \cdot v - \frac{(v \cdot u)^2}{u \cdot u}$$

(c) DOSAŽENÍ

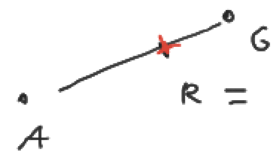
$A = [0, 1, 0]$

$G = [1, 0, 1]$

$C = [1, 1/2, 1]$

$u = \overrightarrow{AG} = (1, -1, 1)$

$v = \overrightarrow{AC} = (1, -1/2, 1)$



$R = A + \frac{5}{6}\overrightarrow{AG}$

totéž co (a)-(b) ✓

$$w = \left(\frac{v \cdot u}{u \cdot u}\right) u \overset{!}{=} \left(\frac{5/2}{3}\right) \cdot u = \frac{5}{6} u$$

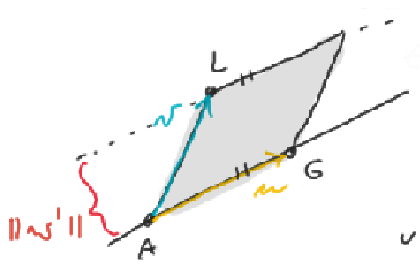
$$w' = v - \frac{5}{6} u = \ldots\ldots$$

RESP.

$$w' \cdot w' = v \cdot v - \frac{(v \cdot u)^2}{u \cdot u} = \frac{9}{4} - \frac{(5/2)^2}{3} = \frac{9}{4} - \frac{25}{12} = \frac{2}{12} = \frac{1}{6}$$

$$\text{vzdál.} = \|w'\| = \sqrt{\frac{1}{6}}$$ ✓

(a) výška rovnoběžníku



$$\text{výška} = \frac{\text{obsah } ▨}{\|u\|}$$

$$\text{vzdál} = \text{výška}$$

$$\begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline u = & (1, & -1, & 1) \\ v = & (1, & -1/2, & 1) \end{array}$$

det $\rightsquigarrow$ $x_1 \underbrace{\begin{vmatrix} -1 & 1 \\ -1/2 & 1 \end{vmatrix}}_{-1/2} - x_2 \underbrace{\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}}_{0} + x_3 \underbrace{\begin{vmatrix} 1 & -1 \\ 1 & -1/2 \end{vmatrix}}_{+\frac{1}{2}}$

TRIK

① "vektorový součin"

$$u \times v = \left(-\tfrac{1}{2}, 0, \tfrac{1}{2}\right) \rightsquigarrow \|u \times v\| \overset{!!}{=} \text{obsah } ▨$$

$$\|u \times v\| = \frac{\sqrt{2}}{2} \rightsquigarrow \text{vzdál} = \text{výška} = \frac{\frac{\sqrt{2}}{2}}{\sqrt{3}} = \frac{1}{\sqrt{2 \cdot 3}} = \frac{1}{\sqrt{6}} \checkmark$$

POSTŘEH

② z předchozího:

$$\text{vzdál} = \|w'\| = \sqrt{w' \cdot w'} = \sqrt{v \cdot v - \frac{(v \cdot u)^2}{u \cdot u}} =$$

$$= \sqrt{\frac{(v \cdot v)(u \cdot u) - (v \cdot u)^2}{u \cdot u}} = \frac{\sqrt{(v \cdot v)(u \cdot u) - (v \cdot u)^2}}{\|u\|} \overset{!!}{=} \frac{\text{obsah } ▨}{\|u\|}$$

NAVÍC :

$$\sqrt{(u\cdot u)(v\cdot v)-(u\cdot v)^2} = \text{OBSAH}$$

rovno-běžník

$$\det\begin{pmatrix} u\cdot u & u\cdot v \\ v\cdot u & v\cdot v \end{pmatrix}$$

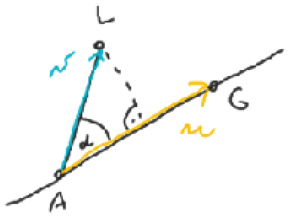
! ... tzv. GRAMOVA MATICE (DETERMINANT)

OBECNĚJI :

$$\boxed{\sqrt{\det\begin{pmatrix} u\cdot u & u\cdot v & u\cdot w \\ v\cdot u & v\cdot v & v\cdot w \\ w\cdot u & w\cdot v & w\cdot w \end{pmatrix}} = \text{OBJEM}}$$

rovno-běžno-stěn

POZN



$$\cos\alpha = \frac{u \cdot v}{\|u\| \cdot \|v\|}$$

$$\sin\alpha = \sqrt{1 - \cos^2\alpha}$$

$$\text{vzdál} = \|v\| \cdot \sin\alpha$$

$$u = \overrightarrow{AG} = (1, -1, 1)$$

$$v = \overrightarrow{AC} = (1, -1/2, 1)$$

$$\cos\alpha = \frac{5/2}{\sqrt{3}\sqrt{9/4}} = \frac{5}{\sqrt{27}}$$

$$\sin\alpha = \sqrt{1 - \frac{25}{27}} = \sqrt{\frac{2}{27}}$$

$$\text{vzdál} = \sqrt{\frac{9}{4} \cdot \frac{2}{27}} = \frac{1}{\sqrt{6}} \checkmark$$

OBECNĚ:

$$\sin\alpha = \sqrt{1 - \frac{(u \cdot v)^2}{\|u\|^2 \|v\|^2}} = \frac{\sqrt{\|u\|^2 \|v\|^2 - (u \cdot v)^2}}{\|u\| \cdot \|v\|}$$

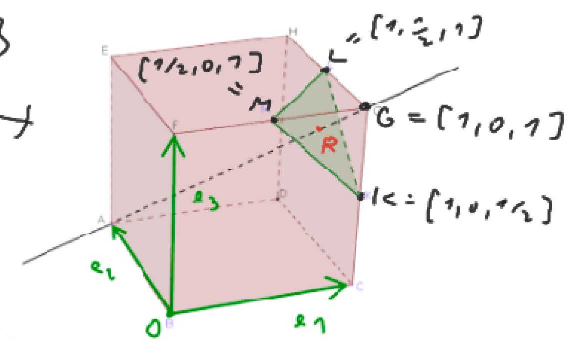
$$\text{vzdál} = \frac{\sqrt{\|u\|^2 \|v\|^2 - (u \cdot v)^2}}{\|u\|}$$

