Chapter 1

Numerals and their kin

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1 Introduction

Numerals are funny words. On the one hand, it seems they share some core component that justifies grouping them together as if they formed a single category, a practice common in both descriptive and theoretic approaches to language.[1] On the other hand, though, morphologically and syntactically they form a very heterogeneous class of expressions with different items often exhibiting distinct properties. This is the case not only when one compares various subclasses of numerals, e.g., cardinals, ordinals and multiplicatives, but also often within the subclass of cardinals different items have adjectival, nominal, or both nominal and adjectival features while in some languages they seem to behave as verbs (Donohue 2005; Ionin & Matushansky 2018).

In generative linguistics, a lot of work has been focused on establishing the syntactic status of numerals with different approaches differing in whether they are lexical or functional categories or whether they are heads or maximal projections. These questions have been studied thoroughly with respect to Slavic numerals with their well-known idiosyncratic properties (e.g., Babby 1987; Pesetsky 1982; Franks 1994; Rutkowski 2002; Klockmann 2012).

On the other hand, since the early days of analytic philosophy and formal semantics a lot of attention has been dedicated to the meaning of numerals. At

[1] This intuition dates back at least to the Neo-Latin grammarian tradition which distinguished three subclasses within the category *nomen*, i.e., *nomen substantivum*, *nomen adjectivum* and *nomen numerale*.

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least from Frege (1884), it has been recognized that capturing the semantics of cardinals is far from trivial. Intuitively, it seems straightforward that they are linguistic expressions that somehow describe a numeric quantity. However, beyond this vague characterization it is much less obvious how to state exactly what they actually are.

In this paper, I will discuss the main formal approaches to the meaning of cardinal numerals. In doing so, I will also focus on examining two issues that only recently have attracted due attention in the semantic literature. Specifically, I will investigate different uses of cardinals as well as various derivationally complex quantifying expressions in Slavic. The two sets of data indicate that cardinals are typally flexibile and that numerals in general are semantically complex expressions. I will argue that a proper approach to the meaning of numerals should describe a compositional mechanism deriving different semantic flavors from the underlying arithmetical meaning.

The paper is outlined as follows. In §2, I will discuss various functions of cardinal numerals. §3 will provide an overview of different semantic approaches to the quantifying function of cardinals, as in *five cats*. Next, §4 will explore the relationship between the quantifying meaning and the ability of cardinals to refer to abstract arithmetical concepts. Finally, in §5, I will review the literature on the morpho-semantics of complex numerical expressions in Slavic which presents evidence calling for a compositional treatment.

2 Cardinals and their various flavors

Let us start with cardinal numerals. One of the challenges in accounting for their meaning is that they can be used in very different ways (Bultinck 2005; Geurts 2006). Consider, for instance, (1). It turns out that a simple word such as English *five* is in fact quite tricky. Of course, it can be used to quantify over entities denoted by the modified NP when it appears prenominally as in (1a) but that is surely not the only function it can have. For instance, when it combines with a measure word as in (1b), it designates a portion of a substance rather than a plurality of objects. It can also appear in a predicative context such as (1c) or as part of a measure phrase like in (1d). And that is not it since it can also be used to refer to an abstract arithmetical concept, see (1e), or to label an entity as in (1f).

(1) a. Five cats meowed.
    b. Five liters of milk got spilled.
    c. These are five cats.

d. That cat is five years old.
e. Five is a Fibonacci number.
f. Tram number five has just left.

Below, I will briefly discuss how the measure, predicative and arithmetical function of cardinal numerals relate to their most deeply studied quantifying use.[2] A thorough investigation into properties of the label function will be undertaken in §??.

2.1 Quantifying vs. measure use

For many years, the mainstream research has been primarily focused on the quantifying use of cardinals exemplified in (1a). This seems justified since counting objects is perhaps the first thing that comes to mind when we think about expressions such as *five*. However, when a numeral appears in a measure phrase such as (1b), it seems to be involved in measuring within a certain dimension, e.g., volume, rather than in counting objects.

Intuitively, the difference between counting and measuring is that the former is about specifying how many discrete objects of a certain kind there are, whereas the latter determines some quantity in relevant measure units. Admittedly, some proposals attempt to reduce measuring to a particular type of counting based on the individuation in terms of quantity (Lyons 1977; Matushansky & Zwarts 2017). Consequently, measuring would simply be counting units determined by measure words. An opposing approach treats counting as a form of measuring, i.e., measuring a quantity of a plural individual in terms of natural or object units (Krifka 1989; 1995). Yet, there is also the third account which views counting and measuring as two independent operations (Rothstein 2017).

The contrast between the two functions becomes clear when one considers classifier constructions involving container nouns, see (2). Such expressions are ambiguous between a measure (content) and counting (container) interpretation (e.g., Landman 2004; Grimshaw 2007; Partee & Borschev 2012). On the measure reading of (2a), Tomek flushed wine to the quantity of two bottles. On the counting reading, on the other hand, he did something quite spectacular since two actual bottles containing wine went down. Moreover, derivational morphology seems to be sensitive to the distinction since the suffix *-ful* selects only for the measure sense of a container noun, compare (2b)–(2c) (Rothstein 2011).

[2] For an exhaustive discussion of numeral NPs interpreted as measure phrases in Slavic, see Matushansky & Ionin (this volume).

(2) a. Tomek flushed two bottles of wine through the toilet.
b. Romek added two { glasses / glassfuls } of wine to the soup.
c. Marek brought two { glasses / #glassfuls } of wine for our guests.

Another argument for distinguishing counting and measuring as two distinct operations comes from distinct restrictions on the domain of quantification (Wągiel 2018). Though both measuring and counting quantify over entities that need to be disjoint (Landman 2016), only the latter requires objects individuated as coherent integrated wholes. To realize the contrast, imagine someone has spilled some wine on the table in such a way that there are two separate blobs *a* and *b* whose volume is exactly one and a half milliliters each. In such a scenario, (3a) is true despite the fact that one of the three milliliters must be split between a portion of *a* and a portion of *b*. On the other hand, (3b) is simply false. This contrast shows that measuring, unlike counting, ignores individuation in terms of integrity.

