Prepositional Aspect and the Algebra of Paths[1]

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The semantics of directional prepositions is investigated from the perspective of aspect. What distinguishes telic PPs (like *to the house*) from atelic PPs (like *towards the house*), taken as denoting sets of paths, is their algebraic structure: atelic PPs are cumulative, closed under the operation of concatenation, telic PPs are not. Not only does this allow for a natural and compositional account of how PPs contribute to the aspect of a sentence, but it also guides our understanding of the lexical semantics of prepositions in important ways. Semantically, prepositions turn out to be quite similar to nouns and verbs. Nominal distinctions (like singular and plural, mass and count) and verbal classes (like semelfactives and degree achievements) have their prepositional counterparts.

0 Introduction

Paths play an important role in the semantics of two empirical domains. They are used in studies of sentential aspect as the spatial or scalar tracks along which events can be temporally measured out (e.g. Verkuyl 1993, Tenny 1994, Jackendoff 1996, Krifka 1998). In the literature on prepositions paths are the kind of entities in terms of which directional prepositions, like *from*, *into*, and *over*, are interpreted (e.g. Jackendoff 1983, Lakoff 1987, Bierwisch 1988, Habel 1989, Piñon 1993, Nam 1995). However, these two empirical domains are not often connected in a systematic way. Prepositional phrases are known to make contributions to the event structure of a sentence:[2]

(1) a. Alex swam (*in/for an hour)
    b. Alex swam to the beach (in/*for an hour)
    c. Alex swam towards the beach (*in/for an hour)

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[2] See for instance Piñon (1993), Jackendoff (1996), Nam (2000), Filip (2003).

A manner of motion verb like *swim* in (1a) has atelic aspect, as shown by the fact that it takes *for* instead of *in* with durative measure phrases. The PP *to the beach* shifts the aspect to telic (1b), but *towards the beach* does not have this effect (1c). Aspectual studies do not go far beyond the aspectual properties of these two prepositions. Spatial studies on the other hand usually do not consider the implications that these phenomena might have for prepositional semantics.

This article intends to relate the two domains of aspect and space in a direct way, by treating the denotation of a directional PP as an algebraically structured set of *paths* that is thematically mapped to a mereology of events along the lines of Piñon (1993), Krifka (1998) and Rothstein (2004). In this way there will be two-way traffic between event structure and spatial structure, as I will show. On the one hand, the lexical semantic definitions of prepositions can be made relevant to event structure in a more principled and more compositional way. On the other hand, the algebraic approach will deepen our insight in the semantics of prepositions because we can exploit the parallelisms with the nominal and verbal domain and use aspectual considerations as a probe into prepositional semantics. The approach taken here is similar in spirit to Piñon (1993) and Nam (2000), but it departs from their proposals in essential respects.

After an introduction to the notion of *prepositional aspect* in section 1, I will describe in section 2 the *algebra of paths* that forms the foundation of the semantics of directional prepositions. How prepositions determine verbal aspect will be the topic of section 3, while section 4 shows how aspectual considerations can be used to both constrain and enrich the lexical semantics of prepositions. Section 5 concludes the article by placing the results in a wider perspective and discussing a few of the remaining questions.

1 Prepositional aspect

1.1 Aspect and prepositions

The spatial prepositions can be divided into locative (or static) and directional (or dynamic) prepositions, that is, between prepositions that are used to indicate where something *is* and prepositions that are used to indicate where something is *going*:

(2) *Locative (static) prepositions*: above, at, behind, below, beside, between, in, in front of, inside, near, on, outside, under

Directional (dynamic) prepositions: across, along, around, away from, down, from, into, off, onto, out of, over, past, through, to, towards, up, via

Locative prepositions can always be used in combination with the copula *be* to form a locative sentence, as in (3a). With directional prepositions this is sometimes possible if the location is understood as the endpoint of a hypothetical journey described by the preposition from an implicit point of view, as in (3b) (Cresswell 1978), or sometimes with a measure phrase, as in (3b'):

(3) a. The car is behind the truck/in the garage/outside the park
b. The car is across the street/around the corner/over the hill (from here)
b'. The car is one mile from the garage/one mile to the east

When we take a closer look at directional prepositions, we see that they can contribute to the aspectual properties of a sentence in different ways. All by themselves manner of motion verbs like *swim*, *walk*, *run* and *drive* are atelic process verbs. Combining these verbs with directional PPs leads to different results:

(4)	a.	Alex walked onto the platform/out of the hotel	in/*for ten minutes
	b.	Alex drove toward the mountains/along the river	*in/for a day
	c.	Alex ran around the lake/through the grass	in/for one hour

The prepositions *onto* and *out of* lead to telic aspect in (4a). In (4b) on the other hand we see two prepositions, *toward* and *along*, that lead to an atelic sentence. Some prepositions behave ambiguously, like *around* and *through* in (4c), allowing either a telic (*in one hour*) or atelic (*for one hour*) interpretation.

In this way, the distinction between 'bounded' and 'unbounded' reference familiar from the verbal and nominal domain shows itself in the prepositional domain too (Jackendoff 1991, Verkuyl and Zwarts 1992, Piñon 1993). Let us call this *prepositional aspect*, using the term *aspect* in a wider sense, for the linguistic presentation of meanings as 'bounded' or 'unbounded', across different syntactic categories (along the lines of Bach 1986, Jackendoff 1991 and Rijkhoff 1991). Analogous to telic and atelic verbs (verb phrases, sentences) we can distinguish telic or bounded prepositions (like *onto*) from atelic or unbounded prepositions (like *along*).[3] Based on the durative adverbs this leads to the following classification:

(5) *Bounded, telic*: to, into, onto, from, out of, off, away from, past, via
Unbounded, atelic: towards, along
(Un)bounded, (a)telic: across, around, down, over, through, up

Even though the judgments about some prepositions may not be sharp and unequivocal, I believe we have a fairly clear empirical domain about which to ask a number of questions. We first of all want to know how boundedness and unboundedness are defined for prepositions and prepositional phrases in general. In other words, what exactly is prepositional aspect? The next question is how individual prepositions come to have the aspect that they have. For example, why is *to* bounded, but *towards* unbounded, even though they are both goal-directed, and how can *across* be ambiguous between a bounded and an unbounded interpretation? If we know what makes a preposition (un)bounded, then the next question is how it transfers this property to the aspect of the verbal domain, i.e. how *to* makes VPs telic and *along* atelic, for example.

Since this article concentrates on the aspect of prepositions, I will completely ignore the role that the noun phrase of a prepositional phrase, the *reference object*, plays in aspect composition and use only definite noun phrases with singular count nouns that refer to stationary objects. How to deal with reference objects that have mass reference, that are conjoined, plural or quantified, as in the examples below, is not the topic of this article.[4]

(6) a. Alex walked through fog
b. Alex drove to Utrecht, Amsterdam and Haarlem
c. Alex jumped over several fences

Another idealization that I make is that the reference objects themselves do not move. Following Talmy's (1983) characterization of the distinction between Figure and Ground in

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[3] I am leaving out the locative prepositions here. They are atelic and can be seen as the prepositional counterpart of states in the verbal domain. The terms *(a)telic* and *(un)bounded* are used interchangeably in this paper.

[4] See Verkuyl and Zwarts (1992) for some discussion of the aspectual effects of extended and plural reference objects and Francez and Steedman (2003) for a treatment of quantified prepositional phrases in general.

spatial relations (also known as Trajector and Landmark), I assume that this idealization is somehow part of the relativistic way in which we conceptualize position and motion in space. Even if a hot air balloon is moving, we understand the path of a bird in a sentence like *The bird is flying around the balloon* in terms of the relative balloon frame and not in terms of the absolute frame of the earth, for instance. This relativism is also shown by the fact that we can describe a constant relation between two moving objects with a locative preposition: *Alex was driving behind the police car* (based on an example from Kracht 2002).

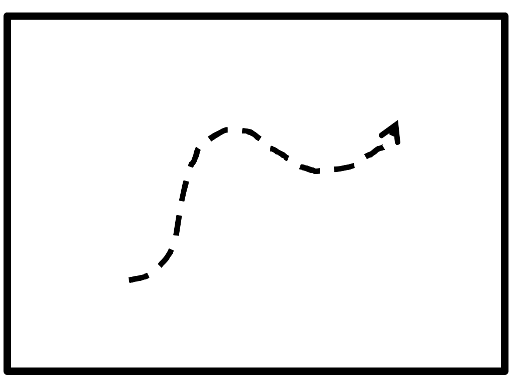
Although I focus on prepositions here, I believe that much of what concerns directional prepositions applies also to directional *adverbs* (many of which also function as directional prepositions anyway, like *up* and *away*) and I will occasionally refer to adverbs to make a point about aspect or paths.

1.2 Prepositions and paths

In the approach taken here, paths are crucial in the same way in which events are crucial in an account of verbal aspect (Bach 1986, Krifka 1989). But what are paths and how are they used? We need to make a few basic assumptions about paths before considering their role in a theory of aspect. More assumptions and refinements will be added along the way.

A path is intuitively a directed stretch of space, typically the trajectory or orbit along which an object moves. Geometrically, it corresponds to a curve with an arrow at one end, as shown in the picture in (7):

(7) A path as a directed curve



A path has a starting point, an end point and points in between, on which the path imposes an ordering, but this ordering is non-temporal. Eschenbach et al. (2000:127) refer to paths as ‘bounded linear oriented structures’.