... což nám něco připomíná!

cv (24) $v(G,\alpha) = ?$

$\alpha = \{K + t\vec{KL} + s\vec{KM} \mid t, s \in \mathbb{R}\}$

$= \{x_1 - x_2 + x_3 - \frac{3}{2} = 0\}$



(a) minimum

$P \in \alpha \rightsquigarrow \vec{GP} = \vec{GK} + t\vec{KL} + s\vec{KM} = \left(-\frac{s}{2}, \frac{t}{2}, -\frac{1}{2} + \frac{t}{2} + \frac{s}{2}\right)$

$\rightsquigarrow f(t,s) = \vec{GP} \cdot \vec{GP} = \dots$ kvadr. polynom v t a s

$\rightsquigarrow |GP| = \min \iff f(t,s) = \vec{GP} \cdot \vec{GP} = \min$

$\iff \boxed{\frac{\partial f}{\partial t} = \frac{\partial f}{\partial s} = 0}$ parciální derivace podle t, s

2 lin. rovnice / 2 neznámé

$\Updownarrow$

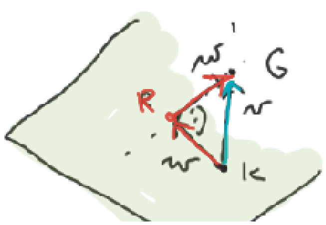
(b) pata kolmice

$P \in \alpha \rightsquigarrow \vec{GP} \perp \alpha \iff \boxed{\vec{GP} \cdot \vec{KL} = \vec{GP} \cdot \vec{KM} = 0}$ (*)

$\dots \rightsquigarrow \left[\begin{matrix} \frac{t}{2} + s = \frac{1}{2} \\ t + \frac{s}{2} = \frac{1}{2} \end{matrix}\right] \rightsquigarrow t = s = 1/3 \rightsquigarrow P = R = [5/6, 1/6, 5/6]$

$\rightsquigarrow$ vzdál $= |GR| = \dots = \frac{\sqrt{3}}{6}$ ✓

(c) kolmý průmět

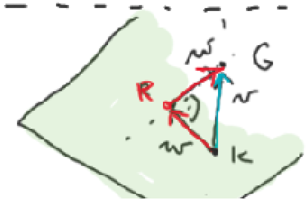


[$w = \perp$ průmět v do $\vec{\alpha} = \langle \vec{KL}, \vec{KM} \rangle$] $\iff$ (*)

[$w' = v - w = \perp$ průmět v do $\vec{\alpha}^{\perp}$] ... okoč

vzdál $= \|w'\|$

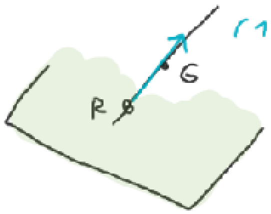
(c) ZKRATKA



[w = ⊥ průmět v do $\vec{\alpha} = \langle \vec{KL}, \vec{KM} \rangle$ ← dim 2]

$w' = v - w$ = ⊥ průmět v do $\vec{\alpha}^{\perp} = \langle u \rangle$ ← dim 1 !

vzdál = $\|w'\|$



$(1,-1,1) = u$

TOTÉŽ!

$$w' = \left(\frac{v \cdot u}{u \cdot u} \right) \cdot u$$

$$\|w'\| = \sqrt{\frac{(v \cdot u)^2}{u \cdot u}} = \frac{|v \cdot u|}{\|u\|}$$

$$= \frac{1/2}{\sqrt{3}} = \frac{\sqrt{3}}{3 \cdot 2} = \frac{\sqrt{3}}{6}$$ ✓

POZN.

$$\alpha = \{ x_1 - x_2 + x_3 - \tfrac{3}{2} = 0 \}$$

$$v(G, \alpha) = \frac{|1 - 0 + 1 - \frac{3}{2}|}{\sqrt{3}} = \frac{1/2}{\sqrt{3}}$$

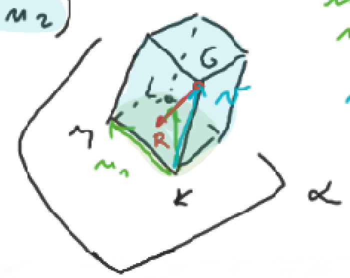
(d) OBJEM / OBSAH

objem(v, u_1, u_2)

$\parallel$

obsah(u_1, u_2)

$\times$ výška



$u_1 = \overrightarrow{KM} = (-1/2, 0, 1/2)$

$u_2 = \overrightarrow{KL} = (0, 1/2, 1/2)$

$v = \overrightarrow{KG} = (0, 0, 1/2)$

$$\frac{1}{4} \cdot \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

OBECNĚ . . . .

$$\begin{pmatrix} u_1 \cdot u_1 & u_1 \cdot u_2 & u_1 \cdot v \\ u_2 \cdot u_1 & u_2 \cdot u_2 & u_2 \cdot v \\ v \cdot u_1 & v \cdot u_2 & v \cdot v \end{pmatrix}$$

det $\rightarrow$ objem(u_1, u_2, v)

$\ldots = \sqrt{\frac{1}{64}} = \frac{1}{8}$

det $\rightarrow$ obsah(u_1, u_2)

$\ldots = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}$

vzdál $= \frac{1}{2\sqrt{3}} = \frac{1}{\sqrt{6}}$ ✓

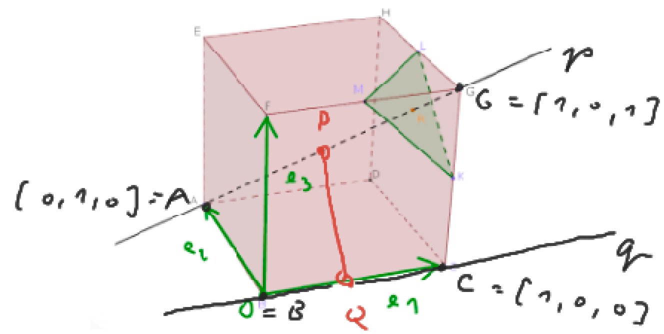
SPEC . . .

