$$\sum_{n=1}^{\infty}\left(\frac{n+3}{n+7}\right)^n(2x+1)^n \qquad\qquad \sum a_n(x-x_0)^n$$

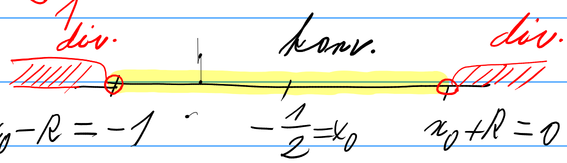
$$\left(\frac{n+3}{n+7}\right)^n(2x+1)^n=\left(\frac{n+3}{n+7}\right)^n\left(2\left(x+\frac{1}{2}\right)\right)^n=\boxed{\left(\frac{n+3}{n+7}\right)^n 2^n}\left(x+\frac{1}{2}\right)^n$$

$$a_n=\left(2\frac{n+3}{n+7}\right)^n \qquad x_0=-\frac{1}{2}$$

$$\frac{1}{R}=\lim_{n\to+\infty}\sqrt[n]{|a_n|}$$

$$\sqrt[n]{|a_n|}=\sqrt[n]{\left(2\frac{n+3}{n+7}\right)^n}=2\cdot\frac{n+3}{n+7}\to 2\,,\quad n\to+\infty$$

$$\frac{1}{R}=2,\qquad R=\frac{1}{2}.$$



$x=0$: $$\sum_{n=1}^{\infty}\left(\frac{n+3}{n+7}\right)^n(2\cdot 0+1)^n=\sum_{n=1}^{\infty}\left(\frac{n+3}{n+7}\right)^n\to \text{diverguje (nutná podm.)}$$

$$\left(\frac{n+3}{n+7}\right)^n=\left(\frac{n+7-4}{n+7}\right)^n=\left(1-\frac{4}{n+7}\right)^n=$$

$$\lim_n\left(1+\frac{a}{n}\right)^n=e^a \qquad\Bigg|\qquad =\left(1-\frac{4}{n+7}\right)^{(n+7)\cdot\frac{n}{n+7}}=\left(\underbrace{\left(1-\frac{4}{n+7}\right)^{n+7}}_{\downarrow\, e^{-4}}\right)^{\frac{n}{n+7}\nearrow 1.}$$

$x=-1$: $$\sum_{n=1}^{\infty}\left(\frac{n+3}{n+7}\right)^n\cdot(-2+1)^n=\sum_{n=1}^{\infty}(-1)^n\cdot\left(\frac{n+3}{n+7}\right)^n$$

(nutná podm. neplatí)

$\hookrightarrow$ ne konv.

Obor konv. $(-1,0)$.

$$\sum_{n=1}^{\infty} \frac{1}{(n+3)3^n} \cdot x^n$$

$$\sum_n a_n (x-x_0)^n$$

$a_n = \frac{1}{(n+3)3^n}$, $x_0 = 0$

$$\frac{1}{R} = \lim_{n\to+\infty} \sqrt[n]{|a_n|} \qquad \left(\lim \left|\frac{a_{n+1}}{a_n}\right|\right)$$

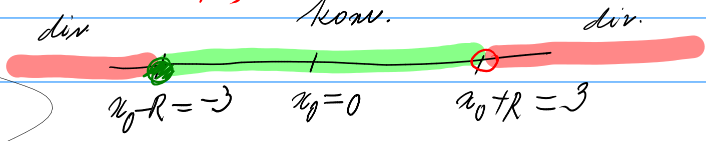
$$\sqrt[n]{|a_n|} = \sqrt[n]{\frac{1}{(n+3)3^n}} = \frac{1}{\sqrt[n]{n+3}\cdot 3} = \frac{1}{3}\cdot\frac{1}{\sqrt[n]{n+3}} \to \frac{1}{3}, \quad n\to+\infty$$

$\sqrt[n]{n+3} \to 1$

$\lim_{n\to+\infty} \sqrt[n]{n} = 1$

$\hookrightarrow R = \frac{1}{\frac{1}{3}} = 3.$

$(-3, 3)$ – obor konv.



$x=3$ $\quad \sum_{n=1}^{\infty} \frac{1}{(n+3)\cancel{3^n}} \cdot \cancel{3^n} = \sum_{n=1}^{\infty} \frac{1}{n+3}$ $\quad$ harm. ř., diverguje.

$x=-3$ $\quad \sum_n \frac{1}{(n+3)3^n} \cdot (-3)^n = \sum \frac{1}{(n+3)\cancel{3^n}} \cdot (-1)^n \cancel{3^n} = \underbrace{\sum (-1)^n \cdot \frac{1}{n+3}}_{\text{altern.}}$

$\sum (-1)^n c_n$, $c_n > 0$, $c_n \downarrow 0$

Podle Leibn. kr. konv.