Let us assume that directional PPs are interpreted as sets of paths and directional prepositions as functions that map objects to sets of paths. At this point in the article I do not want to say more about how paths and prepositions are related to events and verbs (see section 3) and I will not give definitions of individual prepositions until section 4. For now, we will approach the aspect of prepositions in the same way as the aspect of nouns and verbs is treated in the literature. We can make judgments about the unboundedness of words like *wine* and *walk* and form hypotheses about the algebraic structure of their denotations (in terms of quantities or events) without worrying about their lexical semantic definitions. The fact that wine is a liquid and walking a cyclic pattern of certain body part motions is somehow the source of the unbounded aspect, but this is not made explicit at the level of analysis that aspectual theories operate on. Prepositions can be treated in the same way, but what makes them different from nouns and verbs is that they are a closed class defined over the relatively well-defined conceptual domain of space. This makes it possible to study the relation between lexical definitions and aspectual structures in detail.

Another important assumption that I make is that all directional PPs are interpreted with respect to one and the same domain of paths. We do not have different kinds of semantic objects for different types of directional prepositions. Bohnemeyer (2003) suggests that some prepositions (like *to*) refer to 'vectors', while others (like *around*) refer to 'path shapes'. In this article, there is no such distinction. There is an empirical and a methodological argument for this uniformity. The empirical argument is that different types of directional PPs co-occur as verbal adjuncts modifying one and the same motion event:[5]

(8) a. Alex walked through the daylight over the yard to the shed
    b. Alex drove across the bridge from the park towards the mountains

Notice that the sequencing of PPs does not express a sequencing of movements. Alex in (8a) did not first walk through the daylight and then over the yard and then to the shed. We want to analyze (8a) and (8b) in roughly the following way:

(9) a. The path of Alex' walking event is through the daylight and over the yard and to the shed

[5] Although there are constraints on what directional expressions can be combined in one clause, as Goldberg (1991), Tenny (1994), and Bohnemeyer (2003) show.

b. The path of Alex' driving event is across the bridge and from the park and into the wilderness

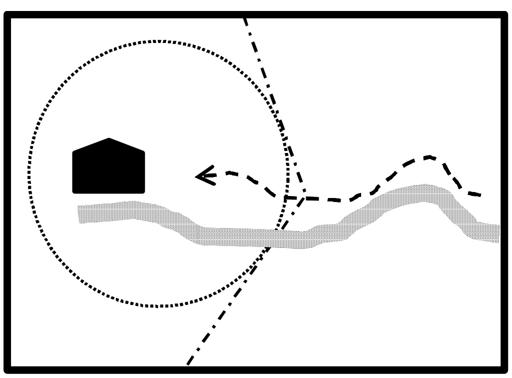
The directional PPs in these sentences are all predicates of one path, the trajectory of the theme, the moving object, of the event. The PP denotations must intersect and this can only work if they are all of the same type.

The methodological argument for uniform interpretation is that it provides a common ground for comparing prepositions, defining classes of prepositions, and making generalizations. This is especially important when aspectual classes are involved. If *around* is of a fundamentally different type from *to*, then it is much harder to see how these words can both lead to telic aspect than when they define subsets from one and the same domain of paths.

1.3 Paths and aspect

How are paths going to help us to explain the aspectual contribution of prepositions? One approach that won't work is to look at individual paths, because a path, all by itself, cannot be telic or atelic. To see this, consider the path in the following figure and imagine it to be the trajectory of a driving car.

(10) A path with multiple descriptions



This path can be described in each of the following ways:

(11) The car drove ...

a.	into the valley (*for hours)	(telic)
b.	towards the house (for hours)	(atelic)
c.	across the border (*for hours)	(telic)
d.	along the river (for hours)	(atelic)

As the durative adverbs indicate, two of the PPs that describe the path are telic, two are atelic. This shows that (a)telicity is a property of a *description* of objects, not of the objects themselves. The same observation has repeatedly been made in the nominal and verbal domain. The same object can be described as mass (*bread*) or as count (*a loaf*) (Chierchia 1998) and the same activity of running can be described as a process (*Alex ran*) or as an event (*Alex ran a mile*) (Krifka 1998). It is important to realize that prepositional aspect is no different in this respect.

This implies that, in spite of the suggestive term *telicity* (from Greek *telos* ‘end, boundary, goal’) it cannot be the presence or absence in a path of endpoints or culmination points that determines prepositional aspect in a simple way. Prepositional phrases can be telic without having an endpoint in a clear sense and atelic while being bounded by the reference object. Consider the following examples.

(12)
a. Alex ran away from the accident
b. Alex swam towards the island
c. Alex jogged three times around the village
d. Alex drove round and round the hotel

(12a) can be telic even though *away from the accident* does not specify a particular endpoint. What the reference object *the accident* specifies is the starting point, but where the paths end is left open, as long as it is *not near* the accident. The reverse is seen in (12b): atelic aspect in spite of the presence of a goal.[6] This means that there is no aspectual asymmetry between source prepositions and goal expressions, contrary to what is suggested in Filip (2003). Both source and goal prepositions can be bounded and unbounded; we are dealing with two

[6] One could say that *the island* in *towards the islands* is a kind of limit that the path never reaches or that the path is a topologically open set bounded by the island (Jackendoff 1991:37). A simple definition of *towards* that does not require such devices is given in section 3.

independent dimensions of the semantics of prepositions. In (12c) the telicity comes from the cardinality expression *three times*, not from the specification of an endpoint and (12d) is atelic because the number of circumventions of the hotel is unspecified, but not the endpoints of the individual paths around the hotel (which are identical to the starting point).

This article intends to give a unified account of these phenomena by taking an algebraic approach to aspect, locating telicity and atelicity in the algebraic structure of PP denotations. In order to do this we first need to lay a formal foundation for an algebra of paths that can help us to formulate these properties.

2 An algebra of paths

In a model-theoretic approach there are different ways to formalize the informal notion of path that we used in the previous section. One way is to take paths as *primitives* in the universe of discourse and develop a system of axioms that characterizes their properties. This is the approach of Piñon (1993) and especially of Krifka (1998) and Eschenbach et al. (2000). Instead of taking paths as primitives they can be *constructed* out of other elements, as nested sets or sequences of places (Bierwisch 1988, Verkuyl and Zwarts 1992, respectively) or as functions from some ordered domain to places (Cresswell 1978, Habel 1989, Nam 1995). The axiomatic approach allows one to introduce just as much assumptions as one needs and it avoids the ‘filmstrip model’ of movement as simply a sequence of static snapshots (a point made by Jackendoff 1996 and Krifka 1998). The constructive approach on the other hand has the advantage of making the relation between paths and places maximally explicit and of being closer to our geometric intuitions. Both approaches are compatible with the kind of path algebra that I will adopt here. Nevertheless, for the sake of concreteness, I will assume constructed paths, defined as continuous functions from the real unit interval [0,1] (the ‘indices’) to positions in some model of space. The relation between paths and positions is straightforward: the starting point of path $\mathbf{p}$ is $\mathbf{p}(0)$, the end point is $\mathbf{p}(1)$ and for any $i \in [0,1]$ $\mathbf{p}(i)$ is the corresponding point of the path. The set of paths in the universe of discourse will be denoted by $\mathbf{P}$. The appendix of this article gives a more precise definition of this formal notion of path, for which I certainly do not want to claim cognitive reality, but that seems at least explicit and restricted enough to model the relevant spatial and aspectual properties of directional prepositions.

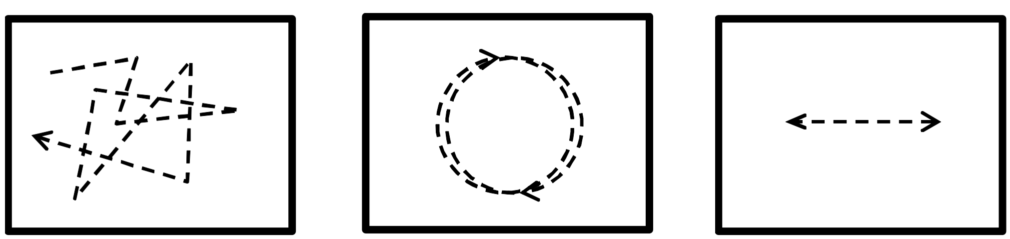
In earlier work I argued that positions and other spatial properties are best understood as *relative positions*, modeled by *vectors* (Zwarts 1997, Zwarts and Winter 2000, Zwarts 2003).

Nothing of what I say in this article hinges on that directly, but the relative nature of positions (and therefore also of paths) is important to address a problem that was raised by a anonymous reviewer. Because paths are atemporal stretches of space they require a fixed, stationary reference object. As I already mentioned in section 1.1, the reference object may in fact be moving, as in *The bird is flying around the balloon*. When the balloon is moving along a straight line, the atemporal path of the bird is not a circular path enclosing the balloon, but a sinusoidal path along the straight path of the balloon. Instead of following the reviewer's suggestion to make paths time-dependent, I would assume that the path of the bird is represented as a sequence of vectors that take the balloon as their origin and not as a sequence of positions in an absolute space. The balloon defines the centre of its own vector space, that is embedded (either stationary or moving) in an absolute space, along the lines of Zwarts and Winter (2000). But for the remainder of this article I will assume reference objects that are fixed.

Notice that when we define a path as a sequence of positions, we can get paths that cross themselves one or more times, that cover the same ground more than once and that backup and traverse the same stretch of space again backwards. The following examples demonstrate clearly that this should not be ruled out:

(13) a. Alex walked *all around* the city centre
b. Alex ran *round and round* the track
c. Alex paced *back and forth* the alley

(14) a. path crossing b. path overlapping c. path backing up

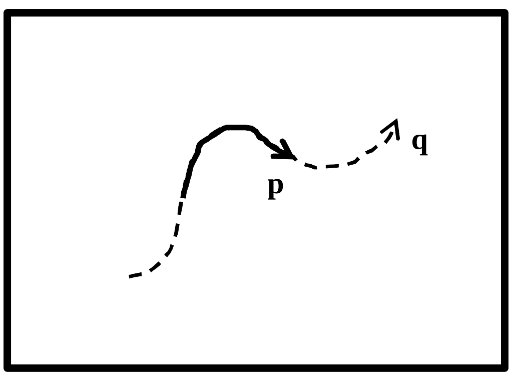


Alex' path crosses itself in (13a), repeats itself in (13b), and contains backups in (13c), as illustrated in the corresponding pictures in (14). It is part of the lexical semantics of these complex prepositions that it happens this way, and not a matter of how simple paths are traversed in temporally complicated ways. Therefore, we have to deal with it within the semantics of prepositions and cannot relegate it to the semantics of verbs or time.