"VNĚJŠÍ SOUČIN"

$$\begin{vmatrix} -1/2 & 0 & 1/2 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 \end{vmatrix} \xrightarrow{\det} \boxed{\text{OBJEM}(u_1, u_2, v)}$$

$\det = -\frac{1}{8} \rightsquigarrow$ OBJEM $= \frac{1}{8}$ ✓

cv (14) $v(p,q) = ?$



(a)-(b)

$P = A + t\,\overrightarrow{AG} \in p\,,\quad Q = B + s\,\overrightarrow{BC} \in q$

$\overrightarrow{PQ} \perp p$ a $\overrightarrow{PQ} \perp q \iff \overrightarrow{PQ}\cdot\overrightarrow{AG} = \overrightarrow{PQ}\cdot\overrightarrow{BC} = 0$

$\ldots \rightsquigarrow \begin{bmatrix} -2t + 2s = 0 \\ 6t - 2s = 2 \end{bmatrix} \rightsquigarrow t = s = \frac{1}{2} \rightsquigarrow P = [\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}]$

$Q = [\tfrac{1}{2}, 0, 0]$

$\rightsquigarrow$ vzdál $= |PQ| = \frac{\sqrt{2}}{2}$ ✓

(c) $v = \overrightarrow{AB} = (0,1,0)$

$\underbrace{n \in \langle \overrightarrow{AG}, \overrightarrow{BC} \rangle^{\perp}}$ $\ldots$ $n = (0,1,1)$

$v' = \perp$ průmět v do n $\ldots$ $v' = \left(\frac{v\cdot n}{n\cdot n}\right) n = \frac{1}{2} n$

$\rightsquigarrow$ vzdál $= \frac{1}{2}\|n\| = \frac{\sqrt{2}}{2}$ ✓

(d) $u_1 = \overrightarrow{AG} = (1,-1,1)$

$u_2 = \overrightarrow{BC} = (1,0,0)$

objem $(u_1, u_2, v) = \ldots = 1$

obsah $(u_1, u_2) = \ldots = \sqrt{2}$

$\rightsquigarrow$ vzdál $= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ✓

cv (27) $\quad \mathcal{B} = \left\{ \begin{array}{r} x_2 - x_4 = 2 \\ x_3 = 1 \end{array} \right\}, \quad \mathcal{C} = \left\{ \underbrace{\begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}}_{B} + t \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{u} \;\middle|\; t \in \mathbb{R} \right\}$ $\quad$ vzdál. & vzáj. poloha?

$$\mathcal{B} = \left\{ \underbrace{\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}}_{C} + r \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{v_1} + s \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}}_{v_2} \;\middle|\; r, s \in \mathbb{R} \right\}$$

(a)-(b) $\quad P = C + r v_1 + s v_2 \in \mathcal{B}, \quad Q = B + t u \in \mathcal{C}$

$\vec{PQ} \perp \mathcal{B}$ a $\vec{PQ} \perp \mathcal{C} \iff \boxed{\vec{PQ} \cdot v_1 = \vec{PQ} \cdot v_2 = \vec{PQ} \cdot u = 0}$

$\underbrace{\ldots}$ $\rightsquigarrow \left[\begin{array}{r} -2t + 4s = 2 \\ 4t - 2r - 2s = 2 \\ -2t + 2r = 2 \end{array} \right] \rightsquigarrow \left[\begin{array}{l} r = 6 \\ s = 3 \\ t = 5 \end{array} \right] \rightsquigarrow \begin{array}{l} P = [6, 5, 1, 3] \\ Q = [6, 5, 3, 3] \end{array} \rightsquigarrow$ vzdál $= |PQ| = \underline{\underline{2}}$

- vzdál $\neq 0 \implies \mathcal{B} \cap \mathcal{C} = \emptyset$
- $\dim\{$řešení$\} = 0 \implies \vec{\mathcal{B}} \cap \vec{\mathcal{C}} = \{o\}$

$\Big\}\ \mathcal{B} \not\parallel \mathcal{C}$ ... <u>MIMOBĚŽNÉ</u>

(c) $\quad v = \vec{BC} = (-1, 2, -2, -3)$

$\underbrace{w \in (\vec{\mathcal{B}} + \vec{\mathcal{C}})^{\perp}}$ $= \langle u, v_1, v_2 \rangle^{\perp}$ ... $\boxed{w = (0, 0, 1, 0)}$

$v' = \perp$ průmět v do w ... $v' = \left(\frac{v \cdot w}{w \cdot w} \right) w = -2w \rightsquigarrow$ vzdál $= 2\|w\| = \underline{\underline{2}}$ ✓

(d) $\quad$ objem $(v_1, v_2, u, v) = \ldots = 2$

objem $(v_1, v_2, u) = \ldots = 1$

$\rightsquigarrow$ vzdál $= \frac{2}{1} = \underline{\underline{2}}$ ✓

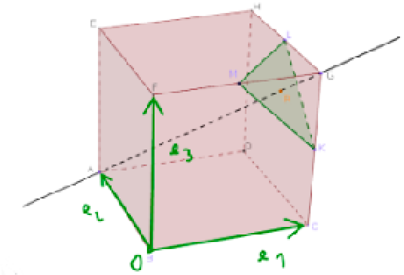
<u>POZN</u> $\quad w = (0, 0, 1, 0)$ je "náhodou" stejný jako ve cv (19)

$\rightsquigarrow$ jiné řešení ...

(28) V úloze (o) určete objemy všech rovnoběžnostěnů, jejichž 4 z 8 vrcholů jsou K, L, M, G,

:)

mnoho různých doplnění, OBJEM STÁLE STEJNÝ!!

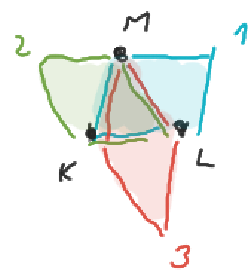


← KRYCHLE S HRANOU 1

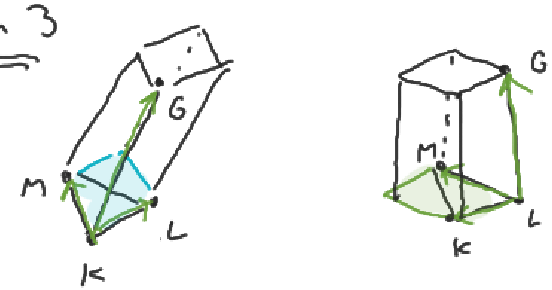
$G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \quad L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

dim 2



dim 3



atd...

např.