Obor konv. $\langle -3, 3)$.

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n^2}$$

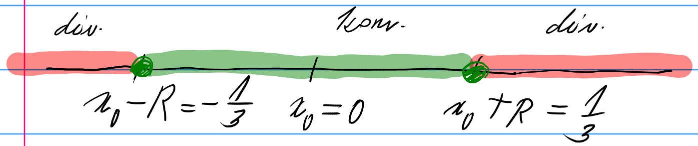
$$\sum a_n (x-x_0)^n$$

$x_0 = 0, \quad a_n = \frac{3^n}{n^2}$

$$\frac{1}{R} = \lim \sqrt[n]{|a_n|} \qquad \left(= \lim_n \left|\frac{a_{n+1}}{a_n}\right| \right)$$

$$\sqrt[n]{|a_n|} = \sqrt[n]{\frac{3^n}{n^2}} = \frac{3}{\sqrt[n]{n^2}} = \frac{3}{(\underset{\downarrow 1}{\sqrt[n]{n}})^2} \to 3, \quad n \to +\infty, \quad R = \frac{1}{3}.$$

$$\left[\left|\frac{a_{n+1}}{a_n}\right| = \frac{3^{n+1}}{(n+1)^2} \cdot \frac{n^2}{\cancel{3^n}} = 3 \cdot \Big(\underbrace{\frac{n}{n+1}}_{\downarrow 1}\Big)^2 \to 3, \quad n \to +\infty \right].$$



$x = \frac{1}{3}$: $\sum_{n=1}^{\infty} \frac{\cancel{3^n}}{n^2} \cdot \cancel{\left(\frac{1}{3}\right)^n} = \sum \frac{1}{n^2}$ zobecn. harm., konv.

$x = -\frac{1}{3}$: $\sum \frac{3^n}{n^2} \cdot \left(-\frac{1}{3}\right)^n = \sum \frac{\cancel{3^n}}{n^2} (-1)^n \cdot \frac{1}{\cancel{3^n}} = \sum \frac{(-1)^n}{n^2}$ – konv.

(– alt., Leibn. kr.

– $\sum \frac{1}{n^2}$)

Obor konvergence je $\langle -\frac{1}{3}, \frac{1}{3} \rangle$.

$$\sum_{n=1}^{\infty}\left(\frac{2n-7}{n+3}\right)^n$$

$2n-7>0, \quad n>3$

$\sum a_n \quad \lim_{n\to+\infty}\sqrt[n]{a_n} = L > 1$

$$\sqrt[n]{a_n} = \sqrt[n]{\left(\frac{2n-7}{n+3}\right)^n} = \frac{2n-7}{n+3} = \frac{\cancel{n}\left(2-\frac{7}{n}\right)}{\cancel{n}\left(1+\frac{3}{n}\right)} \to 2, \quad n\to+\infty$$

$\frac{7}{n}\to 0$, $\frac{3}{n}\to 0$

<u>divergira</u>

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} \qquad \sum a_n, \ \lim_{n\to+\infty} \frac{a_{n+1}}{a_n} = L > 1$$

$$\frac{a_{n+1}}{a_n} = \frac{(2(n+1))!}{((n+1)!)^2} \cdot \frac{(n!)^2}{(2n)!} = \frac{(2n+2)! \ \cancel{(n!)^2}}{(n+1)^2 \cdot \cancel{(n!)^2} \ (2n)!} = \frac{(2n+2)(2n+1) \cdot \cancel{(2n)!}}{\cancel{(2n)!} \ (n+1)^2} =$$

$$= \frac{(2n+2)(2n+1)}{(n+1)^2} \to 4, \ n \to +\infty$$

diverguje

$$\frac{(2n+2)(2n+1)}{(n+1)^2} = \frac{4n^2+4n+\dots}{n^2+2n+1} = n^2(4+$$

$$\sum_{n=1}^{\infty} \underbrace{\left(\frac{3n+2}{3n+4}\right)^n}_{= a_n}$$

$\sum a_n$

$$\sqrt[n]{a_n} = \frac{3n+2}{3n+4} \to 1, \quad n \to +\infty$$

$$a_n = \left(\frac{3n+2}{3n+4}\right)^n = \left(\frac{3n+4-2}{3n+4}\right)^n = \left(1 - \frac{2}{3n+4}\right)^n =$$

$$= \left(1 + \frac{-2}{3n+4}\right)^{(3n+4)\cdot\frac{n}{3n+4}} = \left(\boxed{\underbrace{\left(1 + \frac{-2}{3n+4}\right)^{3n+4}}_{\to e^{-2}}}\right)^{\frac{n}{3n+4} \to \frac{1}{3}} \xrightarrow[n \to +\infty]{} \left(e^{-2}\right)^{\frac{1}{3}} = e^{-2/3}.$$

$$\lim_{n \to +\infty} \left(1 + \frac{a}{n}\right)^n = e^a$$

$$\lim_{n \to +\infty} a_n \neq 0$$

diverguje.