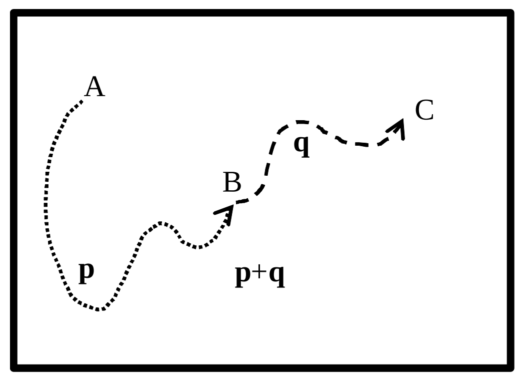
What we need to distinguish carefully in this respect is a path **p** (which is a directed spatial entity) and the space covered by that path (which is a non-directed spatial entity) and which is simply defined as the range of the function **p**. A path that goes around the house once and a path that goes around it five times can have the same range, but they are different paths, having different aspectual implications, as we will see in section 4.

The set **P** of paths is partially ordered by a *subpath* relation, as shown in the left-hand picture below. Roughly speaking, **p** is a subpath of **q** (**p** ≤ **q**) if **p** is the same path as **q** restricted to part of its domain. See the appendix for the definition of ≤ and its proper variant <. A natural sum operation over paths is *concatenation* (Habel 1989, Nam 1995), illustrated in the second picture.

(15) a. **p** is a subpath of **q**

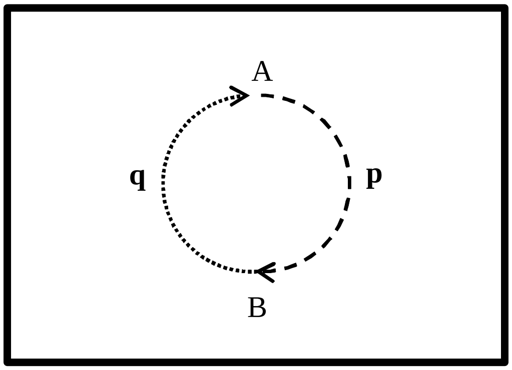


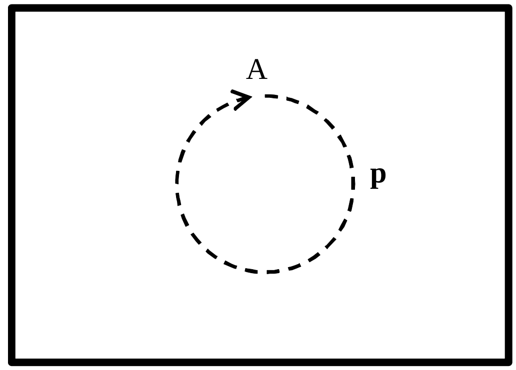
b. the concatenation of **p** and **q**



If **p** is a path from A to B and **q** is a path from B to C, i.e. if **p**(1)=**q**(0), then **p**+**q** is the path that takes **p** to get from A to B and **q** to get from B to C. Obviously, the 'concatenants' are always subpaths of the concatenation. Concatenation is a partial operation, only defined when the second path starts where the first ends. It is associative, (**p**+**q**)+**r**=**p**+(**q**+**r**), but it is neither commutative (**p**+**q**≠**q**+**p**) nor idempotent (**p**+**p**≠**p**). This can be seen in the following two figures:

(16) a. non-commutativity of concatenation b. non-idempotence of concatenation





Picture (16a) illustrates that **p**+**q** is a path that leads from A back to A, while **q**+**p** is the path that goes the other way round, from B to B, so **p**+**q**≠**q**+**p**. Picture (16b) shows a path **p** going around once. The path **p**+**p** is a different path, going around twice. Hence **p**≠**p**+**p**.

Armed with the subpath relation and the concatenation operation we can now take a closer look at the structure of PP denotations.

3 The algebra of paths and verbal aspect

3.1 How to define prepositional aspect algebraically

In the literature, two closure properties have been proposed to account for unbounded reference in the verbal and nominal domain, one ‘downward looking’ closure property based on a proper part relation and one ‘upward looking’ closure property based on a sum relation. When applied to directional PP denotations, as sets of paths, these definitions run as follows:

(17) a. A set of paths **X** is *divisive* (or *homogeneous*) iff for all **p**, **q** ∈ **X**, if **q** < **p**, then **q** ∈ **X**.

b. A set of paths **X** is *cumulative* iff

(i) there are **p**, **q** ∈ **X** such that **p**+**q** exist and

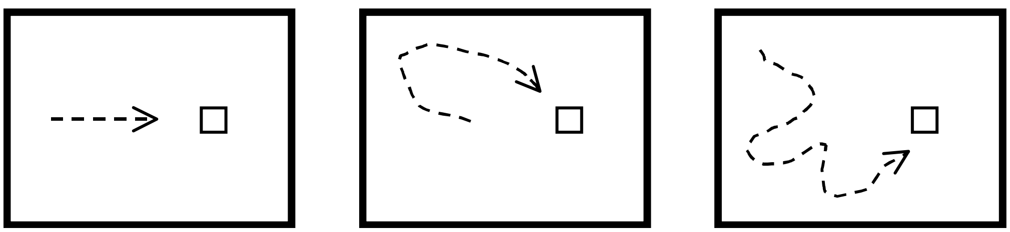
(ii) for all **p**, **q** ∈ **X**, if **p**+**q** exists, then **p**+**q** ∈ **X**.

The definition of cumulativity for paths is slightly more complex than for other domains, because concatenation of paths is a partial operation. Cumulative sets must contain paths that connect head-to-tail and it requires the concatenations of these paths to be found in the same set.[7]

Piñon (1993) and Nam (2000) propose that what characterizes an atelic PP like *towards the station* is divisive reference. At first sight this seems intuitively right: when **p** is a path towards the station, then every subpath of **p** is also a path towards the station. It works like that in picture (18a). *To the station*, on the other hand, is not divisive, because a path to the station may have proper subpaths that do not reach the station.

(18) Three paths that can be labeled *towards the station*

a. b. c.



However, picture (18b) and (18c) show that divisivity does not hold when paths curve in particular ways. Every subpath of the path in (18a) is also towards the station, but not all of the subpaths in (18b) and (18c). Some subpaths (like initial parts of the path in the second picture) are actually pointing away from the station. So *towards the station* does not have divisive reference even though it is clearly an atelic PP. There are other atelic PPs that do not have divisive reference either:

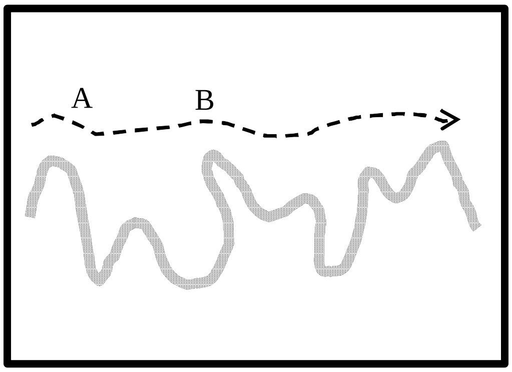
(19) a. Alex drove around the city centre (for/*in a day)
b. Alex walked round and round the block (for/*in a day)
c. Alex drove along the river (for/*in a day)

When Alex is driving round the city centre in a crisscross fashion (which is the reading of (19) intended here), then there are subpaths of the path of movement that we would no longer describe as *around the city centre* (e.g. when he is driving down one particular street). We can only use this PP for paths that ‘fill up’ the interior of a reference object in a certain sense and

[7] I am thankful to Carola Eschenbach and an anonymous reviewer for making me realize that without clause (i), sets without any connecting paths (like the denotation of *into the house* in 4.1.1) would be vacuously cumulative, which is not the intention, of course.

have a certain level of convolutedness. If a path is in the denotation of the atelic PP *round and round the block*, then it has subpaths that are not in this set (paths that go around the block once, around a corner, etc.). The non-divisivity of *along the river* can be illustrated in the following way:

(20) A path *along the river*



Even though a path can be along the river at a certain scale, at a smaller scale this may no longer hold. It is not clear, for example, whether or not the subpath leading from A to B still counts as along the river.

The conclusion must be that divisivity is not the algebraic property that characterizes unbounded PPs. This does not mean that there are no unbounded PPs that have divisive reference. Take *through the tunnel* in its unbounded sense (*walk through the tunnel for hours*): every subpath of a path through the tunnel will also be through the tunnel. But even here there is a limit to divisibility, as an anonymous reviewer pointed out. There may be parts of paths through the tunnel that move across rather than forward and that we would hesitate to call *through the tunnel*. In general, we can say that a PP is only divisive if its definition depends purely on location, i.e. *through the tunnel* will only be divisive if it refers to paths consisting of positions *in* the tunnel without involving orientation or direction. Being this narrow, divisivity does not qualify as the defining property for unboundedness and it may not even be a relevant property at all. This is not a surprising outcome, given what we know from the way mass nouns refer in the nominal domain and process verbs in the verbal domain. That mass nouns have divisive reference seems natural for *water* (ignoring its molecular structure), but not for *furniture* (a part of a piece of furniture is no longer furniture). The parts of a process that we call *walk* will not always be called *walk* themselves (e.g. lifting the left foot a few inches from the ground).