$\vec{KL} = \begin{pmatrix} 0 \\ 1/2 \\ 1/2 \end{pmatrix} \quad \vec{KM} = \begin{pmatrix} -1/2 \\ 0 \\ 1/2 \end{pmatrix} \quad \vec{KG} = \begin{pmatrix} 0 \\ 0 \\ 1/2 \end{pmatrix} \quad \rightsquigarrow \quad \vec{KL} = \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \quad \vec{KM} = \frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \quad \vec{KG} = \frac{1}{2}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

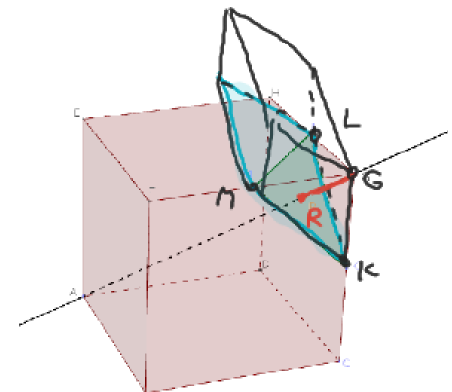
(a) Gramův det:

$$\det\begin{pmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/4 \end{pmatrix} \overset{!}{=} \det\left(\frac{1}{2^2}\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}\right) \overset{!}{=} \frac{1}{4^3}\underbrace{\det\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}}_{1}$$

$$\rightsquigarrow \text{objem} = \sqrt{\frac{1}{4^3}} = \frac{1}{8}$$

(b) VNĚJŠÍ SOUČIN

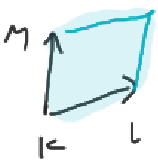
$$\det \begin{array}{c} \begin{pmatrix} 0 & -1/2 & 0 \\ 1/2 & 0 & 0 \\ 1/2 & 1/2 & 1/2 \end{pmatrix} \\ \begin{array}{ccc} \overrightarrow{KC} & \overrightarrow{KM} & \overrightarrow{KG} \end{array} \end{array} = \frac{1}{2^3} \det \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix} = \frac{1}{8} = \text{objem} \checkmark$$



(c) "PODSTAVA × VÝŠKA"

↑ kolmý průmět ...

$\ldots = |RG| = \frac{\sqrt{3}}{6}$



(máme z dřívějška)

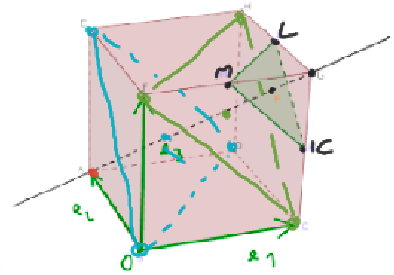
$\text{obsah}(\overrightarrow{KC}, \overrightarrow{KM}) = \cdots = \frac{\sqrt{3}}{4}$

$\rightsquigarrow$

$\text{objem}(\overrightarrow{KC}, \overrightarrow{KM}, \overrightarrow{KG}) = \frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{6} = \frac{1}{8} \checkmark$

(28) V úloze (o) určete objemy všech

- rovnoběžnostěnů, jejichž 4 z 8 vrcholů jsou K, L, M, G,
- čtyřstěnů, jejichž 3 ze 4 vrcholů jsou K, L, M a zbylý vrchol je vrcholem krychle.

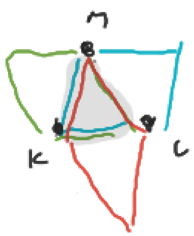


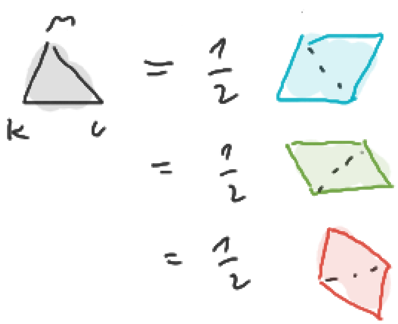
← KRYCHLE S HRANOU 1

$G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}, \quad L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}, \quad M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

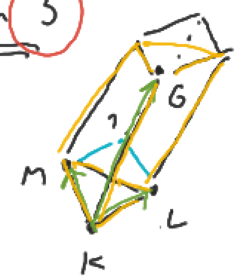
dim 2

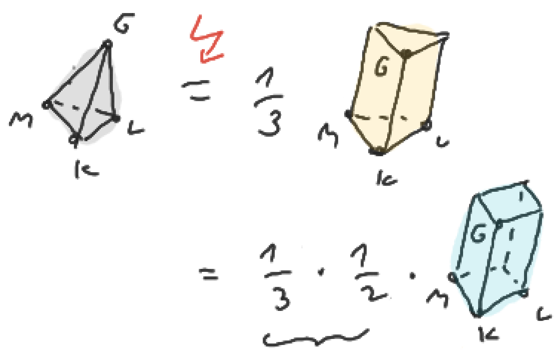




$\frac{1}{2}!$

dim 3

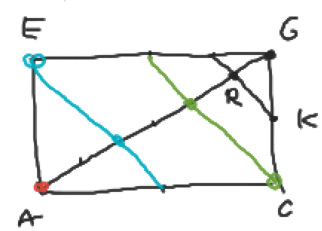




$\frac{1}{6} = \frac{1}{3!}$

- objem $KLMG = \frac{1}{6} \cdot \frac{1}{8} = \frac{1}{48}$
- objem $KLMC$ = objem $KLMH$ = = objem $KLMF$ = ... = $1/48$
- objem $KLME$ = ... = $1/16$
- objem $KLMA$ = ... = $5/48$

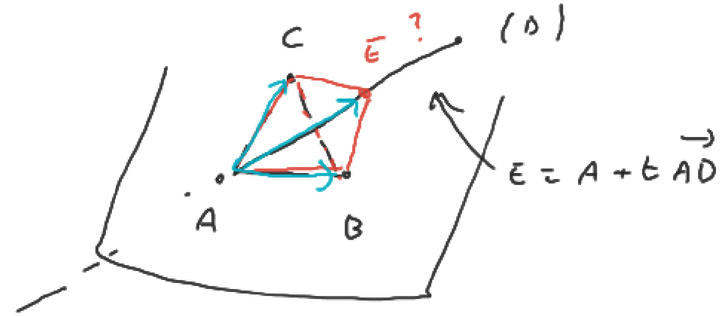
viz též →



(29) V 3-dim prostoru, pro

$A = [0,0,1]$, $B = [2,1,1]$, $C = [1,2,1]$, $D = [1,1,2]$,

- určete bod E na přímce AD tak, aby simplex $ABCE$ měl objem 1.