$$\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n+1}{\sqrt{n^4+3}}$$

$$\frac{n+1}{\sqrt{n^4+3}} \sim \frac{n}{\sqrt{n^4}} = \frac{n}{n^2} = \frac{1}{n}$$

$$\sum (-1)^n c_n$$

$$c_n = \frac{n+1}{\sqrt{n^4+3}} > 0$$

$c_n \downarrow 0$ Leibniz. kr. $\Rightarrow$ konv.

$$\frac{c_{n+1}}{c_n} \qquad f(s) = \frac{s+1}{\sqrt{s^4+3}} \quad \text{pro } s \text{ velká}$$

$$f'(s) = \frac{\sqrt{s^4+3} - (s+1)\frac{1}{2\sqrt{s^4+3}} \cdot 4s^3}{s^4+3} = \frac{s^4+3 - (s+1)\cdot 2s^3}{(s^4+3)\sqrt{s^4+3}} =$$

$$= \frac{s^4+3-2s^4-2s^3}{(s^4+3)\sqrt{s^4+3}} = \frac{-s^4-2s^3+3}{(s^4+3)\sqrt{s^4+3}}$$

$$\sum c_n$$

$$c_n > 0 \qquad c_n = \frac{n+1}{\sqrt{n^4+3}}$$

$$d_n \overset{def}{=} \frac{1}{n}.$$

$\sum c_n \qquad \sum d_n$

$c_n > 0, \quad d_n > 0$

$\exists \lim_{n\to+\infty} \frac{c_n}{d_n} = L < +\infty$

$\Rightarrow \sum c_n$ konv. $\Leftarrow$ $\sum d_n$ konv.

$$\frac{c_n}{d_n} = \frac{\frac{n+1}{\sqrt{n^4+3}}}{\frac{1}{n}} = \frac{(n+1)n}{\sqrt{n^4+3}} = \frac{\sqrt{(n+1)^2 n^2}}{\sqrt{n^4+3}} =$$

$$= \sqrt{\frac{(n+1)^2 n^2}{n^4+3}} \to 1, \ n\to\infty \Rightarrow \sum \frac{n+1}{\sqrt{n^4+3}} \text{ diverg., neboť}$$

diverguje $\sum \frac{1}{n}$ (sr. kr. - lim.)

$$\sum_{n=1}^{\infty} \underbrace{\sqrt[n]{0{,}001}}_{a_n}$$

$\lim\limits_{n\to+\infty} a_n \neq 0$; $\quad a_n = \sqrt[n]{0{,}001} = \left(\frac{1}{1000}\right)^{\frac{1}{n}} = \left(10^{-3}\right)^{\frac{1}{n}} = 10^{-\frac{3}{n}} \xrightarrow[n\to+\infty]{} 1$, $\frac{3}{n} \to 0$

nutná podm., diverguje.

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{2^{n^2}} \qquad \sum a_n \qquad a_n > 0$$

$$\frac{a_{n+1}}{a_n} = \frac{((n+1)!)^2}{2^{(n+1)^2}} \cdot \frac{2^{n^2}}{(n!)^2} = \frac{(n+1)^2 \cdot \cancel{(n!)^2} \cdot 2^{n^2}}{2^{n^2+2n+1} \cancel{(n!)^2}} =$$

$$= \frac{(n+1)^2 \cdot \cancel{2^{n^2}}}{\cancel{2^{n^2}} \cdot 2^{2n+1}} = \frac{(n+1)^2}{2^{2n+1}} \longrightarrow 0 \,, \; n \to +\infty$$

$$\lim_{x \to +\infty} \frac{(x+1)^2}{2^{2x}} \quad (\text{L'Hôp.})$$

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = L = 0 < 1.$$

konverguje. (pod. kr.).