(3) Scenario: There are exactly two 1.5 ml blobs of wine on the table.
a. There are three milliliters of wine on the table. TRUE
b. There are three objects on the table. FALSE

Though monotonic systems of measurement track certain part-whole relations (Schwarzschild 2002), they do not seem to be sensitive to the spatial arrangement of parts making up a whole. This suggests that despite sharing a common core counting and measuring are two distinct things.

2.2 Quantifying vs. predicative use

It is well known that cardinals used as quantifiers give rise to scalar implicatures. For instance, *five cats* in (4a) allows for an *at least* reading, as witnessed by the compatibility with the *in fact* clause. Similarly, (4b) is typically interpreted in a way that Rumcajs must take at least three cards.

(4) a. Five cats live in the barn; in fact, six cats live there.
b. Rumcajs must take three cards.

However, numeral NPs in predicate position systematically lack a lower-bounded interpretation (Partee 1986; Landman 2003; Geurts 2006). For instance, (5a) can only get the *exactly* reading. This is further corroborated by the infelicity of sentences such as (5b).

(5) a. These are five cats.

b. # The guests are three filmmakers; in fact, they are four filmmakers.

Furthermore, it has been observed that also bare cardinals can appear as predicates in predicate position, see (6a)–(6b) (Rothstein 2017). This resembles the behavior of adjectives rather than determiners.

(6) a. The apostles are twelve.
    b. The planets are eight.

Similarly to numeral NPs, bare cardinals in predicate position lack an *at least* reading. Thus both (6a) and (6b) are true if there are exactly twelve apostles and exactly eight planets, respectively. Furthermore, sentences such as (7) are infelicitous.

(7) # My reasons for saying this are four; in fact they are five.

The contrasts discussed above indicate that meaning of the predicative use of cardinals differs from their semantics in the quantifying function.

2.3 Quantifying vs. arithmetical use

It is not always the case that cardinals designate a plurality or measure. In (1e), *five* refers to an abstract number concept rather than to a collection of entities. In this use, cardinals have different properties than in their quantifying function (Bultinck 2005; Rothstein 2017; Wągiel & Caha 2020). Specifically, number concepts can have special properties such as being prime, see (8a), and they can occur in mathematical statements such as (8b) and dedicated grammatical construction as in (8c). When compared, the dimension of comparison is based on their relative ordering, see (8d).

(8) a. Five is prime.
    b. Ten divided by five equals two.
    c. Hanna can count up to five.
    d. Five is bigger than four.

This behavior contrasts with cardinals in their quantifying function, which lack the properties mentioned above. (9a) is odd since being prime is not a property that can be attributed to a collection of things. Similarly, expressions denoting a plurality of entities are illicit in constructions calling for a numeric value such as (9b)–(9c). Finally, (9d) has different truth conditions than (8d), e.g., it would not be true if one compared five cherries with four watermelons.

(9) a. # Five things are prime.
b. # Ten things divided by five things equals two things.
c. # Hanna can count up to five things.
d. # Five things are bigger than four things.

Furthermore, cardinals referring to number concepts are incompatible with numeral modifiers such as *more than* and *at least*, see (10a). Finally, the arithmetical function does not give rise to scalar implicatures and always get an *exactly* reading, as witnessed in (10b).

(10) a. # More than five is prime.
b. # Two multiplied by five equals ten, if not more.

As we have seen, cardinal numerals can be used in various ways each of which has different semantic characteristics. In the next two sections, I will review major approaches to the meaning of cardinals and compare how they account for at least some aspects of the flexibility discussed above.

3 Theories of cardinals

Given the variety of uses discussed in the previous section, any quest for 'the' meaning of cardinal numerals is probably a misunderstanding. Rather, a theory of cardinals should be able to capture the relationship between meanings of cardinals in their various functions. And yet, the mainstream research has been mainly focused on the quantifying use often ignoring how it relates to other uses with Bylinina & Nouwen's 2020 systematic inquiry being a notable exception).

3.1 Cardinals as determiners

Let us start with the earliest and most prominent formal account of the meaning of cardinal numerals, namely the standard Generalized Quantifiers (GQ) approach (Barwise & Cooper 1981; though it can be traced back via Montague to Frege). The intuition behind this analysis is that a cardinal such as English *five* is in fact a determiner similar to *some*, *most* or *all*, as suggested by its occurrence in prenominal position as in (11).

(11) { Some / Most / All / Five } cats meowed.

In the GQ theory, a determiner expresses a particular relation holding between two sets (type $\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$), i.e., a set denoted by the NP and a set denoted by the VP. For instance, in (11) *some* yields the truth value True if the intersection between the set of cats and the set of entities that meowed is non-empty, see (12a). Similarly, on the GQ analysis *five* returns True if the cardinality of the set of cats and the set of entities that meowed equals 5, see (12b).

(12) a. $[\![\text{some}]\!] = \lambda P \lambda Q[P \cap Q \neq \varnothing]$
b. $[\![\text{five}]\!] = \lambda P \lambda Q[|P \cap Q| = 5]$

A nice thing about the GQ approach is that it aims to develop a unified semantics of all quantified DPs. Furthermore, it can be enriched with a type-shifting mechanism which allows for systematic mapping between different semantic types (Partee 1986). Consequently, the theory can account for predicative uses of numeral NPs such as (1c) by lowering the type of a general quantifier ($\langle\langle e,t\rangle,t\rangle$) to the type of a predicate ($\langle e,t\rangle$).

However, for some time it has been realized that the GQ approach is most probably not a good way of capturing what cardinals are (e.g., Krifka 1999; Landman 2003). In the next sections, I will briefly discuss problematic data and alternative approaches.

3.2 Cardinals as predicates

There are several problems with the GQ approach to cardinals. First of all, there is a well-known asymmetry between DPs with numerals and DPs with regular determiners in predicate position. For instance, DPs headed by *every* cannot be used predicatively, see (13a). Similarly, examples such as (13b) are infelicitous on a non-partitive interpretation. Given the felicity of sentences such as (1c) and others discussed in §2.2, this fact is puzzling (e.g., Landman 2003).

(13) a. # Hanna is every filmmaker at the party.
b. # The guests are { most / some } filmmakers.