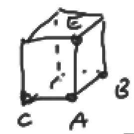


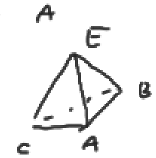
$\vec{AB} = (2,1,0)$ $\quad$ $\vec{AC} = (1,2,0)$ $\quad$ $\vec{AE} = (t,t,t) = t \cdot \vec{AD}$

neznámá t ?

(a) vnější součin

$$\det \begin{pmatrix} 2 & 1 & t \\ 1 & 2 & t \\ 0 & 0 & t \end{pmatrix} = 3t = \pm \text{ objem rovnob.}$$



objem simplexu 

$= \pm \frac{1}{6} \cdot 3t = \pm \frac{1}{2} t = 1$!

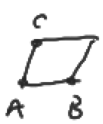
(b) Gram. det

[pracnější]

tj. $t = \pm 2 \rightsquigarrow$ buď $E = [-2,-2,3]$,

nebo $E = [-2,-2,-1]$.

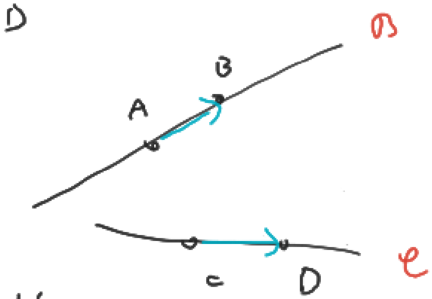
(c) "podstava × výška"

[obsah 

$= \dots = 3$, výška $= \dots = t$]

(31) $A = [2,1,0,-3]$, $B = [5,4,0,-3]$ $\rightsquigarrow$ $\mathcal{B} = A + B$

$C = [2,0,1,-1]$, $D = [9,0,2,-1]$ $\rightsquigarrow$ $e = C + D$



- vzáj. polohu $\mathcal{B}$ a e
- rovnic. vyjádření $\mathcal{B} + e$

$\left.\begin{array}{l}\vec{AB} = (3,3,0,0)\\ \vec{CD} = (7,0,1,0)\end{array}\right\}$ nezávislé $\Rightarrow$ NEJSOU $=$ ANI $\parallel$

$\vec{AC} = (0,-1,1,2)$ } lin. závislé $\Rightarrow$ $\times$ .. RŮZNOB. ?

nezávislé $\Rightarrow$ $\not\times$ ... MIMOB.

(a) úpravy .... ✓

(b) "VZOREČKY"

- Gramův det $= 0 \iff$ ZÁVISLÉ
- vektorový součin $= 0 \iff$ ZÁVISLÉ

!

vektorový součin

$$\boxed{[\vec{AB}, \vec{CD}, \vec{AC}, lib] = \underline{\underline{w}} \cdot lib}$$ ← def.

$$\det \begin{vmatrix} \vec{AB} = (3, 3, 0, 0) \\ \vec{CD} = (7, 0, 1, 0) \\ \vec{AC} = (0, -1, 1, 2) \\ lib = (x_1, x_2, x_3, x_4) \end{vmatrix} = -x_1 \underbrace{\begin{vmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 1 & 2 \end{vmatrix}}_{6} + x_2 \underbrace{\begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix}}_{6} - x_3 \underbrace{\begin{vmatrix} 3 & 3 & 0 \\ 7 & 0 & 0 \\ 0 & -1 & 2 \end{vmatrix}}_{-42} + x_4 \underbrace{\begin{vmatrix} 3 & 3 & 0 \\ 7 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix}}_{-21+3=-18}$$

Laplaceův rozvoj

$$\underline{\underline{w}} = \vec{AB} \times \vec{CD} \times \vec{AC} = (-6, 6, +42, -18) = 6 \cdot (-1, 1, +7, -3)$$

- $w \neq (0, 0, 0, 0) \Rightarrow$ NEZÁVISLÉ ✓
- $w \perp \vec{AB}, \vec{CD}, \vec{AC}$
- $\|w\| =$ objem 

← navíc připomínáme

$\vec{AB}, \vec{CD}, \vec{AC}$ nezávislé $\Rightarrow$ $\mathcal{B}$ a $\mathcal{C}$ MIMOBĚŽNÉ

$\Rightarrow$ $\dim \mathcal{B}+\mathcal{C} = 3$

1 rovnice $(4-3=1)$

rovnicové vyjádření:

$$\mathcal{B}+\mathcal{C} = \{ -x_1 + x_2 + 7x_3 - 3x_4 = 8 \}$$

koef. vlevo:

$\underline{\underline{w}} = (-1, 1, 7, -3)$

= normála nadroviny

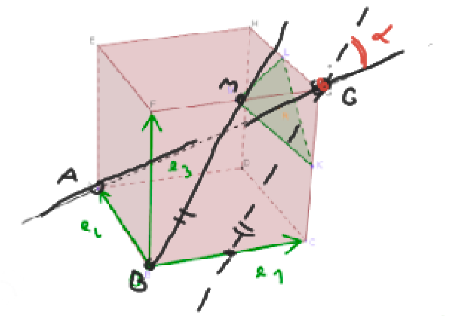
koef. vpravo:

dosaď $A, B, \ldots$

$\parallel$

$[2, 1, 0, -3]$

(32) ODCHYLKA přímek BM a AG



$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$, $L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}$, $M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

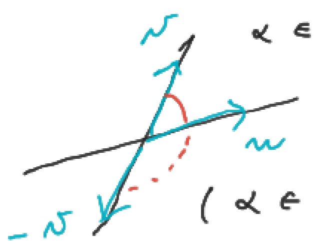
$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

všude jenom VEKTORY!

