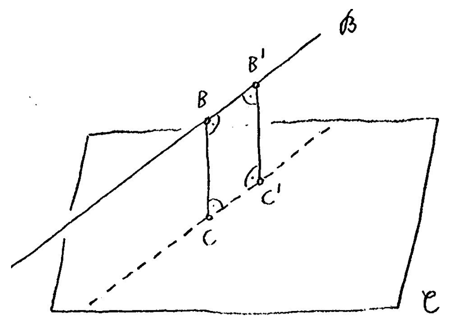
VZDÁLENOSTI

- obecná definice
- geom. charakterizace
- souvislost se vzáj. polohami



VZDÁLENOSTI

- VZDÁL. lib. PODMNOŽIN v lib. METRICKÉM prostoru:

$$v(\mathcal{B}, \mathcal{C}) = \inf \{ |BC|, \text{ kde } B \in \mathcal{B} \text{ a } C \in \mathcal{C} \}$$

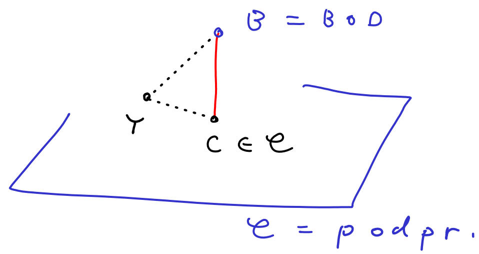
- Pro PODPROSTORY v EUKLEIDOVSKÉM prostoru:

$$\dots \inf = \min \dots$$

- ZŘEJMĚ: $\boxed{v(\mathcal{B}, \mathcal{C}) = 0 \iff \mathcal{B} \cap \mathcal{C} \neq \emptyset}$

- První GEOM. **charakterizace**:

$$\boxed{\begin{aligned} v(B, \mathcal{C}) = |BC|, \text{ tj. } |BC| &= \min \\ &\iff \overrightarrow{BC} \perp \mathcal{C}. \end{aligned}}$$



- **Důkaz** (Pythagorova věta):

(a) předp. $\overrightarrow{BC} \perp \mathcal{C}$ a $Y \in \mathcal{C}$ lib $\rightsquigarrow$ $|BY|^2 = |BC|^2 + |CY|^2 > |BC|^2$

(b) předp. $\overrightarrow{BC} \cancel{\perp} \mathcal{C}$ a $Y \in \mathcal{C}$, $\overrightarrow{BY} \perp \mathcal{C}$ $\rightsquigarrow$ $|BC|^2 = |BY|^2 + |CY|^2 \cancel{<} |BY|^2$.

OBECNÁ CHARAKTERIZACE

- $\mathcal{B}, \mathcal{C}$ ... lib. podprostory, $B \in \mathcal{B}$, $C \in \mathcal{C}$:

(1) $v(\mathcal{B}, \mathcal{C}) = |BC|$, tj. $|BC| = \min$
$\iff \overrightarrow{BC} \perp \mathcal{B}$ a $\overrightarrow{BC} \perp \mathcal{C}$.

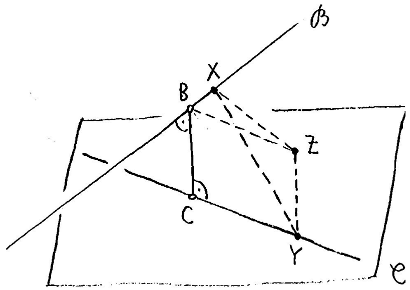
(2) předchozí dvojice B, C je určena jednoznačně
$\iff \overrightarrow{\mathcal{B}} \cap \overrightarrow{\mathcal{C}} = \{o\}$.

- Důkaz:

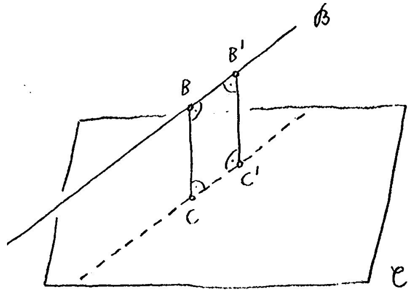
Pro $v(\mathcal{B}, \mathcal{C}) = 0$, tj. $\mathcal{B} \cap \mathcal{C} \neq \emptyset$, všechno zřejmé ...