$$\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}$$

$\sum a_n \quad a_n > 0$

$$\lim_{n\to+\infty} \frac{a_{n+1}}{a_n}$$

$$\frac{a_{n+1}}{a_n} = \frac{2^{n+1}\,(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{2^n \cdot n!} = \frac{\cancel{2^n} \cdot 2(n+1)\cdot \cancel{n!} \cdot n^n}{(n+1)^{n+1} \cdot \cancel{2^n} \cdot \cancel{n!}} =$$

$$= 2\,\frac{(n+1)n^n}{(n+1)^{n+1}} = 2\,\frac{n^n}{(n+1)^n} = 2\cdot\left(\frac{n}{n+1}\right)^n = 2\left(\frac{n+1-1}{n+1}\right)^n =$$

$$= 2\left(1-\frac{1}{n+1}\right)^n = 2\left(1+\frac{-1}{n+1}\right)^{(n+1)\cdot\frac{n}{n+1}} =$$

$$\lim_{n\to+\infty}\left(1+\frac{a}{n}\right)^n = e^a$$

$$= 2\left(\underbrace{\left(1+\frac{-1}{n+1}\right)^{n+1}}_{\to e^{-1}}\right)^{\overbrace{\frac{n}{n+1}}^{\to 1}} \xrightarrow[n\to+\infty]{} 2\cdot e^{-1} = \frac{2}{e} < 1$$

$e > 2$

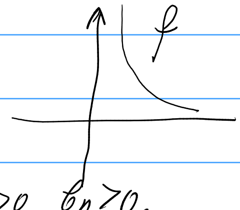
$$\lim \frac{a_{n+1}}{a_n} = \frac{2}{e} < 1 \Rightarrow \text{konv.}$$

(podíl. kr.)

$$\sum_{n=3}^{\infty} \underbrace{\frac{1}{(3n-5)\cdot(\ln(4n-7))^2}}_{a_n} = \sum a_n$$

$$f(t) = \frac{1}{(3t-5)\cdot(\ln(4t-7))^2}$$

$$\ldots = \sum_{n=3}^{\infty} f(n),$$



$$\int_3^{+\infty} f(t)\,dt = \int_3^{+\infty} \frac{dt}{(3t-5)(\ln(4t-7))^2}$$

$a_n>0,\ b_n>0.$

$\lim \frac{a_n}{b_n} = L < +\infty$

$\hookrightarrow$ $\sum a_n$ konv. $\Leftarrow \sum b_n$ konv.

$$\sum b_n, \quad b_n = \frac{1}{(4n-7)\cdot(\ln(4n-7))^2}$$

$$\frac{a_n}{b_n} = \frac{1}{(3n-5)\cancel{(\ln(4n-7))^2}}\cdot(4n-7)\cancel{(\ln(4n-7))^2} = \frac{4n-7}{3n-5} =$$

$$= \frac{\cancel{n}\left(4-\frac{7}{n}\right)}{\cancel{n}\left(3-\frac{5}{n}\right)} = \frac{4-\underbrace{\frac{7}{n}}_{\to 0}}{3-\underbrace{\frac{5}{n}}_{\to 0}} \longrightarrow \frac{4}{3}, \quad n\to+\infty$$

$$g(t) = \frac{1}{(4t-7)(\ln(4t-7))^2}, \qquad \int_3^{+\infty} g(t)\,dt = \int_3^{+\infty} \frac{dt}{(4t-7)(\ln(4t-7))^2}$$

$\int \frac{du}{u(\ln u)^2}$

$$\int_3^{A} \frac{dt}{(4t-7)(\ln(4t-7))^2} = \left|\begin{array}{l} \ln(4t-7) = s \\ ds = \frac{1}{4t-7}\cdot 4\,dt, \quad \frac{1}{4t-7}dt = \frac{1}{4}ds \\ t=3, \quad s=\ln(4\cdot 3-7) = \ln 5 \\ t=A: \quad s = \ln(4A-7) \end{array}\right.$$

$\frac{1}{4}ds$

$$= \frac{1}{4}\int_{\ln 5}^{\ln(4A-7)} \frac{1}{s^2}\,ds = \frac{1}{4}\int_{\ln 5}^{\ln(4A-7)} s^{-2}\,ds = \frac{1}{4}\left[-\frac{1}{s}\right]_{\ln 5}^{\ln(4A-7)} =$$

$$= \frac{1}{4}\left(\frac{1}{\ln 5} - \underbrace{\frac{1}{\ln(4A-7)}}_{\to 0}\right) \to \frac{1}{4\ln 5}, \quad A\to+\infty.$$

$A\to+\infty$?

int. konv. $\Rightarrow$ konv. řada $\sum b_n$ (integr. kr.).

$\Rightarrow$ $\sum a_n$ konv.
(srovn. kr.)