Furthermore, as witnessed in (14a) bare determiners also cannot be used predicatively. This contrasts with examples such as (6), in which cardinals pattern with adjectives, see (14b). Since there is no standard type-shifting rule allowing for mapping the type of determiners ($\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle$) onto the type of predicates ($\langle e,t\rangle$), the predicative use of bare cardinals is unaccounted for. But even if such a rule were postulated, it would still fail to explain the contrast between cardinals and determiners such *most* and *all*.

(14) a. # My reasons for saying this are { most / all / every }.

b. My reasons for saying this are { serious / personal / five }.

The last problematic data point to be discussed here concerns the fact that cardinals can appear along with bona fide determiners within a single DP (e.g., Rothstein 2017). From the GQ perspective, the compatibility of *five* with the definite article in (15a) and with *every* in (15b) is unexpected and raises serious questions regarding the semantic contribution of each of the elements.

(15) a. The five cats that I saw meowed.

b. Every two students got to share a hotel room.

The evidence discussed above motivated developing an alternative approach which treats cardinals as predicates (Landman 2003; 2004). Within such a framework, cardinals get an interpretation very similar to that of intersective adjectives, specifically they express a cardinality property. For this approach to work, it is required to distinguish between two types of entities within the domain of individuals, namely atoms, i.e., singular entities, and pluralities (Link 1983). While atoms are minimal elements of a nominal denotation, pluralities are formed from atoms via a pluralization operation * which closes the predicate under sum, i.e., takes a set of atomic entities and returns that set extended with all the pluralities that can be formed by summing the atoms. In addition, the two types of individuals are associated with each other via a part-whole relation defined in terms of mereological parthood, e.g., an atomic entity a is part of a plurality $a + b$.[3]

According to the semantics provided in (16a), *five* denotes a set of pluralities each of which contains 5 atomic entities. Similarly to intersective adjectives, cardinals combine with NPs they modify via Predicate Modification (Heim & Kratzer 1998). Therefore, when *five* is composed with *cats*, see (16b), the resulting phrase denotes the intersection of a set of pluralities of cats and a set of plural individuals that consist of five atomic entities, i.e., a set of sums of five cats.

(16) a. $[\![\text{five}]\!] = \lambda x[\#(x) = 5]$

b. $[\![\text{five cats}]\!] = \lambda x[{}^{*}\textsc{cat}(x) \wedge \#(x) = 5]$

The approach discussed above gives a straightforward explanation for the contrasts between cardinals and determiners such *every* and *most* in predicate position as well as the well-formedness and felicity of constructions such as those in (15). However, it faces some challenges as well.

[3]For an introduction to mereological theories of plurality, see, e.g., Champollion & Krifka (2016).

3.3 Cardinals as predicate modifiers

A problem with treating cardinals as cardinal properties is that cross-linguistically the use of bare cardinals in predicate position is very restricted (Ionin & Matushansky 2018: 33–34). For instance, in Russian (17) is ungrammatical. Similarly, Dutch (18) can only mean that the children are two years old and not that they are two in number. And even in English the predicative use of bare cardinals is heavily restrained, as witnessed by the fact that examples such as (19) are highly degraded. If cardinals are assigned type $\langle e, t\rangle$, then this is rather startling.

(17) * Deti byli { dva / dvoe }. — Russian
children were two two.GNDR
Intended: 'The children were two in number.'

(18) # De kinderen zijn twee. — Dutch
the children are two
Intended: 'The children are two in number.'

(19) ?? The chairs in this room are twelve.

Another issue concerns the fact that what counts as 'one' seems to be highly dependent on the meaning of the NP the cardinal combines with. Arguably, atoms should be defined relative to a particular property rather than in absolute terms. To see why, let us consider partitive constructions such as those in (20) (cf. Chierchia 2010; Wągiel 2018). Example (20a) is weird since the numeral NP designates triples of body parts and cannot be understood as referring to three pluralities of boys. Yet, in (20b) quantification over subgroups of boys is possible. Importantly, neither *parts of the boys* nor *parts of the group* denote entities that are atomic in any absolute sense. The first is true of portions of matter whose parts are also parts of the boys. Similarly, the latter designates subgroups which themselves can also consist of subgroups of boys. This suggests that the cardinal determines what counts as 'one' relative to the denotation of the modified NP (and possibly some contextual hints, see Rothstein 2010) and it is not obvious how this fact could be captured if cardinals were interpreted simply as intersective predicates.

(20) a. # Three parts of the boys were sleeping.
b. Three parts of the group were sleeping.

A prominent alternative to treating cardinals as cardinal properties is to analyze them as predicate modifiers (type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$) equipped with a pluralization operation * and a measure function $\#(P)$ (Krifka 1989; 1995). In (21a), * closes the predicate under sum, i.e., takes a set of atomic entities and returns that set extended with all the pluralities one can form by summing atoms. On the other hand, the operation # returns for a property P a measure function that yields pluralities of five individuals having that property. In other words, *five* maps pluralities of entities onto the natural number 5.[4] When the cardinal combines with an NP, after the predicate slot is saturated we obtain a set of relevant plural individuals (type $\langle e,t \rangle$). For instance, *five cats* denotes a set of pluralities of cats each of which consists of five cats, see (21b). Thus, what counts as 'one' in this system is relativized to the denotation of the NP.

(21) a. $[\![\text{five}]\!] = \lambda P_{\langle e,t \rangle} \lambda x_e [{}^*P(x) \wedge \#(P)(x) = 5]$
b. $[\![\text{five cats}]\!] = \lambda x_e [{}^*\textsc{cat}(x) \wedge \#(\textsc{cat})(x) = 5]$

Another possibility is to interpret cardinals as predicate modifiers providing the cardinality of a partition of a plural individual (Ionin & Matushansky 2006; 2018). A partition of a plurality x is a cover of x, i.e., a set of entities such that the sum of all those entities forms x; in addition, it is a cover whose cells do not overlap, i.e., do not share any parts (cf. Gillon 1987; Schwarzschild 1996). In (22a), S is a partition Π of an individual x and the cardinality of S equals 5. The cardinal combines with the NP via standard Function Application and the result is of type $\langle e,t \rangle$. For instance, *five cats* gets the meaning in (22b), i.e., it denotes a set of pluralities divisible into 5 non-overlapping entities each of which has a property of being a cat.[5]