The property that gives us an adequate characterization of unboundedness for prepositions (and for nouns and verbs as well) is cumulativity. If two paths are *towards the station*, then so is their sum, *if it exists*. This is an important proviso. Remember that sum of paths is defined as concatenation, which is only possible if the paths 'connect' head to tail. Cumulative reference is also the right property for the PPs in (19). A crisscross path concatenated with a crisscross path gives another crisscross path, adding up repeated circles around the block lead to repeated circles around the block, concatenating one path along the river with another path along the river just makes a longer path along the river.

On the other hand, none of the bounded PPs has cumulative reference and there are two general reasons for this. Some PPs lack cumulative reference because no two paths in their denotation can be concatenated, which is the case with *to*. The end point of a *to* path is always outside the reference object, the starting point never is. A PP like *around the house* on the other hand, in its basic notion of one single circular path enclosing the reference object, allows paths to be concatenated, but the result of that concatenation is not a path enclosing the house once, but a path that encloses it twice. The conclusion is:

(21) a. A PP is unbounded if and only if it has cumulative reference.
b. A PP is bounded if and only if it does not have cumulative reference.

Do we have to say more than this? Boundedness in other domains is identified with the properties of being quantized and being telic (Krifka 1998). When applied to paths these definitions are as follows:

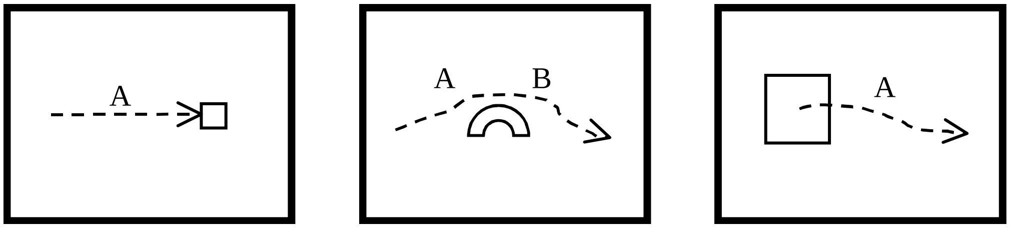
(22) a. A set of paths $\mathbf{X}$ is *quantized* iff for all $\mathbf{p}$, $\mathbf{q} \in \mathbf{X}$, not $\mathbf{p} < \mathbf{q}$.
b. A set of paths $\mathbf{X}$ is *telic* iff for all $\mathbf{p},\mathbf{q} \in \mathbf{X}$, if $\mathbf{p} \leq \mathbf{q}$, then $\mathbf{p}(0)=\mathbf{q}(0)$ and $\mathbf{p}(1)=\mathbf{q}(1)$.

Is quantization the right property for characterizing bounded PPs? Phrased differently, do paths in a bounded PP denotation never have subpaths in the same denotation? It seems they *do* in fact have subpaths. Consider the following examples:

(23) a. Alex ran to the house (in/*for a minute)
b. Alex walked over the bridge (in/*for two minutes)
c. Alex crawled out of the room (in/*for three minutes)

Intuitively, we can draw paths from the PP denotations of these sentences as follows:

(24) a. to the house b. over the bridge c. out of the room



If the path in picture (24a) is in the denotation of *to the house*, then so is the proper subpath that starts at A and ends at the house. The path in (24b) also has a subpath, from A to B, that goes over the bridge. In (24c) the indicated path out of the room has a subpath from inside the house to point A that is also in the denotation. So, bounded PPs are not quantized.

Bounded PPs could have been quantized if we would have set up our semantics in a different way, if the paths of *to the house*, *over the bridge* and *out of the room* would be the smallest paths satisfying the constraints of the preposition (the 'atoms' in a sense). *Over the bridge*, for example, would denote in this view any path of which $\mathbf{p}(0)$ and $\mathbf{p}(1)$ are not on the bridge while every i in the open interval (0,1) is on the bridge. No subpaths of such paths are in the denotation of *over the bridge*. For *to the house* we would have to assume that $\mathbf{p}(0)$ is not at the house, $\mathbf{p}(1)$ is at the house and every point in between is undefined, because we are dealing with an instantaneous transition here. I do not think that paths can have such a 'minimal' structure for reasons that I mentioned in section 1.2. Combinations of PPs can be used to denote paths, e.g. *from the barn to the house*, which we want to compositionally interpret as the intersection of the two PPs. This is only possible if paths are *not* minimal, i.e. if the denotation of *to the house* includes a path for every possible starting point (e.g. the barn) and not just for those starting points just outside the house.

The property of telicity, under Krifka's definition in (22b), does not help us either in defining bounded PPs, because telicity does not give us much more than quantization. Being quantized implies being telic and the only way in which a PP denotation can be telic without being quantized is when the paths are circular, as in (16b), because only then can a proper subpath have the same starting point and end point as the containing path, e.g. $\mathbf{p} < \mathbf{p} + \mathbf{p}$.[8] The problem is, however, that we have a PP that clearly behaves in an unbounded way, namely

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[8] Thanks to an anonymous reviewer for pointing this out to me.

round and round the block, with the telic reference property defined in (22b). This shows that Krifka's telicity is not the right property for characterizing bounded PPs (although I will continue using the term telic a-theoretically, as a synonym of bounded).

3.2 How to transfer prepositional aspect

We have set up the outlines of a system in which the aspect of directional PPs is represented in terms of closure under concatenations (cumulativity). Atelic PPs are closed under concatenation, telic PPs are not closed under concatenation. The next step is to show how these closure properties are transferred to the verbal system.

We can essentially follow the line of Verkuyl (1993), Piñon (1993), Jackendoff (1996), Krifka (1998) in assuming that aspectual properties are transferred from the PP denotation to the verbal denotation by a thematic role with homomorphism properties. The basic link between verbs and directional PPs is performed by a thematic function that maps events to their spatial trace (similar to the spatial trace function in Link 1998 and the movement relation in Krifka 1998). I will call this function TRACE. If *e* is a motion event, then TRACE(*e*) is the path followed by the theme of *e*. The theme is usually an explicit argument of the verb, but it can also be an implicit participant, like the understood projectile in a sentence like *Alex shot the pianist through the window* (see Nam 1995 and Kracht 2002 for further discussion). TRACE is a function over the set of motion events, because every motion event has a unique path. This doesn't mean that *Alex ran around the park* and *Alex ran along the fence* cannot both be true for the same event. As we saw earlier, one path can be described in different ways. TRACE is not a one-to-one function, because different events might in principle be mapped to the same path. The TRACE function allows us to formulate a simple compositional rule for combinations of a verb and a PP:

(25) $[\![\text{V PP}]\!] = \{ e \in [\![\text{V}]\!] : \text{TRACE}(e) \in [\![\text{PP}]\!] \}$

The PP restricts the denotation of the verb (a set of events) to those events that have paths in the PP denotation as their trace. Here is how this works for the tenseless VP *walk to the station*:

(26) ⟦ walk to the station ⟧ = { e ∈ ⟦ walk ⟧: TRACE(e) ∈ ⟦ to the station ⟧ } =

{ e ∈ ⟦ walk ⟧: TRACE(e) ∈ { **p**: **p**(1) is at the station } }=

{ e ∈ ⟦ walk ⟧: TRACE(e)(1) is at the station }

The subject comes in through another thematic role, the THEME role, and results in the addition of an additional restriction on the set of events denoted by the tenseless sentence:[9]

(27) ⟦ Alex walk to the station ⟧ =

{ e ∈ ⟦ walk ⟧: TRACE(e)(1) is at the station and THEME(e)=alex }

Something similar happens when the theme is the object of a sentence (e.g. the pram in *push the pram to the station*). Discussing the linking of arguments to paths would lead us too far from our topic. See Nam (1995) and Jackendoff (1996) for accounts in relation to theories of paths and aspect.

This gives us the rough core of the compositional process, leaving out many interesting phenomena, including those where objects instead of events are mapped to paths (Talmy 1996, Fong 2000, Gawron 2004 among many others):

(28) a. the road to the station
b. the bus to the station
c. The road leads to the station

I will assume that in addition to the TRACE function for events we need another function that maps out objects along paths in a particular way (see Zwarts 2003a for a formulation in the spirit of this article).

In the algebraic event semantics of Bach (1986), Krifka (1998) and others, a sum operation is assumed for events, analogous to the concept of sums in the lattice-based semantics of plurals and mass terms (Link 1998). Given two events e and e' of running, there is also the mereological sum of e and e'. This kind of summation of events is a much less restricted operation than the concatenation operation of paths defined in this article. While the concatenation of two paths can only be formed when the paths connect head-to-tail, the sum

[9] When tense applies to this tenseless sentence it introduces an existential quantifier over events and a temporal location for the events relative to contextually given temporal parameters, a fairly standard assumption.

of two events also exists when the events are separated in time or when they overlap. This is because the mereological sum *aggregates* events into a kind of plural object but it does not *integrate* them into one single unitary event. This makes it less straightforward to relate our algebra of paths in a transparent way (namely by a homomorphism) to the usual type of event structure. Two events of running e and e' with their corresponding traces **p** and **q** will always have an event sum, but if the two events are spatially separated, then the TRACE of this event sum is not defined. I will therefore follow Rothstein (2004) in assuming a more restricted operation on events, closer to the algebra of paths: if two events are spatiotemporally adjacent, then this operation fuses them into another 'singular' event. So, if John swam from 1am to 2am from A to B and from 2am to 3am from B to C, then we can concatenate these events into one swimming event (from 1am to 3am, from A to C). I will represent this particular operation on events with the same symbol + as concatenation of paths, because this brings out the analogy and it will always be clear from the context whether we are concatenating paths or events. In addition, I also assume a subevent relation ≤, which can be defined in terms of the concatenation operation +.