$\sphericalangle(BM, AG) = \sphericalangle(\overrightarrow{BM}, \pm\overrightarrow{AG}) =: \alpha \rightsquigarrow \boxed{\cos\alpha = \frac{|u \cdot v|}{\|u\| \cdot \|v\|}}$

menší ze dvou možností:



$\alpha \in [0°, 90°] \rightsquigarrow \cos\alpha \geq 0$

$(\alpha \in [90°, 180°] \rightsquigarrow \cos\alpha \leq 0)$

$u = \overrightarrow{AG} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$v = \overrightarrow{BM} = \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix}$

$\cos\alpha = \frac{3/2}{\sqrt{3}\sqrt{5/4}} = \frac{\sqrt{3}}{\sqrt{5}}$

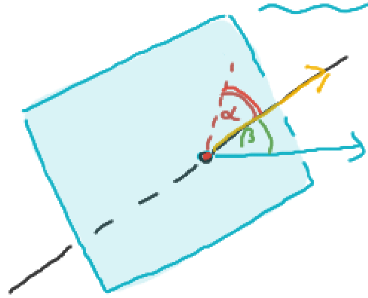
$(\ldots \doteq 0{,}77,$

tj. $\alpha \doteq 39{,}23°)$

(32) ODCHYLKA přímky AG a roviny BGE

(a) pomocí kolmého průmětu ⇝ viz cv. (33)

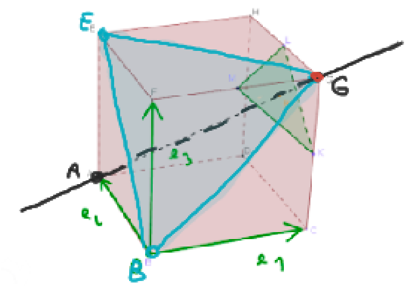
(b) pomocí NORMÁLY:



$m = \overrightarrow{AG}$

n = normála BGE

$$\boxed{\alpha + \beta = 90^\circ}$$



$E = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

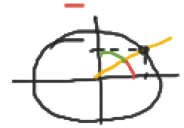
$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

umíme:

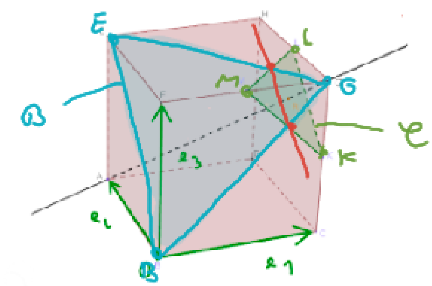
- $n = \overrightarrow{BG} \times \overrightarrow{BE} \quad \dots \quad (-1, -1, 1)$
- $\sphericalangle(m, n) = \beta \quad \dots \quad \cos\beta = \dfrac{|m \cdot n|}{\|m\| \cdot \|n\|} = \dots = \dfrac{1}{3}$

$$\underbrace{\alpha = 90^\circ - \beta}$$ $\rightsquigarrow \quad \sin\alpha = \cos\beta = \dots = \dfrac{1}{3}$



(tj. $\alpha \doteq 19{,}47^\circ$)

(32) ODCHYLKA rovin BCE a KLM

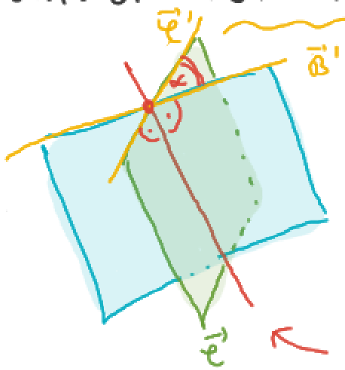


$K = \begin{bmatrix} 1 \\ 0 \\ 1/2 \end{bmatrix}$, $L = \begin{bmatrix} 1 \\ 1/2 \\ 1 \end{bmatrix}$, $M = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$

$B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $G = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$E = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

(a) pomocí MENŠÍCH PODPR :



$\vec{\mathcal{B}} \cap \vec{\mathcal{E}} \neq \{o\}$

$$\boxed{\begin{array}{l} \vec{\mathcal{B}}' \subset \vec{\mathcal{B}} \ \text{ a } \ \vec{\mathcal{B}}' \perp \vec{\mathcal{B}} \cap \vec{\mathcal{E}} \\ \vec{\mathcal{E}}' \subset \vec{\mathcal{E}} \ \text{ a } \ \vec{\mathcal{E}}' \perp \vec{\mathcal{B}} \cap \vec{\mathcal{E}} \end{array}}$$

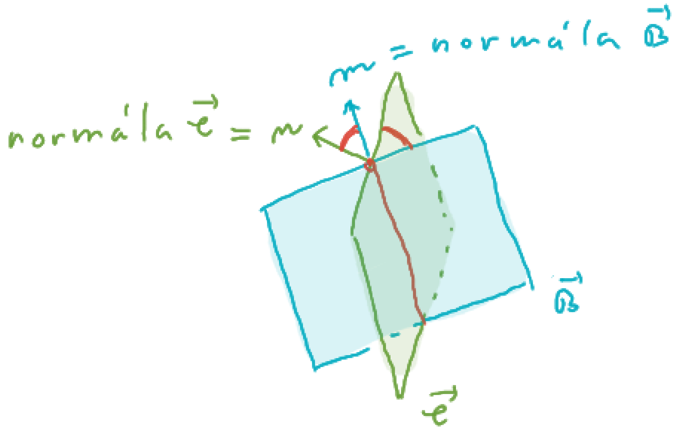
!

