Solubility product

The solubility product constant, K_{sp} is the equilibrium constant for a solid substance dissolving in an aqueous solution. It represents the level at which a solute dissolves in solution. The more soluble a substance is, the higher the K_{sp} value it has.

Consider the general dissolution reaction below (in aqueous solutions):

$$\mathrm{M_mA_a} \longrightarrow \mathrm{m\ M^+ + a\ A^-}$$

To solve for the K_{sp} it is necessary to take the molarities or concentrations of the products $\mathrm{c(M^+)}$ $\mathrm{c(A^-)}$ and multiply them. If there are coefficients in front of any of the products, it is necessary to raise the product to that coefficient power (and also multiply the concentration by that coefficient). This is shown below:

$$K_{sp} = [\mathrm{M}]^m \cdot [\mathrm{A}]^a$$

Note that the reactant, $\mathrm{M_mA_a}$ is not included in the K_{sp} equation. Solids are not included when calculating equilibrium constant expressions, because their concentrations do not change the expression; any change in their concentrations are insignificant, and therefore omitted. Hence, K_{sp} represents the maximum extent that a solid that can dissolved in solution.

Example 1. Calculate the solubility product of $\mathrm{CaF_2}$. Concentration of its saturated solution is $1.888 \times 10^{-4}\ \mathrm{mol \cdot dm^{-3}}$.

The relevant equilibrium is:

$$\mathrm{CaF_2} \longrightarrow \mathrm{Ca^{2+} + 2\,F^-}$$

so the associated equilibrium constant is

$$K_{sp} = [\mathrm{Ca^{2+}}] \cdot [\mathrm{F^-}]^2$$

If we consider the concentration of $\mathrm{CaF_2}$ is c, then concentration of $\mathrm{Ca^{2+}}$ is c and concentration of $\mathrm{F^-}$ is $2c$. Based on this we can calculate

$$\begin{aligned} K_{sp} &= c \cdot (2c)^2 = c \cdot 4c^2 = 4c^3 \\ K_{sp} &= 2.692 \times 10^{-11} \end{aligned}$$

Example 2. Calculate the solubility product of As_2S_3. Concentration of its saturated solution is 8.191×10^{-7} mol $\cdot$ dm^{-3}.

The relevant equilibrium is:

$$As_2S_3 \longrightarrow 2\,As^{3+} + 3\,S^{2-}$$

so the associated equilibrium constant is

$$K_{sp} = [As^{3+}]^2 \cdot [S^{2-}]^3$$

If we consider the concentration of As_2S_3 is c, then concentration of As^{3+} is $2c$ and concentration of S^{2-} is $3c$. Based on this we can calculate

$$\begin{aligned} K_{sp} &= (2c)^2 \cdot (3c)^3 = 4c^2 \cdot 27c^3 = 108c^5 \\ K_{sp} &= 3.981 \times 10^{-29} \end{aligned}$$

Example 3. Calculate the solubility of $BaSO_4$. Solubility product is $K_{sp} = 1.096 \times 10^{-10}$.

$$BaSO_4 \longrightarrow Ba^{2+} + SO_4^{2-}$$

$$\begin{aligned} K_{sp} &= [Ba^{2+}] \cdot [SO_4^{2-}] \\ K_{sp} &= c^2 \\ c &= \sqrt{K_{sp}} \\ c &= 1.047 \times 10^{-5} \text{ mol} \cdot \text{dm}^{-3} \end{aligned}$$

Example 4. Calculate the solubility of $Fe(OH)_3$. Solubility product is $K_{sp} = 3.715 \times 10^{-40}$.

$$Fe(OH)_3 \longrightarrow Fe^{3+} + 3\,OH^-$$

$$\begin{aligned} K_{sp} &= [Fe^{3+}] \cdot [OH^-]^3 \\ K_{sp} &= c \cdot (3c)^3 = 27c^4 \\ c &= \sqrt[4]{\frac{K_{sp}}{27}} \\ c &= 6.090 \times 10^{-11} \text{ mol} \cdot \text{dm}^{-3} \end{aligned}$$

Example 5. Calculate the solubility product of $Bi(OH)_3$. Concentration of its saturated solution is 1.102×10^{-8} mol $\cdot$ dm^{-3}. (3.98×10^{-31})

Example 6. Calculate the solubility of Cu_2S if the solubility product of this salt is $K_{sp} = 2.512 \times 10^{-48}$. $(8.56 \times 10^{-17}$ mol $\cdot$ dm$^{-3})$