Given the two structures of events and paths, TRACE can be characterized as a homomorphism from events to paths. It is a homomorphism because it preserves structure: $e \leq e'$ implies $\text{TRACE}(e) \leq \text{TRACE}(e')$ and $\text{TRACE}(e+e') = \text{TRACE}(e)+\text{TRACE}(e')$, if $e+e'$ is defined. If a walking event e is a subevent of a walking event e', then the path of e is a subpath of the path of e' and the trace of two events is the concatenation of the traces of the individual events.

Verbs like *walk*, *drive*, *swim*, and *push* are always cumulative in reference. The VP that results from combining such a verb with a non-cumulative PP like *to the house* is non-cumulative in reference. This is because TRACE requires every event in the VP denotation to be mapped to a path in the PP denotation. If e and e' are in ⟦ walk ⟧, so is their concatenation $e+e'$, if it exists. If their paths $\text{TRACE}(e)$ and $\text{TRACE}(e')$ are in ⟦ to the house ⟧, then e and e' are in ⟦ walk to the house ⟧, but their concatenation will never be, because $\text{TRACE}(e+e')$, which is identical to $\text{TRACE}(e)+\text{TRACE}(e')$, is not in the non-cumulative ⟦ to the house ⟧. This is different with a cumulative PP like *along the river*. ⟦ walk along the river ⟧ is cumulative because the cumulativity of ⟦ along the river ⟧ ensures that if two walking events have a trace along the river, then their concatenation, if defined, has a trace along a river.

From the point of view of compositionality it is important to treat PP denotations as members of an independent spatial algebra that are related to the event algebra by one general thematic role. In Krifka (1998) directional prepositions are treated as three-place relations between paths, objects and events, as shown in (29):

(29) a. Alex walk to the capitol

b. { *e*: there is a path **p** such that WALK(alex,**p**,*e*) and GOAL(**p**,capitol,*e*) }

The tenseless sentence in (29a) is interpreted as a set of events. What the preposition *to* contributes is the thematic relation GOAL between the path **p**, the capitol, and event *e*. However, there is no straightforward denotation for the directional PP *to the capitol*, because both the verbal and the prepositional contribution involve paths *and* events.[10] In this article we want to keep both events and paths where they belong: events in the verbal domain and paths in the prepositional domain, each with their own algebra (although structured along very similar lines) and related by general thematic roles like TRACE. 'Source' and 'goal' are not thematic roles, but extremities of paths (**p**(0) and **p**(1), respectively) that only play a role PP-internally. In our approach, the sentence in (30a) denotes a set of events that is defined as in (30b), which allows the PP *to the capitol* to be defined as the set of paths in (30c):

(30) a. Alex walk to the capitol

b. { *e*: WALK(*e*) and THEME(*e*)=alex and TRACE(*e*)(1) is at the capitol }

c. { **p**: **p**(1) is at the capitol }

4 The algebra of paths and prepositional semantics

Up to this point we have been discussing prepositional semantics in an informal and global way, assuming that we have a rough understanding of how individual prepositions are defined and what kind of paths are in their PP denotations. Our focus has been on the aspectual properties of prepositions. We must now turn to the other half of our mission and investigate the implications that aspect has for the lexical semantics of prepositions. What does the atelicity of *towards* teach us about the definition of this preposition? How are the atelic and telic versions of *through* defined and related? We will first use aspect to *constrain* definitions of prepositions and then consider ways in which prepositional semantics can be *enriched*.

[10] One way to provide *to the capitol* with its own denotation is to make it a relation between paths and events.

4.1 How aspectual considerations constrain prepositional semantics

A widespread assumption is that many directional prepositions can be defined by locating the starting point of the path **p**(0), the end point **p**(1), or an intermediary point **p**(*i*) in a particular region relative to the reference object. This is summarized in the following table (see Jackendoff 1983,1991, Asher and Sablayrolles 1995, Zwarts and Winter 2000, and Kracht 2002 for different versions of this idea and Piñon 1993 for a related typology of path expressions):

(31) *How directional prepositions relate paths to locations*

		'at'	'in'	'on'	'above'
source prepositions	**p**(0)	from	out of	off	
goal prepositions	**p**(1)	to	into	onto	
route prepositions	**p**(*i*)	via, past	through	across, over	over

What defines *into,* for example, is that the end points **p**(1) of its paths are 'in' the reference object. This does not mean that *into* is defined in terms of the preposition *in* (even though in this case *into* happens to consist of *in*+*to*), but that *into* and *in* both involve location in the interior of an object. This explains the following entailment:

(32) Alex will go *into* the vault → Alex will be *in* the vault

Some of these prepositions have additional, sometimes non-spatial, semantic properties that I will have to ignore here. The semantic structure and the polysemy of a preposition like *over* has been the topic of a host of literature (see Lakoff 1987 and many others) that I cannot possibly address. However, the basic analysis of directional prepositions laid down in (31) is beyond dispute.

The route prepositions often describe paths that go from one side of the reference object to the opposite side, i.e. the reference object is located between **p**(0) and **p**(1), as Nam (1995) suggests or they assume that the reference object has a particular shape or orientation with respect to the path (see Talmy 1983 and Landau and Jackendoff 1993 on *across*). Other prepositions do not even fit this description, like *up* and *down*, *away from* and *towards,* and *around* and *along,* and I will come back to these later.

4.1.1 Source and goal prepositions

We will take a closer look now at what seems at first sight to be a natural way to define a directional preposition like *into*:[11]

(33) ⟦ into the house ⟧ = { **p**: **p**(1) is inside the house }

The PP *into the house* denotes the set of paths that have their end points inside the house. This is a very weak definition, as shown by some examples of paths that are included in the denotation of *into the house*, schematically represented:

(34) a.
```
++++++++++++
0          1
```
b.
```
---+++---+++
0          1
```
c.
```
+++------+++
0          1
```
d.
```
-------+++++
0          1
```

The line of plusses and minuses represents for which points of the interval [0,1] the path is inside the house (+) or not (–). According to the definition a path that has all of its points inside the house (34a) would count as *into the house*, as well as a path that goes into the house, leaves the house and goes back again (34b) or a path that starts inside and stops inside but has an outside middle part (34c). The definition allows anything to happen before **p**(1) as long as the path ends inside the house. However, we commonly understand *into the house* as referring *only* to paths that have the two-stage structure in (34d), a negative and a positive 'phase' (Kaufmann 1989, Fong 1997 and Kracht 2002). And in fact, this intuition is supported by the aspectual behaviour of *into.* The weak definition leads to a set of paths that is cumulative, for the simple reason that, if two paths **p** and **q** are in the denotation of *into the house*, then so is their concatenation **p**+**q** because it has the same end point as **q**. Clearly, this is not what we want, because *into* is telic in aspect. This clearly shows that *into* and the other

[11] What it means for a point to be inside the house is not something I will try to define here. The same is true for other locative relations that figure in the definitions of directional prepositions in this article. See Zwarts (1997) and Zwarts and Winter (2000) for a vector-based approach to such relations.

prepositions in (31) need a stricter definition to get the right aspect, a definition that is based on a single transition from one phase to another phase. There are different ways to define this, but here is a relatively transparent one:

(35) ⟦ into the house ⟧ = { **p**: there is an interval $I \subset [0,1]$ that includes 1 and that consists of all the indices $i \in [0,1]$ for which **p**(i) is inside the house }

Under this definition ⟦ into the house ⟧ is not cumulative, because it contains no paths that can be concatenated: **p**(0) and **p**(1) are always in different areas. 0 is always excluded from the 'positive' interval I (and hence, **p**(0) outside the house), while 1 is always included in I (and **p**(1) inside the house).

In this way, aspectual data help us to constrain the lexical semantics of prepositions: they give us a reason to choose a particular analysis. The source and goal prepositions in (31) all have exactly one positive phase that overlaps with either the starting point or end point. Their definitions are given in a branching format in (36):

(36) { **p**: there is an interval $I \subset [0,1]$ including …

… 0 and consisting of all the $i \in [0,1]$ for which **p**(i) is at x } = ⟦ from x ⟧

… 0 and consisting of all the $i \in [0,1]$ for which **p**(i) is on x } = ⟦ off x ⟧

… 0 and consisting of all the $i \in [0,1]$ for which **p**(i) is in x } = ⟦ out of x ⟧

… 1 and consisting of all the $i \in [0,1]$ for which **p**(i) is at x } = ⟦ to x ⟧

… 1 and consisting of all the $i \in [0,1]$ for which **p**(i) is on x } = ⟦ onto x ⟧

… 1 and consisting of all the $i \in [0,1]$ for which **p**(i) is in x } = ⟦ into x ⟧

It might not be necessary to assume that these stricter definitions are the lexical semantic representations of the prepositions. A more interesting avenue to explore is that the strict interpretation is the result of the interaction between a weak definition and pragmatic principles that strengthen it, analogous to what happens with the scalar implicatures for *some* (implying *not all*). If that is true, then pragmatics feeds aspect in an interesting way. But this is the topic for another article.

4.1.2 Route prepositions

With a route preposition like *over the fence* we see something similar. Suppose we define the denotation of this PP in the following way:[12]

(37) ⟦ over the fence ⟧ = { **p**: there is an $i \in [0,1]$ such that **p**(i) is on/above the fence }

Again, this denotation is so weak that it includes paths that are on the fence all the time or that go back and forth, or that start (and/or end) on the fence, in addition to the kind of paths that we want, with just one single positive part in the middle:

(38) a.
```
++++++++++++
0          1
```
b.
```
--++----++--
0          1
```
c.
```
++++++------
0          1
```
d.
```
----++++----
0          1
```

The definition of *over the fence* gives a cumulative reading, because if two paths each have a point on the fence, then their concatenation has a point on the fence too. What makes *over the fence* different from *into the house* is that it is aspectually ambiguous. It does have unbounded readings that correspond to situations in (38). It can refer to paths that are on the fence all the time (like (38a)) or to paths that go back and forth over the fence (like (38b)). But it also has a more prominent bounded reading that corresponds to (38d), a 'singular' version of *over the fence*. It is this last reading that we will define here and I will come back to the other two readings in section 4.2. In order to get the non-cumulative 'singular' denotation, the basic definition of *over* should be more constrained, in the same way as *into*:

(39) ⟦ over the fence ⟧ = { **p**: there is an interval $I \subset [0,1]$ that does neither include 0 nor 1 and that consists of all the $i \in [0,1]$ for which **p**(i) is on/above the fence }

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[12] See Lakoff (1987) for a discussion on the different ways in which the path can relate to the reference object, making contact with the surface ('on') or being aligned with it in the vertical direction ('above').