- $\vec{\mathcal{B}} \cap \vec{\mathcal{E}}$ ... dim 1 ... $\langle (0,1,1) \rangle$ ($\overrightarrow{BE}$)

- $\vec{\mathcal{B}}'$ ... dim 1 ... $\langle \underbrace{(2,-1,1)}_{u} \rangle = \langle (a, b, a+b) \rangle \cap \langle (0,1,1) \rangle^{\perp}$ ($a\overrightarrow{BG} + b\overrightarrow{BE}$)
- $\vec{\mathcal{E}}'$ ... dim 1 ... $\langle \underbrace{(2,1,-1)}_{v} \rangle = \langle (c, d, -c+d) \rangle \cap \langle (0,1,1) \rangle^{\perp}$ ($c\overrightarrow{KL} + d\overrightarrow{KM}$)

$$\rightarrow \ \measuredangle(\underset{2}{\mathcal{B}}, \underset{2}{\mathcal{E}}) \overset{!}{=} \measuredangle(\underset{1}{\vec{\mathcal{B}}'}, \underset{1}{\vec{\mathcal{E}}'}) = \measuredangle(u, v) \ \dots \ = \arccos \frac{2}{6}$$

(b) pomocí NORMÁL



- $m \propto \overrightarrow{BG} \times \overrightarrow{BE} \quad \ldots \quad (-1,-1,1)$
- $n \propto \overrightarrow{KL} \times \overrightarrow{KM} \quad \ldots \quad (1,-1,1)$

$$\angle(\mathcal{B},\mathcal{E}) \overset{!}{=} \angle(m,n) \quad \ldots \quad \arccos\frac{1}{3} \quad ✓$$

POZN:

nápady s normálami lze použít jen u NADROVIN!

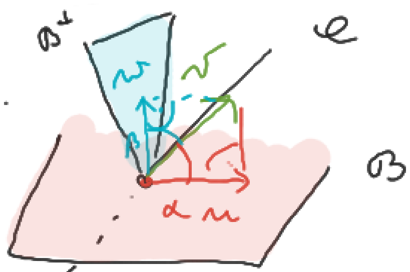
↑ $\dim \vec{\mathcal{E}}^{\perp} = 1$

(33) ODCHYLKA $\mathcal{B} = \left\{ \begin{array}{r} x_2 - x_4 = 2 \\ x_3 = 1 \end{array} \right\}$, $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix} + t \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \;\middle|\; t \in \mathbb{R} \right\}$ ← dim 1

↖ dim 2

$\dim \vec{\mathcal{B}}^{\perp} = 2 \quad (= 4 - 2)$

→ POMOCÍ KOLMÉHO PRŮMĚTU:



$\alpha + \beta = 90°$

$u + w = v$

u = kolmý průmět v do $\vec{\mathcal{B}}$

w = — " — v do $\vec{\mathcal{B}}^{\perp}$

$v = (1, 1, 0, 0)$

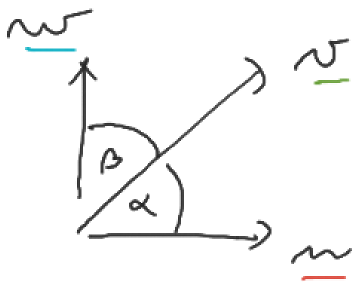
$\vec{\mathcal{B}} = \ldots$

$\vec{\mathcal{B}}^{\perp} = \langle \underbrace{(0,1,0,-1)}_{u_1}, \underbrace{(0,0,1,0)}_{u_2} \rangle$ ← čteme SNADNO ze zadání!

→ $w = a_1 u_1 + a_2 u_2$, $a_1, a_2 \in \mathbb{R}$??

$$\boxed{\begin{array}{l} v - w \perp u_1 \\ v - w \perp u_2 \end{array}} \iff \boxed{\begin{array}{l} w \cdot u_1 = v \cdot u_1 \\ w \cdot u_2 = v \cdot u_2 \end{array}} \iff \left[\begin{array}{l} \overset{2}{(u_1 \cdot u_1)}\, a_1 + \overset{0}{(u_2 \cdot u_1)}\, a_2 = \overset{1}{v \cdot u_1} \\ \overset{0}{(u_1 \cdot u_2)}\, a_1 + \overset{1}{(u_2 \cdot u_2)}\, a_2 = \overset{0}{v \cdot u_2} \end{array} \right.$$

$$\boxed{\begin{aligned} 2a_1 &= 1 \\ a_2 &= 0 \end{aligned}} \longrightarrow \quad \underline{w} = \frac{1}{2} u_1 + 0\, u_2 = \frac{1}{2} \underbrace{(0, 1, 0, -1)}_{u_1}$$



$\underline{v} = (1, 1, 0, 0)$

$$\cos\beta = \frac{v \cdot w}{\|v\| \cdot \|w\|} = \frac{v \cdot u_1}{\|v\| \cdot \|u_1\|} = \frac{1}{\sqrt{2}\,\sqrt{2}} = \frac{1}{2} = \sin\alpha$$

$$\Downarrow$$

$$\underline{\underline{\alpha = 30^\circ}}$$

(34) $u = (1,2,3)$, $v = (2,-1,*)$ dim 3

Doplňte tak, aby $u \in \mathcal{B}$, $v \in \mathcal{C}$ a

(a) $\measuredangle(\mathcal{B},\mathcal{C}) = 90°$ (c) (a) NE (b) ANO

(b) $\vec{\mathcal{B}} \subseteq \vec{\mathcal{C}}^{\perp}$ (d) (a) ANO (b) NE

(a) $\dim \mathcal{B} = \dim \mathcal{C} = 1 \rightsquigarrow v = (2,-1,0)$ ✓

(tak, aby $u \circ v = 0$)

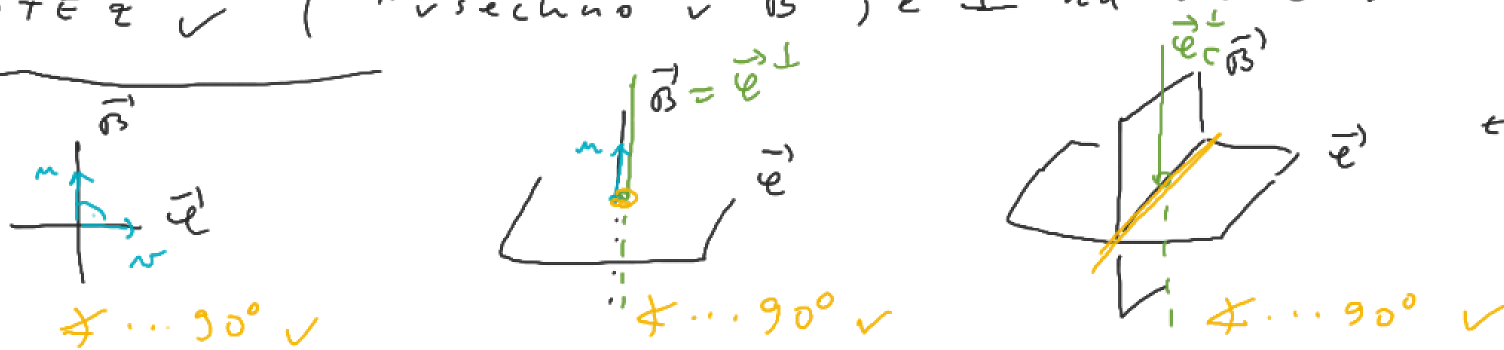
např.