(22) a. $[\![\text{five}]\!] = \lambda P_{\langle e,t \rangle} \lambda x_e \exists S_{\langle e,t \rangle} [\Pi(S)(x) \wedge |S| = 5 \wedge \forall s \in S[P(s)]]$
b. $[\![\text{five cats}]\!] = \lambda x_e \exists S_{\langle e,t \rangle} [\Pi(S)(x) \wedge |S| = 5 \wedge \forall s \in S[[\![\text{cat}]\!](s)]]$

Notice, however, that both approaches discussed above require the denotation of NPs the cardinal combines with to be singular, i.e., to consist only of atoms. It is postulated that the source of plurality is the numeral and the plural marking on the noun is, e.g., due to agreement with no semantic interpretation. Supporting evidence comes from expressions such as those in (23) where the plural is not associated with a plurality (Krifka 1989; Bylinina & Nouwen 2018) as well as

[4]This is a slight simplification since in fact Krifka (1989) postulates a special operation NU for measuring pluralities in terms of 'natural units' they consist of, whereas Krifka (1995) proposes OU for measuring the number of 'object units' realizing a particular kind.

[5]The system is devised this way in order to provide a unified analysis of both simple and complex numerals such as *five hundred*. For a more detailed discussion of the approach, see Matushansky & Ionin (this volume).

from languages such as Finnish and Turkish which display the singular/plural distinction but in which unlike in, say, English cardinals systematically require singular NPs, see (24).

(23) a. zero students
b. 1.0 students

(24) üç { çocuk / *çocuklar } Turkish
three child children
'three children'

Finally, analyzing cardinals as predicate modifiers makes it easier to account for restrictions on bare cardinals in predicate position, see (18)–(19), by appealing to the well-described peculiarities of the copula which would allow for a type-shift only under particular circumstances (Ionin & Matushansky 2018: 33–34).

3.4 Cardinals as degree quantifiers

The last approach to be discussed in this sections builds on two observations regarding the behavior of cardinals. The first observation is that sentences with existential modals such as those in (25) are ambiguous (Kennedy 2013; 2015). On the strong reading, (25a) means that Hanna is allowed to eat five cookies and she is not allowed to eat more. The strong reading is probably the most natural interpretation of (25a). There is, however, also a weak reading which merely states that eating five cookies by Hanna is allowed without saying anything about eating other quantities of cookies. This interpretation becomes more prominent in questions, see (25b).

(25) a. Hanna is allowed to eat five cookies.
b. Is Hanna allowed to eat five cookies?

The second observation is that numerals can take scope independently of the rest of the NP. The evidence comes from decimals in sentences such as (26a) (Kennedy & Stanley 2009). Crucially, it does not entail the existence of families with 2.1 cats. This contrasts with (26b) which is infelicitous due to such an entailment.

(26) a. The average Polish family has 2.1 cats.
b. # A normal Polish family has 2.1 cats.

The data discussed above suggest that cardinal numerals are in fact quantifiers. We have already seen that the analysis of *five* as a determiner is most probably incorrect and obviously treating it as a generalized quantifier over individuals (type $\langle\langle e, t\rangle, t\rangle$) would not make much sense. However, an interesting idea is to interpret cardinals as quantifiers over degrees, i.e., expressions of type $\langle\langle d, t\rangle, t\rangle$ (Kennedy 2013; 2015). Degrees (type d) are objects similar to individuals (type e) with the crucial difference that the domain of degrees D_d, unlike the domain of individuals D_e, is ordered.[6] Hence, on the degree quantifier analysis *five* denotes a set of degree properties whose maximal value equals 5, see (27a). A sentence with a numeral NP would be then interpreted as in (27b), e.g., the maximal number of entities that Hanna had and that are cats is 5.

(27) a. $[\![\text{five}]\!] = \lambda D_{\langle d,t\rangle}[\textsc{max}(D) = 5]$
b. $[\![\text{Hanna had five cats}]\!] =$
$= \textsc{max}\{n \mid \exists x[\textsc{had}(x)(\textsc{hanna}) \wedge {}^{*}\textsc{cat}(x) \wedge \#(x) = n)\} = 5$

The analysis of cardinals as degree quantifiers provides a promising perspective to explain interactions between modals and both unmodified and modified cardinals such as *more than five* and *at least five* (Nouwen 2010). In the next section, I will discuss the arithmetical function of cardinals and how it relates to the theories described so far.

4 Relating cardinalities and number concepts

The theories of cardinal numerals discussed so far focus on the quantifying function. However, as we have already seen in (1e) and §2.3, cardinals can also be used to refer to abstract numeric values.

4.1 Cardinals as names of numbers

On the arithmetical function, cardinals seem to behave as proper names of number concepts (type n) or, alternatively, degrees (type d).[7] From this perspective, the arithmetical meaning of *five* is simply the number 5, see (28a). Expressions such as *prime* would then denote properties of numbers (type $\langle n, t\rangle$). For instance

[6]For more discussion on degrees and an introduction to degree semantics, see, e.g., Morzycki (2016: Ch. 3).

[7]Throughout the paper, I will assume there is no difference between the two notions and following the widespread convention in the literature on the arithmetical use of cardinals I will simply use the label n.

the extension of (28b) would be the set $\{2, 3, 5, 7, 11, 13, \dots\}$. On the other hand, expressions used to formulate mathematical statements such as *plus*, *times*, *divided by* and *equals* seem to describe relations between number concepts. For instance, (28c) is interpreted in terms of addition of two numeric values.

(28) a. $[\![\text{five}]\!] = 5$
b. $[\![\text{prime}]\!] = \lambda n_n[\textsc{prime}(n)]$
c. $[\![\text{plus}]\!] = \lambda n_n \lambda m_n[n + m]$

The key question is what the relationship between names of number concepts, i.e., the arithmetical use, and meanings of cardinals used in the quantifying function is. In the next sections, I will discuss two possible approaches to how to relate the two meanings.

4.2 Deriving number concepts from cardinalities

The mainstream view is to take the quantifying meaning as basic, be it the predicate, predicate modifier or degree quantifier analysis, and to derive the arithmetical meaning from it. Let us discuss three variants of that view.