This definition accounts for our intuitions about *over* and it leads to a non-cumulative denotation. There are paths in the denotation that can be concatenated (this is different from the source and goal definitions), but these concatenations are paths that have more than one interval of [0,1] on the fence and therefore do not fall in the strict, singular denotation of *over the fence*.

The same type of definition (but with a different locative basis) holds for *through* and *across* and maybe for *via* and *past*:

(40) { **p**: there is an interval $I \subset [0,1]$ that does neither include 0 nor 1 and that consists of all the $i \in [0,1]$ for which $\mathbf{p}(i)$ is …
… on/above x } = ⟦ over x ⟧
… in x } = ⟦ through x ⟧
… on x } = ⟦ across x ⟧
… at x } = ⟦ via x ⟧
… near x } = ⟦ past x ⟧

Even though the semantic definition of some of these prepositions might involve more than this, these definitions capture the most important part of their meaning and they are in line with earlier analyses in the literature.

4.1.3 *Towards* and *away from*

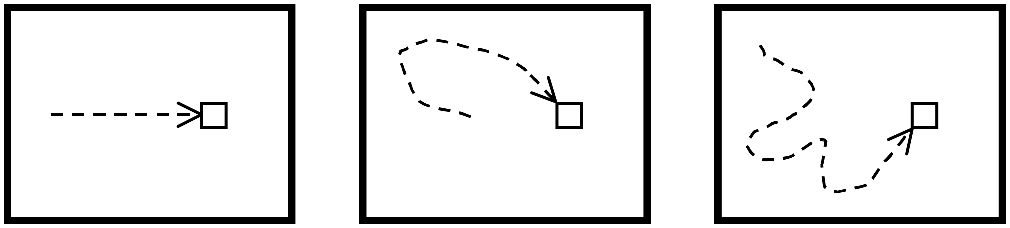
The prepositions *towards* and *away from* provide another example of the important interaction between aspectual and spatial semantics. It has been suggested in Jackendoff (1991), Piñon (1993), and Krifka (1998) that *towards* is a kind of progressive or partitive of *to*, denoting the initial subpaths of the paths of *to*:

(41) ⟦ towards the gate ⟧ =
{ **p**: there is a $\mathbf{q} \in$ ⟦ to the gate ⟧ such that $\mathbf{p} \leq \mathbf{q}$ and $\mathbf{p}(0) = \mathbf{q}(0)$ }

This works well when we are dealing with straight paths, but not with curved paths, as (42) illustrates:

(42) Three paths *to the gate*

a. b. c.

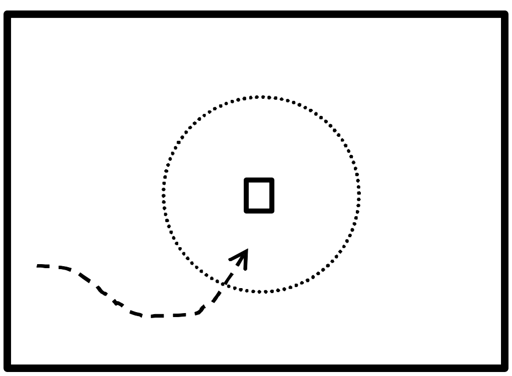


There are initial subpaths in (42b) and (42c) that would never qualify as *towards* (but rather as *away from*). What are alternative ways for defining *towards*? One possibility is to treat it as a genuine goal preposition with 'near' as the underlying location:

(43) $[\![$ towards the gate $]\!]$ = { **p**: there is an $I \subset [0,1]$ that includes 1 and that consists of all the $i \in [0,1]$ for which $\mathbf{p}(i)$ is near the gate }

This is a path that goes into a contextually determined 'outer halo' of the reference object, to use the terminology of Asher and Sablayrolles (1995) who propose an analysis like this.

(44) *Towards* as into an outer halo



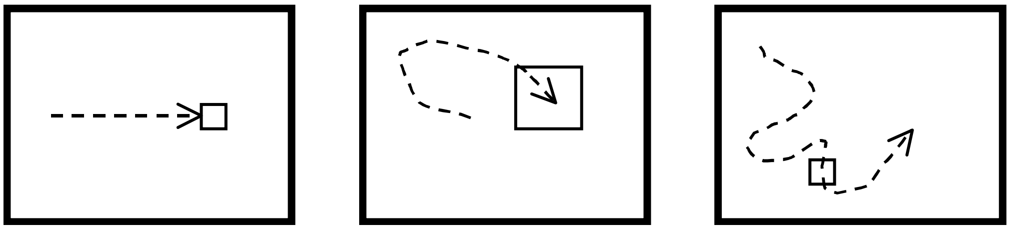
However, defined in this way, *towards* is not cumulative, which it should be, of course. There is another definition, that does give us the right results, and which is based on the comparative location 'nearer' (and an underlying distance function over locations) instead of the absolute location 'near' (Nam 1995):

(45) $[\![$ towards the gate $]\!]$ = { **p**: $\mathbf{p}(1)$ is nearer to the gate than $\mathbf{p}(0)$ }

This set is cumulative in reference, as it should be. It is again a very weak definition. It includes the denotations of the goal preposition *to* and it overlaps with the denotation of a route preposition like *through*:

(46) Three unusual paths in the denotation of *towards*

a. b. c.



There are two solutions to this problem. A semantic solution is to add the constraint that for every $i \in [0,1]$ $\mathbf{p}(i)$ is *outside* the gate. The pragmatic solution is that *towards* does not apply to these cases because there are informationally stronger prepositions available: *to*, *into* and *through*, respectively.[13] I think this pragmatic solution is more interesting, but again, I leave this for future research.

There is a strong intuition that with prototypical instances of *towards* the distance to the reference object decreases monotonically, i.e. for every $i, j \in [0,1]$, if $j > i$, then $\mathbf{p}(j)$ is nearer to the reference object than $\mathbf{p}(i)$. I am not sure whether this should be treated as a pragmatic *strengthening* of the weak meaning given above or whether weaker meanings are *relaxations* of a monotonically decreasing prototype meaning, as suggested by a reviewer.

If *towards* is not the partitive of *to*, then what is the relation between the two? In a sense, they stand to each other as a comparative to a superlative. If *towards* refers to paths that get ‘nearer’, then *to* refers to paths that get ‘nearest’, which is what ‘at’ is. When you are at a place, you can’t get any nearer. Another solution for making the relation between *to* and *towards* explicit, suggested by a reviewer, is to combine the partitive with the comparative analysis:

(47) $[\![$ towards the gate $]\!]$ = { $\mathbf{p}$: there is a $\mathbf{q} \in [\![$ to the gate $]\!]$ such that $\mathbf{p} \leq \mathbf{q}$ and $\mathbf{p}(1)$ is nearer to the gate than $\mathbf{p}(0)$ }

[13] Levinson (2000) suggests that *near* has the scalar implicature ‘not in’, not because that is part of its meaning, but because the stronger preposition *in* exists. There is a clear parallel with *towards* and *into* here.

For some speakers of English *away from* seems to be the reverse of *towards*:

(48) $[\![$ away from the gate $]\!]$ = { **p**: **p**(1) is further from the gate than **p**(0) }
= { **p**: **p**(0) is nearer to the gate than **p**(1) }

This is cumulative. For other speakers *away from* is a bounded preposition and its definition is the opposite of the definition that we rejected for *towards*:

(49) $[\![$ away from the gate $]\!]$ = { **p**: there is an $I \subset [0,1]$ that includes 1 and that consists of all the $i \in [0,1]$ for which **p**(i) is not near the gate }

A path is *away from the gate* if it leaves its outer halo. Such an asymmetry of reverse prepositions seems unlikely, but as Zwarts (1997) and Zwarts and Winter (2000) show, there is an important asymmetry between the notions of proximity and distance in the locative domain that might be related to what we see here in the directional domain.

4.1.4 *Up* and *down*

The prepositions *up* and *down* are both ambiguous between bounded and unbounded readings. In its bounded reading *up the hill* seems similar to a goal PP with *to* (the top of the hill being the goal) but in its unbounded reading *up the hill* seems more like *towards* (getting nearer to the top). What we concluded about *towards* is important here too: we can't take the bounded reading of *up* as basic and derive the unbounded reading as a partitive or imperfective. A path that goes up the hill might start with a path that goes actually down the hill for a while (because of a steep cliff that needs to be avoided, for instance). On the other hand, we do not want to treat the two meanings of *up* and *down* as unrelated either, so there is no option but to take the unbounded meaning as basic. I suggest that these prepositions are a kind of directional comparatives (like *towards*), they compare the initial and final point of the path, but this time it is not the relative nearness to the reference object, but the relative height:

(50) $[\![$ up the hill $]\!]$ = { **p**: for every $i \in [0,1]$, **p**(i) is on the hill and **p**(1) is higher than **p**(0) }
$[\![$ down the hill $]\!]$ = { **p**: for every $i \in [0,1]$, **p**(i) is on the hill and **p**(1) is lower than **p**(0) }

The reference object plays a different role for *up* and *down* than it does for *towards*: all the points of the path are on the surface of the hill. The hill is not a source or goal of the path in any way. The sets of paths defined in (50) are cumulative. Notice that there is no mention in the definitions of the top or foot of the hill. The same is true for related adverbs like *upstream*/*downstream*, *uphill*/*downhill*, *upwind*/*downwind*, although *upstairs*/*downstairs* seem a bit different.