$$\mathcal{B} = \left\{ [\ \vdots\ ] + t \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$$

$$\mathcal{C} = \left\{ [\ \vdots\ ] + s \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} \right\}$$

cokoliv

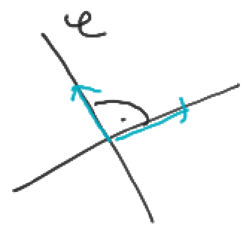
(b) TOTÉŽ ✓ ("všechno v $\vec{\mathcal{B}}$ je $\perp$ na všechno v $\vec{\mathcal{C}}$")



další možnosti

(34) $u = (1, 2, 3, *)$ $\quad v = (2, -1, *, *)$ $\quad$ dim 4

(a)
(b) } obdobné



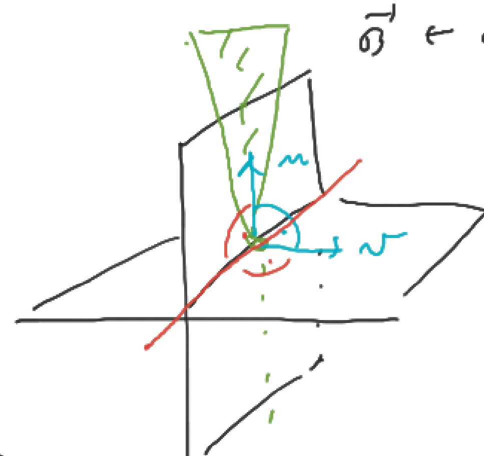
např. $u = (1, 2, 3, 0)$
$v = (2, -1, 0, 0)$

(c) $\measuredangle \neq 90^\circ$ a $\vec{\mathcal{B}} \subseteq \vec{\mathcal{E}}^{\perp}$?

$\vec{\mathcal{E}}^{\perp} \leftarrow$ dim 2 !

... NENÍ MOŽNÉ !

$\vec{\mathcal{B}} \leftarrow$ dim 2



$\vec{\mathcal{E}} \leftarrow$ dim 2

(d) $\measuredangle = 90^\circ$ a $\vec{\mathcal{B}} \not\subseteq \vec{\mathcal{E}}^{\perp}$

$$\mathcal{B} = \left\{ [\,\vdots\,] + t\begin{pmatrix}1\\2\\3\\0\end{pmatrix} + r\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\}$$

$$\mathcal{E} = \left\{ [\,\vdots\,] + s\begin{pmatrix}2\\-1\\0\\0\end{pmatrix} + \check{s}\begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\} \rightsquigarrow \vec{\mathcal{E}}^{\perp} = \left\{ \begin{pmatrix}a\\2a\\b\\0\end{pmatrix} \right\} = \left\{ a\begin{pmatrix}1\\2\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\0\\1\\0\end{pmatrix} \right\} \ \text{... dim 2} \checkmark$$

u $\quad$ v

$\measuredangle = 90^\circ$ ✓

cokoliv $\langle u, v \rangle^{\perp}$

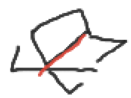
$\vec{\mathcal{B}} \not\subseteq \vec{\mathcal{E}}^{\perp}$ ✓

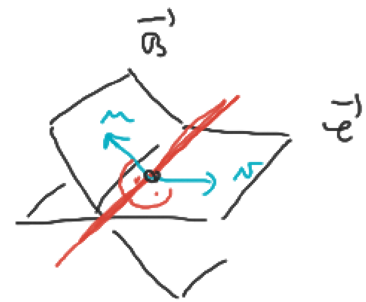
DOPLŇKOVÉ CV. $\qquad \measuredangle(\mathcal{B}, \mathcal{C}) = ?$

$$\mathcal{B} = \left\{ [\,\vdots\,] + t\begin{pmatrix}1\\2\\3\\0\end{pmatrix} + r\begin{pmatrix}1\\2\\3\\4\end{pmatrix} \right\} \qquad \mathcal{C} = \left\{ [\,\vdots\,] + s\begin{pmatrix}2\\-1\\1\\1\end{pmatrix} + s'\begin{pmatrix}-4\\2\\2\\3\end{pmatrix} \right\}$$

MOŽNOSTI

$\vec{\mathcal{B}} \cap \vec{\mathcal{C}}$

- $\{0\} \rightsquigarrow$  $\rightsquigarrow$ NEUMÍME ☹
- dim 1 $\rightsquigarrow$  $\rightsquigarrow$ $\measuredangle = \measuredangle$ menších podpr... ☺
- dim 2 $\rightsquigarrow$  $\rightsquigarrow$ $\measuredangle = 0$ ☺



$$\vec{\mathcal{B}} \cap \vec{\mathcal{C}} = \left\langle \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \right\rangle \ \dots \ \dim 1$$

$u \in \vec{\mathcal{B}}$ a $u \perp \vec{\mathcal{B}} \cap \vec{\mathcal{C}} \rightsquigarrow u = \begin{pmatrix}1\\2\\3\\0\end{pmatrix}$

$v \in \vec{\mathcal{C}}$ a $u \perp \vec{\mathcal{B}} \cap \vec{\mathcal{C}} \rightsquigarrow v = \begin{pmatrix}10\\-5\\1\\0\end{pmatrix} \leftarrow \begin{pmatrix}2s-4s'\\-s+2s'\\s+2s'\\s+3s'\end{pmatrix} \cdot \begin{pmatrix}0\\0\\0\\1\end{pmatrix} = 0$

$$s + 3s' = 0$$

např. $\boxed{s = 3 \quad s' = -1}$

$$\measuredangle(\mathcal{B}, \mathcal{C}) = \measuredangle(u, \pm v) = \alpha$$

$$\dots \cos\alpha = \frac{|u \cdot v|}{\|u\| \cdot \|v\|} = \frac{3}{\sqrt{14}\cdot\sqrt{126}}$$