As we have already seen in §3.2, on the predicate analysis cardinals are interpreted as denoting a cardinal property, see (29a). Rothstein (2017) assumes this semantics to be basic.[8] The arithmetical meaning is then derived via a special type-shifting operation $^\cap$ which when applied to a cardinal property yields a proper name, see (29b).

(29) a. $[\![\text{five}]\!]_{\langle e,t\rangle} = \lambda x_e[\#(x) = 5]$
b. $[\![\text{five}]\!]_n = {}^\cap[\![\text{five}]\!]_{\langle e,t\rangle} = 5$

The proposal is seemingly motivated by an analogy with adjectival and nominal uses of expressions such as *white*. It builds on Chierchia (1985)'s theory of predication according to which every property has an entity correlate of type π derived by $^\cap$. For instance, the predicate *white* can be turned into the name of the property of being white. Similarly, $^\cap$ turns *five* into the name of a property of being five in number.

However, as argued convincingly by Ionin & Matushansky (2018: 31–33) this analogy does not hold since arithmetical environments such as those discussed in §2.3 require the number 5 itself as an argument rather than the property of

[8]In fact, this is a slight simplification since the theory distinguishes typally between lower numerals and 'lexical powers', i.e., multiplicands such as *hundred* in *five hundred*.

being five in number. Consequently, in order to derive the arithmetical function Ionin & Matushansky (2018: 26–27) propose a null suffix NOMNUM which is a nominalizer that when applied to a predicate modifier, turns it into a numeric singular term. More specifically, for any cardinal numeral it yields the corresponding cardinality. An argument for such an account comes from the fact that cardinal numerals as names of numbers display full morpho-syntactic uniformity suggesting they are derived expressions. On the other hand, cardinals in the quantifying function often show variation with respect to ϕ-features, e.g., grammatical gender.

Another mode of relating the arithmetical and quantifying meaning is proposed by Kennedy (2015). The idea is to derive the first from the latter by the standard type-shifting operations BE and IOTA defined in (30a) and (30b), respectively (Partee 1986). BE takes a quantifier and returns a property. On the other hand, IOTA yields the unique entity of which the relevant property is true.

(30) a. BE $= \lambda \mathcal{P}_{\langle\langle e,t,\rangle,t\rangle} \lambda x_e [\mathcal{P}(\lambda y_e[y = x])]$
b. IOTA $= \lambda P_{\langle e,t\rangle} \iota x[P(x)]$

Generalizing the type-shifts defined above so that they can also apply to quantifiers over degrees and properties of degrees (or numbers), respectively, allows to derive the arithmetical meaning by applying BE and IOTA consecutively to the degree quantifier semantics. Specifically, when BE applies to a set of degree properties whose maximal value equals 5, see (27a), the result in (31a) is the singleton set $\{5\}$. The subsequent application of IOTA yields the number 5, as shown in (31b).

(31) a. BE($[\![$five$]\!]$) $= \lambda n_n[n = 5]$
b. IOTA(BE($[\![$five$]\!]$)) = IOTA$(\lambda n_n[n = 5]) = 5$

A nice feature of such an approach is that it allows to relate the quantifying and arithmetical functions using standard independently motivated type-shifting machinery. In the next section, I will discuss accounts that derive cardinalities from the arithmetical meaning.

4.3 Deriving cardinalities from number concepts

Though the view that the quantifying function of cardinals is the basic one seems to have become mainstream, there are a number of approaches that build on a contrary intuition, namely that underlyingly cardinal numerals are simply names of number concepts (Scha 1981; Krifka 1989; Zabbal 2005; Scontras 2013; 2014;

Wągiel 2018). According to that alternative view, cardinals in prenominal position are in fact syntactically and semantically complex expressions derived from the arithmetical meaning via various shifts.

An early theory of cardinals postulating that they are derived from number concepts was developed by Scha (1981). This system distinguishes between numbers, numerals and numerical determiners. The first simply denote number concepts. On the other hand, the NUMERAL head shifts an integer (type n) to a cardinal predicate while the numerical D head transforms such a cardinal predicate into a determiner with a GQ-style semantics, see (32).[9]

(32) $[\ \text{D}_{\langle\langle e,t\rangle,\langle\langle e,t\rangle,t\rangle\rangle}\ [\ \text{NUMERAL}_{\langle e,t\rangle}\ [\ \text{NUMBER}_n\ \textit{five}\ ]\ ]\ ]$

A related idea is the core of the system developed by Hackl (2000) who makes use of a covert quantificational operator MANY in order to turn a number concept into a determiner, see (33a). On this approach, the cardinal on its quantifying use is accompanied with an additional null element that ensures the shift, as depicted in (33b). The result combines with the denotation of the NP to form a generalized quantifier.

(33) a. $[\![\text{MANY}]\!] = \lambda n_n \lambda P_{\langle e,t\rangle} \lambda Q_{\langle e,t\rangle} \exists x_e [\#(x) = n \wedge P(x) \wedge Q(x)]$

b. $[\![\text{five cats}]\!] = [\![[\ [\ 5\ \text{MANY}\ ]\ \text{cats}\ ]]\!]$

Another proposal on how to derive the quantifying meaning from a name of a number concept dates back to Krifka (1995). On this view, all cardinals are assumed to underlyingly designate number concepts but they also involve an additional classifier element that depending on a language can be either overt or covert. There are several variants of this approach but what they all share is that such a classifier element takes an integer and returns a counting device equipped with an appropriate measure function, see (34a) (Scontras 2013; 2014; Wągiel 2018). For instance, in the derivation of the phrase *five cats* the name of a number concept is first turned into a quantifying predicate modifier which then combines with the noun to yield the meaning in (34b).

(34) a. $[\![\text{CL}]\!] = \lambda n_n \lambda P_{\langle e,t\rangle} \lambda x_e [{}^*P(x) \wedge \#(P)(x) = n]$

b. $[\![\text{five cats}]\!] = [\![[\ [\ 5\ \text{CL}\ ]\ \text{cats}\ ]]\!] = \lambda x_e [{}^*\text{CAT}(x) \wedge \#(x) = 5]$

[9] The exact denotation of the numerical determiner depends on whether the whole sentence gets a collective, distributive or cover reading. In other words, there are three distinct Ds each of which deriving a different interpretation.

