Exercise 9: Hypotheses testing

1. Answer the following questions:

    1.1 Are statistical hypotheses defined in statistics, or parameters?

    1.2 What symbol we use for magnitude of type I. error we are willing to tolerate?

    1.4 If $p < \alpha$, do we reject null hypothesis H_0?

    1.5 Je-li $p > \alpha$, do we keep H_0?

    1.6 If H_0 is true, but we rejected it based on our data, what type of error did we make?
        a) type I. error
        b) type II. error
        c) we didn't make any error

    1.7 If H_0 is true and we didn't reject it based on our data, did we make type II. error?

    1.8 If H_0 is false, but we didn't reject it based on our data, we made:
        a) type I. error
        b) type II. error
        c) both type I. and type II.
        d) no error

    1.9 What symbol we use for statistical error of type II.? How is type II. error related to test power?

    1.10 Which three factors increase test power?

    1.11 ___ is to α the same as type II. error is to k ___.

2. Define the following terms::

    2.1 null hypothesis, H_0

    2.2 alternative hypothesis, H_1

    2.3 type I. error

    2.4 type II. error

    2.5 statistical significance level, α

    2.6 test power, $1 - \beta$

3. Which of the following can be considered as statistical hypothesis?
    a) $m = 63.0$
    b) $\mu = 1.2$
    c) $\sigma = 10$
    d) $\rho = 0.50$
    e) $s = 10.00$
    f) $r = 0$

4. Choose correct answer:

   4.1 If $z = 2.0$, can we reject H_0 (does the same answer apply for $t = 2,0$ as well?)
      a) at 1% statistical significance level
      b) at 5% statistical significance level, but not at 1% statistical significance level
      c) nor at 1%, neither at 5% statistical significance level

   4.2 Which of the following is the least probable?
      a) $z = -3.1$
      b) $z = 0.00$
      c) $z = 2.0$
      d) $z = 2.58$

   4.3 If H_0 is true, is probability of obtaining higher z-score than 1.31 the same as probability of obtaining lower z-score than -1.31?
      a) yes
      b) can't be determined, because we don't know degrees of freedom
      c) no

5. Determine correct answer:

   5.1 Which of the following significance levels requires for rejecting the null hypothesis the biggest difference between m a hypothetical value μ?
      a) 0.01
      b) 0.05
      c) 0.10

   5.2 If a confidence interval for μ ranges from 47.2 to 63.4, which of the following statistical hypothesis would be rejected on 5% statistical significance level?
      a) $\mu = 45$
      b) $\mu = 50$
      c) $\mu = 55$
      d) $\mu = 60$
      e) $\mu = 65$

   5.3 Assume that H_0: $\mu = 100$ was rejected at 1% significance level. Is the value 100 inside the 99% confidence interval? And is the value 100 inside 95% confidence interval?

6. Answer the following questions:

   6.1 t statistics would be used for testing hypothesis H_0: $\mu = c$, when we don't know:
      a) n
      b) m
      c) σ
      d) α

   6.2 If $n = 20$, are critical values for t slightly higher than for z?

   6.3 In which of the following cases are the differences between critical values of t and z the highest?
      a) $n = 5$
      b) $n = 10$
      c) $n = 100$
      d) $n = 1000$

   6.4 In testing hypothesis H_0: $\mu = c$, where c is some constant, which of the following statistics is used more often – z or t ? Why?

   6.5 What are the degrees of freedom for the following samples when testing hypothesis H_0: $\mu = c$?
      n = 11
      n = 60
      n = 101

6.6 If $\alpha = 0.05$ and $df = 20$, what are the critical values of t for two-tailed and one-tailed test?

7. Determine correct answer:

7.1 If H_0 is true, what is the probability that we reject the hypothesis at 5% statistical significance level, that is, what is the probability of type I. error?

7.2 If $\alpha = 0.05$ and H_0 is not rejected, do we know the probability of type II. error?

7.3 If $\alpha = 0.05$ and $p < 0.01$, can we reject H_0 at 1% significance level?

7.4 If H_0 was not rejected at 5% significance level, what is the probability of type I. error?

7.5 If H_0 was rejected at 5% significance level, what is the probability of type II. error?

8. Determine correct answer?

8.1 What is statistical significance?
   a) $P(D|H_0)$, e.g. $P(m = 3| \mu = 0)$
   b) $P(H_0|D)$, e.g. $P(\mu = 0|m = 3)$
   c) $P(H_0|H_1)$, e.g. $P(\mu = 0| \mu \neq 0)$
   d) $P(H_1|H_0)$, e.g. $P(\mu \neq 0| \mu = 0)$
   e) $P(H_0|\alpha)$, e.g. $P(\mu = 0| \alpha < 0,01)$
   f) $P(\alpha|H_0)$, e.g. $P(\alpha < 0,01| \mu = 0)$

8.2 Formulate the answer you chose in the previous question by your own words.

8.3 Why it is important to be aware of the fact that $P(D|H_0) \neq P(H_0|D)$?

9. Standardized reading test was administered to a sample of 16 6th grade students included in a program for improving reading abilities. Their average score in the end of the year was 8.0 with SD 1.8. A researcher wants to know whether this mean is different from value 6.8, which is the national average reading score of 6th grade students in the end of the year.

9.1 Formulate H_0

9.2 Will we use as test statistics z or t ?

9.3 What is the value of standard error of mean?

9.4 Compute t.

9.5 What are the critical valued of t for $\alpha = 0.05$ and for $\alpha = 0.01$?

9.6 Can we reject H_0 at 1% or 5% level?

9.7 Compute 95% a 99% confidence interval for μ.

10. Average height (μ) of adult men in USA is 176.53 cm with σ 7.62 cm. Assume that average height in a sample of 25 mentally retarded men is 171.45 cm. The question is, whether is this mean statistically different than $\mu = 176.53$ cm (assume that standard deviation in population of mentally retarded men is also $\sigma = 7.62$ cm).

10.1 Formulate H_0

10.2 Would you use as test statistic z or t ?

10.3 Is it correct to formulate the alternative hypothesis as H_1: $\mu < 176.53$ cm, after we found out that $m = 171.45$ cm?

10.4 What is the value of σ_m?

10.5 What is the test statistic z ?

10.6 Do we reject H_0 at $\alpha = 0.01$?

10.7 Would the critical values of z remain the same, if n increased to 100?

10.8 Would the value of σ_m remain the same, if n increased to 100?

12. In population there isn't difference between men and women in some test. In spite of that, we found a difference in this test between men and women in our study. Probability was 0.03, so we rejected our null hypothesis at level 0.05. What error type did we make?

13. If the level of alfa decreases, what error type decreases as well?

15. If the null hypothesis is false, what error type you can't make?

16. Statistical significance means:
 a) probability of the null hypothesis validity
 b) probability of test statistics given the null hypothesis is valid (true)
 c) generalizability of analysis results