Following the suggestion of the previous section about *to* and *towards* I propose to treat the bounded versions of these prepositions as the *positives* or *superlatives* of the prepositions in (50):[14]

(51) ⟦ up the hill ⟧) = { **p**: for every $i \in [0,1]$, **p**(i) is on the hill and **p**(1) is the highest point of the hill }

⟦ down the hill ⟧) = { **p**: for every $i \in [0,1]$, **p**(i) is on the hill and **p**(1) is the lowest point of the hill }

These definitions are non-cumulative.

The prepositions *up* and *down* show an interesting similarity to directed motion verbs like *ascend* and *fall* and degree achievements like *lengthen* and *cool* (Kennedy and Levin 2002, Rothstein 2004). These verbs are ambiguous between telic and atelic readings. The atelic readings are ‘comparative’ (e.g. *ascend* ‘go higher’), the telic reading are ‘absolute’ (e.g. *lengthen* ‘make long’). This suggests important similarities between paths in a spatial dimension and changes in a scalar dimension. Kennedy and Levin (2002) analyze the verbs as involving a degree of change on some spatial or qualitative scale. The approach that suggests itself here is to take degrees of change as paths on a scale. This would provide interesting possibilities for a unified treatment of change and direction.

4.1.4 *Along* and *past*

Along and *past* are route prepositions based on the locative relation of ‘proximity’ to the reference object. What distinguishes them from the other route prepositions is their unambiguous aspect: *along* is always atelic, *past* always telic. Route prepositions like *through*, *over*, and *across* are ambiguous between the telic reading defined above and an

[14] The highest point and lowest point of a hill are not literally points, of course, but vaguely defined regions.

atelic reading in which the relevant location applies to the whole path. (I ignore the iterative readings here.) A simple explanation is that *past* and *along* lexicalize a distinction that other prepositions leave ambiguous: *past* for telic and *along* for atelic paths:

(52) $[\![$ past the house $]\!]$ = { **p**: there is an $I \subset [0,1]$ that includes neither 0 nor 1 and that consists of all the $i \in [0,1]$ for which $\mathbf{p}(i)$ is near the house }

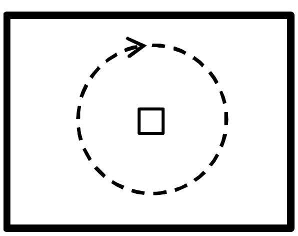
$[\![$ along the river $]\!]$ = { **p**: for all $i \in [0,1]$ $\mathbf{p}(i)$ is near the river }

A path along the river is a path that has all of its points near the river. The universal definition accounts for the cumulativity of this PP, but it also predicts that the PP is divisive in reference, in contrast to what I claimed in section 3.1. What is missing in the definition of *along* is a part that aligns the path with the axis of the river. Clearly, more work is needed on the role that axes and dimensions of objects play in prepositions like *along* (and also *across* and maybe *through*, as we saw in the example with the tunnel in 3.1).

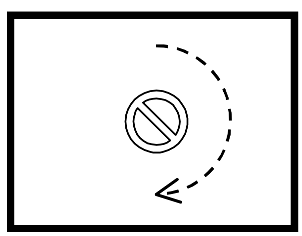
4.1.5 *Around*

Around is rather a polysemous preposition which has various bounded and unbounded senses, some of which we already discussed above.

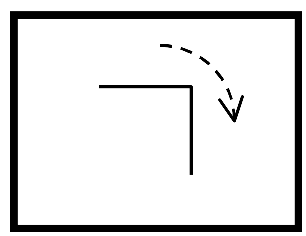
(53) a. around the block



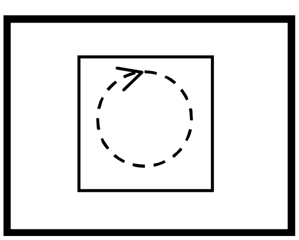
b. around the barrier



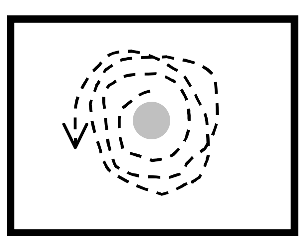
c. around the corner



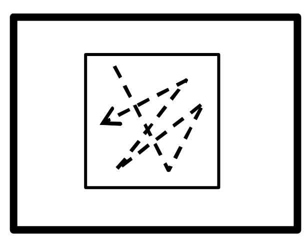
d. around the room



e. around the sun



f. around the city centre



Zwarts (2003b) gives a particular account of how these senses are derived from the meaning of one complete circle. What is important in the context of this article is what aspectual considerations can tell us about the semantics of *around*, more specifically, how we can rule out certain analyses because they lead to the wrong algebraic structure. I will put the iterative reading in (53e) and the crisscross reading in (53f) aside for the time being and focus only on the remaining four meanings, that are all telic. In order to derive this telicity we again need to ensure that there is only one subinterval of the path that has a particular property. Here is one way to do this for the central reading of (53a):

(54) a. ⟦ around the block ⟧ = { **p**: **p** encloses the block }

What it means for a path to enclose the reference object and how this relates to the idea of a circle is something I will not further work out here (see again Zwarts 2003b). The reading in (53d) differs only from this definition in locating the path inside the reference object instead of locating the reference object inside the path. The two paths in (53b) and (53c) can be treated as subpaths of the complete path with additional conditions:

(55) b. ⟦ around the barrier ⟧ = { **p**: there is a path **q** enclosing the barrier such that **p** ≤ **q** and **p**(*i*) and **p**(*j*) are on roughly opposite sides of the barrier }

c. ⟦ around the corner ⟧ = { **p**: there is a path **q** enclosing the corner such that **p** ≤ **q** and **p**(*i*) and **p**(*j*) are on roughly orthogonal sides of the corner }

All of these denotations are non-cumulative because in the basic definition in (54) the path encloses the reference object exactly once.

4.2 How the algebra of paths enriches prepositional semantics

We have discussed the role that concatenations and subpaths can play in distinguishing bounded from unbounded prepositions in algebraic terms. The topic of this section is to show how concatenations and subpaths can play a more explicit role in the lexical semantics of prepositions. This brings out more parallels between prepositions and nouns and verbs.

4.2.1 Non-Boolean *and* conjoining PPs

There is evidence that the conjunction *and* with PPs can be interpreted as concatenation of paths. The conjoined PP *up the stairs and down the stairs* is not the set of paths that are both up the stairs and down the stairs (56a) (because this would always be an empty intersection), but the set of paths that are concatenations of such paths (56b).

(56) (Alex ran) [$_{PP}$ up the stairs and down the stairs]

a. $[\![$ up the stairs $]\!] \cap [\![$ down the stairs $]\!]$ (= $\emptyset$)

b. $[\![$ up the stairs $]\!] + [\![$ down the stairs $]\!]$ =

$\{ \mathbf{p}+\mathbf{q}: \mathbf{p} \in [\![$ up the stairs $]\!] \wedge \mathbf{q} \in [\![$ down the stairs $]\!] \}$

It is not impossible to treat the PP in (56) as a Boolean conjunction of predicate modifiers (see Keenan and Faltz 1985), but this does not account for the strong spatial connection that exists between the conjuncts.

The concatenation operation is also part of the lexical semantics of directional prepositions and adverbs like *up and down*, *in and out*, *back and forth*, and *to and fro*, that concatenate paths of opposite directions in an alternating fashion. There is a non-iterative reading of these conjunctions (a single cycle consisting of only two opposite movements), that is defined in (57a) for *up and down*. The iterative reading in (57b) involves repeated concatenation of opposite paths.

(57) a. $[\![$ up and down $]\!] = [\![$ up $]\!] + [\![$ down $]\!]$

b. $[\![$ up and down $]\!] = X_1 + X_2 + \ldots + X_{n-1} + X_n$

with $n > 1$, $X_i \in \{ [\![$ up $]\!], [\![$ down $]\!] \}$ and X_j, X_{j+1} opposite

Notice that the iterative *up and down* path may very well start with a downward part, which motivates the use of variables over denotations.[15] It follows from these definitions that the iterative *up and down* is unbounded and the single concatenation *up and down* bounded.

4.2.2 Singular and plural PPs

We can also apply the concatenation operator to a set of paths, forming the closure under concatenations, notated by the star operator:

(58) $*(\mathbf{X})$ = the closure of $\mathbf{X}$ under concatenations

[15] I am grateful that an anonymous reviewer pointed out to me that my original definition of iterative *up and down*, based on the concatenative closure of the union of $[\![$up $]\!]$and $[\![$down $]\!]$, was too weak, because it did not require a minimal cycle of at least two opposite movements.

This set *(**X**) is cumulative. As in the nominal domain, the * operator can be used to represent plurality in the prepositional domain. In section 4.1.5 we defined the primary meaning of the PP *around the house* as the set of paths that enclose the house once, a non-cumulative set. When the plural operator * applies we get a PP that refers to paths that consist of one or more cycles around the house, as illustrated in picture (53e). The operator can remain invisible (59b), or it can be made visible by reduplication (59c).

(59)	a.	around the house	'one circle'	(singular)
	b.	around the house	'more than one circle'	(plural)
	c.	round and round the house	'more than one circle'	(plural)

Pluralization of *around the house* is possible because the starting points and end points of its paths can be identical. Hence, there are paths **p** and **q** such that **p**(1)=**q**(0). The same is true for route prepositions like *over*, *through*, and *across*. One *over* path can always start where another *over* path ends. Crucially, this is different with the source and goal prepositions. The end point of one path *to the house* can never be the starting point of another path *to the house*. As a result we do not get the kind of iterated readings with *to the house* that we get with *around the house*.