In yet another variant of the discussed approach, the classifier semantics turns an arithmetical concept into a predicate (Sudo 2016). The classifier takes a numeric value and yields a cardinal property (type $\langle e, t \rangle$) rather than a predicate modifier. In addition, it can introduce a special presupposition concerning the nature of referents of the modified noun. The resulting expression then combines with the NP via Predicate Modification.

From the theoretical perspective both types of relationship between the arithmetical and quantifying function, i.e., the derivation from the quantifying meaning to the arithmetical meaning and vice versa, seem plausible. Thus, the question which one is correct appears to be an empirical issue. In §4.2, we have already seen that certain aspects of cardinals' morphology have been argued to support the quantifying-function-is-basic view. In the next section, I will discuss a different type of evidence that can be useful in determining the direction of the derivation.

4.4 Morphological evidence

It has been observed that many languages distinguish formally between so-called counting, i.e., arithmetical, and attributive, i.e., quantifying, numerals (Hurford 1998; 2001). For instance, in Mandarin the cardinal *èr* ('two') is used in arithmetical environments whereas *liǎng* ('two') is an attributive form appearing in the quantifying use. Similarly, the Maltese numeral *tnejn* ('two') indicates the arithmetical function while *żewġ* ('two') has the quantifying meaning. Though the distinction in question is typically restricted to a subset of cardinals in a language and there are a number of patterns of how the two forms can differ, it is a relatively widespread phenomenon across languages.

Interestingly, the two families of theories discussed above make different predictions regarding the morphological make-up of cardinal numerals cross-linguistically. On the assumption that morphology expresses meaning, if the arithmetical function is derived from the quantifying function, see §4.2, then we would expect that across languages a pattern with arithmetical cardinals being morphologically more complex than quantifying cardinals should be relatively widespread. This is because an additional affix is expected to introduce a shift from a counting device to a number concept. On the other hand, if the quantifying function is derived from the arithmetical one, see §4.3, then we would expect an inverse pattern to be common. Namely, quantifying cardinals should include an additional morpheme encoding a shift from number concepts to quantifying modifiers.

It turns out that the cross-linguistically widespread asymmetry attested, e.g., in many obligatory classifier languages, is of the latter type (Wągiel to appear; Wągiel & Caha 2020). For instance, bare cardinals in Japanese cannot modify NPs and require an additional element, traditionally referred to as a classifier, see (35a).[10] However, such an element is illicit in an arithmetical environment like (35b) despite the fact that *ko* being a general classifier could be used to indicate any type of inanimate entity.

(35) a. { *go-no / go-ko-no } ringo — Japanese
five-GEN five-CL-GEN apple
'five apples'

b. juu waru { go-wa / #go-ko-wa } ni-da.
ten divided five-TOP five-CL-TOP two-COP
'Ten divided by five equals two.'

Therefore, it seems that the approaches that derive the arithmetical semantics from the quantifying one make incorrect predictions with respect to meaning/form correspondences in cardinal numerals cross-linguistically. However, there is a twist here since a pattern with more complex arithmetical cardinals is also attested, though scarce. For instance, in German the arithmetical cardinal for 1 consists of an additional morpheme compared to its quantifying equivalent, see (36).

(36) a. { ein / *ein-s } Mädchen — German
one one-NBR girl
'one girl'

b. Zehn geteilt durch { *ein / ein-s } ist gleich zehn.
ten divided by one one-NBR is equal ten
'Ten divided by one equals ten.'

The pattern attested in German is puzzling and seemingly indicates that both types of the relationship between the two functions in question are possible. Nevertheless, it can be accounted for by postulating a more complex morpho-semantic structure of cardinals along with a spell-out mechanism based on late insertion (Wągiel & Caha 2020). On this analysis, though the arithmetical meaning is more basic, it is still derived from an even more primitive concept of a

[10] Typically, classifiers also introduce certain restrictions on the referents of the NP modified by the cardinal, e.g., being a round object, a flat object, a plant, a big animal etc. This aspect of their meaning can be captured by accommodating various presuppositions concerning the nature of the denoted individuals (Sudo 2016).

number scale. The quantifying meaning is then obtained by turning a number concept into a quantificational modifier. The pattern in (36) can be then explained by postulating that *ein* denotes the concept of a number scale, the suffix *-s* forges the name of the numerical value and that there is yet another phonologically null morpheme -∅ that shifts the number scale into a counting device. Crucially, not only does such an account capture cross-linguistic variation but also it explains why the German pattern is rare. On the other hand, no alternative explanation of the typological facts that would build on the quantifying meaning as the primitive component has been proposed so far.

To conclude, the patterns discussed above suggest that the arithmetical meaning of cardinals is the basic one and that other uses can be derived from it. In the next section, I will discuss complex numerical expressions in Slavic and suggest how this type of evidence can provide further hints on which of the two ways of relating number concepts and cardinalities discussed so far is on the right track.

5 Complex numerical expressions

While the mainstream semantic research focuses mostly on cardinal numerals, there is a growing body of formal literature acknowledging that cardinals are in fact only a subset of various numerical expressions. This fact has been well known in Slavic descriptive traditions. Since Slavic languages have a rich morphology, it is not surprising that one can find abundant formal complexity also in numerals. In this section, I will briefly review recent developments in the study of different types of derivationally and semantically complex numerical expressions. Though the focus of the section is on Slavic data, some parallels with other languages will be drawn.

5.1 Ordinals

The cross-linguistically most widespread type of derived numerical expressions are ordinal numerals, i.e., forms such as English *fifth* and Russian *pjatyj* ('fifth'). Intuitively, ordinals represent position or rank in a sequential order. If a language distinguishes morphologically a class of ordinals, they are always derived by an additional affix (Stolz & Veselinova 2013).[11]

Perhaps somewhat surprisingly, the semantics of ordinals has been typically linked to superlatives. Cross-linguistically, the two types of expressions often

[11]Unless an ordinal is suppletive, e.g., *one* ~ *first* in English.

share identical or related morphemes (Hurford 1987; Veselinova 1998)[12] and have the same syntax (Barbiers 2007). Furthermore, the acquistion of ordinals seems to be influenced by superlative morphology (Meyer et al. 2018). Even more importantly, ordinals, just like superlatives, give rise to an ambiguity between an absolute and comparative interpretation (Bhatt 2006; Sharvit 2010). On the absolute reading of (37a), John gave Mary a telescope that is older than all other telescopes. On a comparative reading, however, he might have given her a relatively new telescope than happens to be older than the telescopes other people gave her, i.e., John's gift is compared to other gifts rather that to telescopes in general. Similarly, (37b) can mean either that Mary got a telescope that was older than other telescopes, or that John's telescope was the first Mary has received.

