

$$I_{I_1}^1 - I_{I_2}^2 = 2 - I_{I_1}^1 - 2 + I_{I_2}^2 = -(I_{I_1}^1 - I_{I_2}^2) < 0;$$

$$\begin{aligned} I_{I_8}^8 - I_{I_6}^6 &= \sqrt{(1 - R^{I_1})^2 + (1 - P^{I_1})^2} - \sqrt{(1 - R^{I_2})^2 + (1 - P^{I_2})^2} \\ &= \sqrt{\left(1 - \frac{8}{10}\right)^2 + \left(1 - \frac{8}{20}\right)^2} - \sqrt{\left(1 - \frac{6}{10}\right)^2 + \left(1 - \frac{6}{12}\right)^2} \\ &= \sqrt{\left(\frac{2}{10}\right)^2 + \left(\frac{6}{10}\right)^2} - \sqrt{\left(\frac{4}{10}\right)^2 + \left(\frac{5}{10}\right)^2} \\ &= \sqrt{\frac{40}{100}} - \sqrt{\frac{41}{100}} < 0; \end{aligned}$$

$$I_{I_9}^9 - I_{I_7}^7 = 1 - I_{I_9}^9 - 1 + I_{I_7}^7 = -(I_{I_9}^9 - I_{I_7}^7) > 0;$$

$$\begin{aligned} I_{I_{10}}^{10} - I_{I_{11}}^{11} &= \frac{r^{I_2}}{2C + 2N^{I_2} - 3r^{I_2}} - \frac{r^{I_1}}{2C + 2N^{I_1} - 3r^{I_1}} \\ &= \frac{6}{20 + 24 - 18} - \frac{8}{20 + 40 - 24} \\ &= \frac{6}{26} - \frac{8}{36} = \frac{216 - 208}{26 \cdot 36} > 0; \end{aligned}$$

$$\begin{aligned} I_{I_{11}}^{11} - I_{I_{13}}^{13} &= \frac{2r^{I_2}}{C + N^{I_2}} - \frac{2r^{I_1}}{C + N^{I_1}} \\ &= \frac{12}{10 + 12} - \frac{16}{10 + 20} \\ &= \frac{12}{22} - \frac{16}{30} = \frac{360 - 352}{22 \cdot 30} > 0. \end{aligned}$$

Thus, we see that in situation 1 and situation 2 the corresponding differences have opposite signs, but the only parameter distinguishing two situations is the number of pertinent documents in the search collection, that is, only  $C$ . Hence, the signs of all the preceding differences depend on the value of  $C$ . This means that complex search characteristics  $I_1, I_6, I_8, I_9, I_{10}$ , and  $I_{11}$  do not possess the order preservation property.

Now, let us show that complex search characteristic  $I_{12}$  also does not have this property. We will do this with the same assumptions we used to demonstrate a similar statement of characteristics  $I_1, I_6$ , and so on with the only addition

being that the search collection contains 1052 documents; that is,  $N_0 = 1052$ . Let us consider two possibilities, as we did previously.

1. The search collection contained 100 pertinent documents ( $C = 100$ ).

In this situation,

$$I_{I_1}^1 = N^{I_1} - r^{I_1} = 12; \quad b_{I_1} = C - r^{I_1} = 92;$$

$$d_{I_1} = N_0 - C - I_{I_1}^1 = 940;$$

$$I_{I_2}^2 = N^{I_2} - r^{I_2} = 6; \quad b_{I_2} = C - r^{I_2} = 94;$$

$$d_{I_2} = N_0 - C - I_{I_2}^2 = 946.$$

Then,

$$\begin{aligned} I_{I_{12}}^{12} - I_{I_{13}}^{13} &= \frac{r^{I_1} d_{I_1} - I_{I_1}^1 b_{I_1}}{\sqrt{(r^{I_1} + I_{I_1}^1)(r^{I_1} + b_{I_1})(b_{I_1} + d_{I_1})(I_{I_1}^1 + d_{I_1})}} \\ &\quad - \frac{r^{I_2} \cdot d_{I_2} - I_{I_2}^2 \cdot b_{I_2}}{\sqrt{(r^{I_2} + I_{I_2}^2)(r^{I_2} + b_{I_2})(b_{I_2} + d_{I_2})(I_{I_2}^2 + d_{I_2})}} \\ &= \frac{8 \cdot 940 - 12 \cdot 92}{\sqrt{20 \cdot 100 \cdot 1032 \cdot 952}} - \frac{6 \cdot 946 - 6 \cdot 94}{\sqrt{12 \cdot 100 \cdot 1040 \cdot 952}} \\ &= \frac{4}{\sqrt{100 \cdot 952}} \left( \frac{1880 - 276}{\sqrt{20 \cdot 1032}} - \frac{1419 - 141}{\sqrt{12 \cdot 1040}} \right) \\ &= \frac{4}{\sqrt{100 \cdot 952}} \left( \frac{1604}{\sqrt{20 \cdot 12 \cdot 2 \cdot 43}} - \frac{1278}{\sqrt{12 \cdot 20 \cdot 2 \cdot 26}} \right) \\ &= \frac{8}{\sqrt{100 \cdot 952 \cdot 480}} \left( \frac{802}{\sqrt{43}} - \frac{639}{\sqrt{26}} \right) \\ &= \frac{8}{\sqrt{100 \cdot 952 \cdot 480}} \\ &\quad \times \frac{\sqrt{802 \cdot 802 \cdot 26} - \sqrt{639 \cdot 639 \cdot 43}}{\sqrt{43 \cdot 26}} \\ &= \frac{8}{\sqrt{100 \cdot 952 \cdot 480}} \cdot \frac{\sqrt{16723304} - \sqrt{17557803}}{\sqrt{43 \cdot 26}} < 0 \end{aligned}$$

2. The search collection contained 10 pertinent documents ( $C = 10$ ). In this situation,

$$I_{I_1}^1 = N^{I_1} - r^{I_1} = 12; \quad b_{I_1} = C - r^{I_1} = 2;$$

$$d_{I_1} = N_0 - C - I_{I_1}^1 = 1030;$$

$$I_{I_2}^2 = N^{I_2} - r^{I_2} = 6; \quad b_{I_2} = C - r^{I_2} = 4;$$

$$d_{I_2} = N_0 - C - I_{I_2}^2 = 1036.$$