In an important sense, route prepositions like *around*, *over*, *through*, and *across* are similar to semelfactive verbs like *jump*, *kick* and *flash*. The status of semelfactives in aspectual classification has always been a bit unclear. On the one hand they behave like events, on the other hand they easily allow activity readings. What makes them special is that they describe events that return to their original state after passing through an intermediate state. If you jump, you start with your feet on the ground, go through a state in which they are off the ground, and return to your initial state. This cyclic three stage structure makes semelfactives crucially different from achievements and accomplishments, that always describe a two stage transition from one state to another. Because their initial and final states overlap, the individual events of a semelfactive verb can be concatenated and lead to iterative activity readings, but this is not possible with achievements and accomplishments (Rothstein 2004).

4.2.3 Count and mass PPs

In the nominal domain the 'universal grinder' is the operation that turns count nouns into mass nouns:

(60) a. There is an apple in the salad

b. There is apple in the salad

Intuitively, the mass denotation of *apple* consists of the bits and pieces of the individual apples in the count denotation. In the prepositional domain grinding has been proposed to derive *towards* from *to* in Jackendoff (1991) and Piñon (1993), but I have argued in section 4.1 that this analysis is not correct. Does the universal grinder apply in other cases?

We already discussed two meanings of the route prepositions *through*, *over*, *across* and *around*: the bounded meaning and the iterative meaning that we treated as a plural of this bounded meaning. The third meaning occurs in the following examples:

(61) a. Alex walked through the tunnel for hours

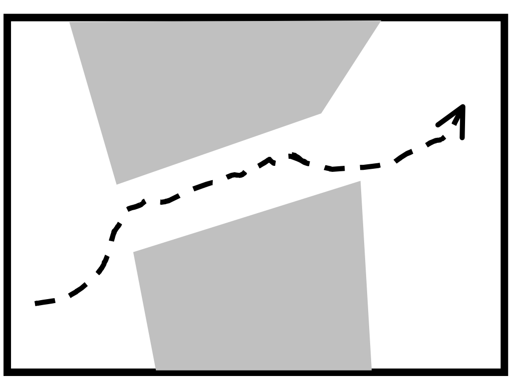
b. Alex crawled across the table for hours

c. Alex flew over the yard for hours

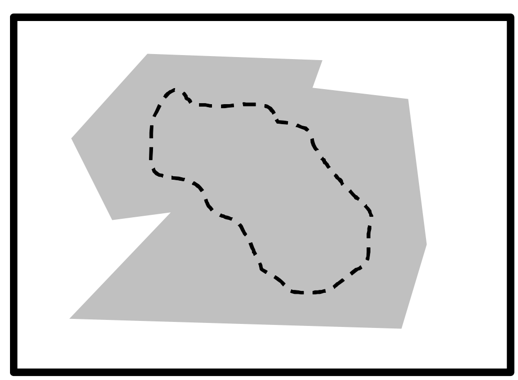
d. Alex drove around the city centre for hours

The paths denoted by these PPs stay in the tunnel, on the table, above the yard and in the city centre all the time. The framework of this article suggests that the prepositions here are used as mass prepositions derived by taking singular count prepositions through the grinder. Let us focus on *through the tunnel* to see how this could work. A typical path in the singular count denotation of this PP is given in (62a):

(62) a. A path *through the tunnel*



b. A path *round the city centre*



Applying the grinder means extracting subpaths from this path, but not from the whole path. We want only subpaths from the *internal* section, not from the parts that are sticking out. This requires us to first extract the set of minimal paths from a PP denotation, i.e. the paths in **X** that have no proper subpath in **X**:

(63) $min(\mathbf{X}) = \{ \mathbf{p} \in \mathbf{X}$: there is no $\mathbf{q} \in \mathbf{X}$ such that $\mathbf{q} < \mathbf{p} \}$

The definition of prepositional grinding then involves taking parts of these minimal paths:

(64) $gr(\mathbf{X}) = \{ \mathbf{p}$: there is a $\mathbf{q} \in min(\mathbf{X})$ such that $\mathbf{p} \leq \mathbf{q} \}$

The set *gr*(⟦ through the tunnel ⟧) is cumulative. Notice that the definition of *through* in 4.1.2 allows any shape of path as long as one middle part is inside the reference object. In the same way, we get the definitions for *across*, *over*, and *around*.

For *around the city centre* something more sophisticated might be needed to derive the crisscross meaning. If the primary bounded meaning of this PP consists of single closed paths like the one in (62), then the grinder *gr* will never give complicated crisscross patterns and the resulting PP denotation will not be cumulative. The solution is to allow combinations of the plural operator and the grinder operator:

(65) *gr*(*(⟦ around the city centre ⟧)) or *(*gr*(⟦ around the city centre ⟧))

Either by grinding concatenations of single paths or by concatenating bits of pieces of single paths we get the rich pattern of path shapes and the cumulativity that we need for the ‘mass’ use of *around*.

5 Conclusions

The domain of prepositions is parallel in many ways to the nominal and verbal domain. This article has explored some of these parallels and their role in understanding both the aspectual properties of prepositions and their spatial properties, as well as the relation between the two. We have seen that an adequate account of prepositional aspect requires an account of prepositional ‘number’. Most prepositions have a primary singular meaning that leads to telic aspect and under certain conditions plural and mass meanings can be derived with atelic effects. Many verbal aspectual classes also have their prepositional counterparts, like stative verbs (locative prepositions), semelfactives (*over*, *through*, *across*, and *around*), and degree achievements (*up* and *down*). Beavers (2002) and Denis, Kuhn and Wechsler (2003) point out interesting contrasts between *to* and *into* that suggest a finer aspectual classification. *To* and *into* are both possible with a verb like *walk*, but we can note a clear difference when we replace *walk* with *step* or when we imagine a context where Alex was standing just outside the room:

(66) a. Alex walked to/into the room
b. Alex stepped *to/into the room
c. Alex walked *to/into the room (when she was standing just outside the room)

This might suggest that *to* is an 'accomplishment' preposition (durative) and *into* an 'achievement preposition' (punctual). The suggestion of Beavers and Denis, Kuhn, and Wechsler is that *to*-paths have internal structure that *into*-paths lack. *Into*-paths have only a starting point and an end-point, with no intermediate points, i.e. they are 'minimal' paths. *To*-paths have more internal structure, they are 'longer', 'extended'. Obviously, my approach does not allow paths to be distinguished in their internal structure along these lines, so I leave the incorporation of such contrasts to future work.

Many other relevant phenomena have to remain unexplored here, like the role of the reference object in determining prepositional aspect (compare *over the fence* with *over the bridge*), of measure phrases (*ten miles towards the border*) and other modifiers (*halfway along the river*), as well as the interaction between PPs and specific types of nouns and verbs (like *walk* and *step* in the examples above or verbs of perception like *look*). On the theoretical side there are still many questions about the nature and role of paths (continuous versus discrete, one-dimensional versus more-dimensional, finite versus infinite, atemporal versus temporal, directed versus non-directed). Another important area is the balance between semantics and pragmatics and the way denotations are shaped by the competition between different prepositions (see Zwarts 2003c). The path-based algebraic model developed in this article seems a good starting point for approaching such issues and deepen our understanding of the role of aspect and space in natural language.

Appendix: A path algebra and an event algebra

The path algebra used in this article is a pair ⟨**P**,+⟩, where **P** is the set of paths and + the concatenation operation on paths. **P** is defined as the set of continuous functions from the real unit interval [0,1] to $|R^3$ that have constant speed, i.e. such that the first derivative of **p** is constant. I am assuming the standard definition of continuity and derivation for functions in terms of limits. The restriction to constant speed is necessary to abstract away from the velocity with which the function traverses the path. Such paths are then said by mathematicians to be *parametrized by arc length* (Marsden and Tromba 1981). In this way, every path is represented by one unique function in **P**.

The concatenation of paths is defined in the following way:

(67) *The definition of concatenations of paths*

For **p**, **q**, **r** ∈ **P**, **p** + **q** = **r** iff there is an h ∈ [0,1] and there is a monotone increasing bijection λ from [0,h] to [0,1] such that for all *i* ∈ [0,h] **r**(*i*) = **p**(λ(*i*)) and there is a monotone increasing bijection ρ from [h,1] to [0,1] such that for all *i* ∈ [h,1] **r**(*i*) = **q**(ρ(*i*)).

The domain of **r** is divided up into two parts, that are made to correspond to the concatenants **p** and **q** by the mappings λ and ρ, respectively. These functions only exist for any given **p** and **q** when **p**(1) = **q**(0) (i.e. when **p**(λ(h)) = **q**(ρ(h))), otherwise **p** + **q** is undefined. This makes + a partial operation and the path algebra ⟨**P**,+⟩ a partial algebra.

The subpath relation ≤ can be defined in terms of +:

(68) *The definition of the subpath relation on paths*

p ≤ **q** iff there are **r** and **r′** such that **r** + **p** + **r′** = **q**.

This relation is reflexive, antisymmetric, and transitive. The relation is proper (<) when **p** and **q** are not identical: **p** < **q** iff **p** ≤ **q** and **p** ≠ **q**. Constant paths (that assign the same point to every index) have no proper subpaths. They are the least elements of this partial ordering and the identity elements for the concatenation operation.

The path domain is restricted to the closed unit interval [0,1] for reasons that go beyond the scope of this article. If paths are defined as functions from [0,1] into vector spaces (as proposed in Zwarts and Winter 2000), then the operation of vector addition can be defined over paths by pointwise addition of their component vectors. This allows for the definition of another algebra of paths, orthogonal to the one assumed here, and important for extending the spatial monotonicity constraints of Zwarts (1997) from locative prepositions to directional prepositions.

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