(37) a. John gave Mary the oldest telescope.
 b. John gave Mary the first telescope.

However, it has been observed that ordinals, unlike superlatives, do not give rise to so-called upstairs *de dicto* readings in sentences with intensional operators such as *want* (Bylinina et al. 2015). For instance, (38a) can have a particular comparative interpretation which makes it true in the scenario below. Yet, that reading is unavailable for (38b).

(38) Scenario: There are many trains throughout the day. John wants to take a train. Any of the trains between 3 pm and 4 pm is fine for him. Similarly, Bill and Steve want to take a train, and they are fine as far as the departure time is between 5 pm and 6 pm and between 7 pm and 8 pm, respectively. These people do not know anything about one another.
 a. John wants to take the earliest train. TRUE
 b. John wants to take the first train. FALSE

As a result, Bylinina et al. (2015) argue that superlatives and ordinals differ with respect to where they are interpreted. According to that proposal, while superlatives involve movement, ordinals are always interpreted *in situ*.

5.2 Collective numerals

One of the key topics discussed in the literature on plurality regards distributive and collective interpretations of sentences including plural DPs such as (39)

[12]Notice that this is also the case in some Slavic languages. For instance, Polish *pierw-szy* and Ukrainian *per-šyj* (both 'first') contain the comparative markers *-szy* and *-šyj* which also appear in superlatives, e.g., *naj-star-szy* and *naj-star-šyj* (both 'the oldest'), respectively.

(e.g., Scha 1981; Link 1983; Landman 1989; 2000; Schwarzschild 1996). For instance, (39b) is ambiguous between a reading on which a total of three letters have been written, i.e., a distributive interpretation, and a reading on which only one letter has been written, i.e., a collective interpretation. The source of the ambiguity has often been located inside the VP (e.g., Hoeksema 1983; Link 1984; Schwarzschild 1991; see also Lasersohn 1995: Ch. 7 for an overview).

(39) a. The boys wrote a letter.
b. Three boys wrote a letter.

Interestingly, Slavic languages have special numerical expressions which force a collective interpretation of the whole sentence. For instance, consider the Czech sentences in (40) (Dočekal 2012; 2013).[13] While (40a) patterns with (39b) in that it is ambiguous between a distributive and a collective reading, the sentence in (40b) rules out the distributive interpretation, and thus the sole possible reading is that only one letter has been written.

(40) a. Tři chlapci napsali dopis. Czech
three boys wrote letter
'Three boys wrote a letter.' ✓COLLECTIVE, ✓DISTRIBUTIVE
b. Trojice chlapců napsala dopis.
three.COLL boys.GEN wrote letter
'A group of three boys wrote a letter.' ✓COLLECTIVE, # DISTRIBUTIVE

Since the difference between the two sentences boils down to the alternation between the basic cardinal numeral *tři* ('three') and the morphologically complex derived form *trojice* ('group of three'), the suffix *-ice* has been proposed to be a morphological reflex of a semantic operation forcing a collective interpretation, specifically the group-forming operator ↑ proposed by Landman (1989).[14]

Importantly, collective numerals cannot be used to refer to number concepts, as witnessed in (41). This fact is surprising if the arithmetic meaning were derived from the quantifying one, see §4.2. Despite the collective meaning component described above, NPs involving collective numerals denote pluralities and one would expect the same shift that is posited to turn the quantificational meaning

[13] In the literature, expressions such as *trojice* ('group of three') have been referred to as collective or group numerals as well as denumeral group nouns (due to their nominal-like behavior).

[14] The affix *-oj-* appears only in morphologically complex numerical expressions derived from the roots for 2 and 3 and is usually assumed to have a purely structural function, i.e., it marks non-cardinal stems.

of a cardinal into the name of a number concept should be applicable also in this case, contrary to facts.

(41) a. # Trojice je prvočíslo. Czech
three.COLL is prime.number
Intended: 'Three is prime.'

b. # Šestice děleno trojicí je dvojice.
six.COLL divided three.COLL.INS is two.COLL
Intended: 'Six divided by three is two.'

Collective numerals have been identified also in other Slavic languages. In particular, Polish *trójka* ('group of three') patterns with (40b) (Wągiel 2015) and similar examples include Slovak *trojica*, BCS *trojka* and Russian *trojka* (all 'group of three').[15] The existence of the discussed phenomenon in Slavic is unexpected since it is typically postulated to hold cross-linguistically that if a language has a scopal quantifier, it is a quantifier forcing a distributive interpretation (Gil 1992).[16] Hence, the Slavic data discussed above compel to revise that generalization. In any case, a systematic theory-driven cross-linguistic research of collective numerals has not been carried out so far.

5.3 *Both*

An interesting feature of many languages including Slavic is that alongside a regular cardinal numeral corresponding to English *two* there is also another form corresponding to English *both*. In an early GQ account, *both* has been analyzed as a quantificational determiner with the same meaning as *the two* (Barwise & Cooper 1981). However, it has been observed that the two expressions are not equivalent (Ladusaw 1982). For instance, *both* is incompatible with collective predicates, see (42a), and cannot appear inside partitives, see (42b). As witnessed in (43), the same behavior is also attested in Slavic (Łazorczyk & Pancheva 2009).

(42) a. { The two / #Both } students are a happy couple.

b. One of { the two / #both } children sneezed.

[15] Russian shows a strong tendency to lexicalize collective numerals, e.g., *trojka* is also a word for a carriage drawn by a team of three horses abreast.

[16] In Hebrew, there are derived expressions that at first blush resemble Slavic collective numerals, e.g., *šlišiya* ('threesome of'). However, they have been reported to differ in that they do not force a collective interpretation (Gil 1992).

(43) a. # Obe ženščiny prišli vmeste. Russian
both woman.GEN came together
Intended: 'Both women came together.'

b. Kto iz etix {dvoix / *oboix} sdelal to, čto xotel otec?
who from these two.GEN both.GEN did this what wanted father
'Which of these two did what his father wanted?'