Pro $v(\mathcal{B}, \mathcal{C}) \neq 0$:

(1) "pravoúhlé $\triangle$" ...



(2) "obdélníčky" ...

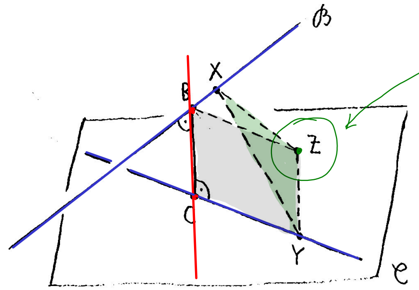


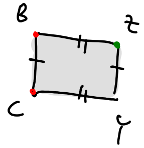
DETAILY K DŮKAZU

(1)

- Předp. $|BC| = \min \rightsquigarrow \overrightarrow{BC} \perp \mathcal{B}$ a $\overrightarrow{BC} \perp \mathcal{C}$. (viz s. 83)

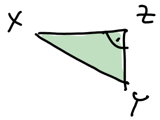
- Předp. $\overrightarrow{BC} \perp \mathcal{B}$ a $\overrightarrow{BC} \perp \mathcal{C} \overset{?}{\rightsquigarrow} |XY| \geq |BC|$ pro lib. $X \in \mathcal{B}$, $Y \in \mathcal{C}$



tak, aby 

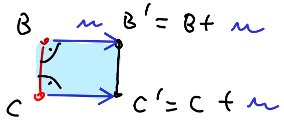
, t.j. $\overrightarrow{ZY} = \overrightarrow{BC}$,

$\overrightarrow{BC} \perp \mathcal{B}$ a $\overrightarrow{BC} \perp \mathcal{C} \implies \overrightarrow{ZY} = \overrightarrow{BC} \perp \overrightarrow{ZB} + \overrightarrow{BX} = \overrightarrow{ZX}$,

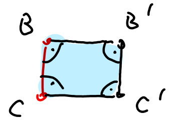
Tedy 

$\implies |XY|^2 = |XZ|^2 + |ZY|^2 \geq |ZY|^2 = |BC|^2$.

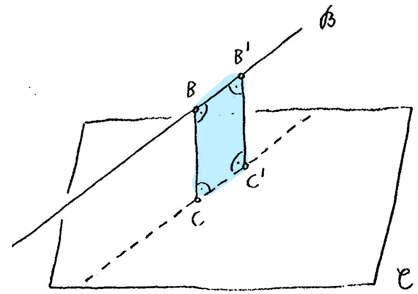
(2)

- předp. $u \in \vec{\mathcal{B}} \cap \vec{\mathcal{C}} \implies$ 

$\implies |BC| = |B'C'|$.

- Předp. $|BC| = |B'C'|$, t.j. 

$\implies \overrightarrow{BB'} = \overrightarrow{CC'} \in \vec{\mathcal{B}} \cap \vec{\mathcal{C}}$.



SOUVISLOST SE VZÁJ. POLOHAMI

- Ozn:

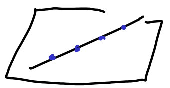
r = vzdálenost $\mathcal{B}, \mathcal{C}$

d = dim {řešení odp. soustavy} = dim $\vec{\mathcal{B}} \cap \vec{\mathcal{C}}$

m = min {dim $\mathcal{B}$, dim $\mathcal{C}$}

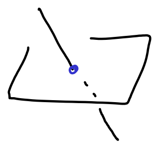
- Např:

$\mathcal{B} \subseteq \mathcal{C}$



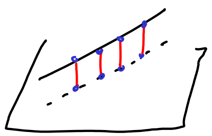
$r = 0$
$d = m$

$\mathcal{B} \times \mathcal{C}$



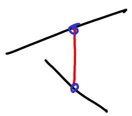
$r = 0$
$d < m$

$\mathcal{B} \parallel \mathcal{C}$



$r \neq 0$
$d = m$

$\mathcal{B} \not\times \mathcal{C}$



$r \neq 0$
$d < m$

- OBECNĚ:

$\vec{\mathcal{B}} \cap \vec{\mathcal{C}}$ / $\mathcal{B} \cap \mathcal{C}$		$d = m$	$d < m$
		je max	není max
$r = 0$	není $\emptyset$	$\subseteq$	$\times$
$r > 0$	je $\emptyset$	$\parallel$	$\not\times$

viz s. 47 – 48