The data presented above led to proposals postulating that *both*, unlike cardinals, involves an additional distributive component (e.g., Ladusaw 1982; Landman 1989).[17] Interestingly, however, there is yet another respect in which the two differ. Specifically, *both* cannot be used to refer to a number concept in an arithmetical environment, see (44).

(44) a. # Oba to liczba parzysta. Polish
both that number even
Intended: 'Both is an even number.'

b. # Dziesięć dzielone przez oba równa się pięć.
ten divided by both equals REFL five
Intended: 'Ten divided by both equals five.'

Similarly to collective numerals, the behavior of *both* and its Slavic equivalents presented in (44) poses a serious challenge for approaches deriving the arithmetical function of numerals from the allegedly basic quantifying meaning.

5.4 Taxonomic numerals

A unique property of some Slavic languages is that they have taxonomic numerals. Likewise collective numerals, they are derived by a special suffix which triggers a non-trivial semantic effect. Unlike cardinals, such expressions do not quantify over object-level entities, i.e., tokens of a type, but rather over subkinds of a particular kind corresponding to the meaning of the modified noun.For instance, let us consider the taxonomic numeral *dvojí* ('two kinds of') in Czech (Dočekal 2012; 2013; Grimm & Dočekal to appear). The phrase in (45a) does not refer to two musical instruments but rather to two types of violin, e.g., a set of classical violins and a set of electric violins. Likewise, (45b) denotes two kinds of the beverage, e.g., white and red wine, irrespective of its amount.

[17] For the recent research on bona fide distributive numerals, see, e.g., Gil (2013), Cable (2014), Wohlmuth (2019).

(45) a. dvojí housle Czech
two.TAX violin
'two kinds of violin'

b. dvojí víno
two.TAX wine
'two kinds of wine'

Unlike cardinals, taxonomic numerals specify the domain of quantification to be within the realm of kinds exclusively. Therefore, the suffix *-í* has been proposed to introduce a quantificational operation dedicated to counting subkinds, specifically the KU (for 'kind unit') operation proposed by Krifka (1995).

Though taxonomic numerals seem to be best preserved in Czech, Polish *dwojaki* and *dwoisty*, Bulgarian *dvojak* and Russian *dvojakij* (all 'twofold') are arguably expressions of a similar type. However, there appear to be certain differences with respect to their distribution and behavior that have not been described sufficiently so far. Crucially, none of the taxonomic numerals can be used to refer to an abstract number concept.

5.5 Aggregate numerals

Another intriguing type of Slavic derivationally complex numerical expressions is sometimes referred to as aggregate numerals, e.g., forms such as Czech *dvoje* ('two collections') (Dočekal 2012; 2013) as well as BCS *dvoji* (Lučić 2015). Likewise collective numerals, they are derived by a special suffix which triggers a non-trivial semantic effect and lack the arithmetical meaning. While cardinals simply count atomic individuals in the denotation of the modified NP, aggregate numerals are used to quantify over collections of entities that typically form a particular spatial configuration. For instance, (46a) refers to two aggregates of cards, e.g., two decks of cards. Similarly, (46b) denotes two pairs of shoes.

(46) a. dvoje karty Czech
two.AGGR cards
'two sets of cards'

b. dvoje boty
two.AGGR shoes
'two pairs of shoes'

Aggregate numerals are not limited to Slavic. For instance, similar expressions seem to have existed in Latin, e.g., *trēs* ('three') ~ *trīnī* ('three collections') (Ojeda

1997), and Estonian uses pluralized numerals for this purpose, e.g., *kaks* ('two') ~ *kahed* ('two collections') (Norris 2018). However, it is the Slavic data what has been taken as evidence for the relevance of mereotopology in natural language, specifically the significance of topological relations holding between atomic members of a plurality within a part-whole structure.[18] In particular, the suffix *-e* in phrases such as (46) is sensitive to how parts making up a whole are arranged, and thus it has been proposed to introduce a semantic operation dedicated to counting clusters, i.e., topologically structured spatial groupings of entities (Grimm & Dočekal to appear).

5.6 Gender-sensitive numerals

All Slavic languages mark grammatical gender on low numerals whereas some of them, e.g., Bulgarian, Polish and Slovak, distinguish between virile (personal masculine) and non-virile forms also in higher numerals. (Cinque & Krapova 2007; Pancheva 2018; Wągiel to appear). However, what is probably even more interesting is the existence of complex expressions which I will refer here to as gender-sensitive numerals.[19] Such expressions introduce non-trivial restrictions with respect to the natural gender of individuals making up a plurality.

Let us consider Polish gender-sensitive numerals such as *troje* ('three (at least one male and at least one female)') (Wągiel 2014; 2015). Unlike virile and non-virile cardinals in (47a) and (47b), respectively, they do not show grammatical gender agreement with modified NPs. Instead, they are neuter, as demonstrated in (47c). And yet they introduce a gender-related effect. While (47a) denotes a plurality of either males or students whose natural gender is not known to the speaker and (47b) refers to three females, (47a) is interpreted as designating a mixed-gender group.

(47) a. tych trzech studentów — Polish
these.ACC.V two.V students.GEN.V
'these three students (male or indeterminate)'

b. te trzy studentki
these.ACC.NV three.NV students.NOM.NV
'these three female students'

[18] For an introduction to mereotopology, see Grimm (2012: Ch. 4), Wągiel (2018: Ch. 6).

[19] In the traditional Slavic linguistics such forms are typically termed 'collective numerals'. In order to avoid confusion with expressions discussed in §5.2, I will not use that term.

c. to troje studentów
this.NOM.N three.GNDR.N students.GEN.V
'these three students (at least one male and at least one female)'

Based on the evidence discussed above, numerals such as Polish *troje* have been proposed to introduce a presupposition that a plurality of individuals referred to by the whole phrase is heterogeneous and consists of both male and female individuals. Though similar expressions seem to be only attested in BCS, e.g., *dvoje* ('two (one male and one female)'), there are also other types of gender-sensitive numerals in Slavic. For instance, BCS *dvojica* and Russian *dvoe* (both 'two (male)') presuppose a homogeneous group consisting of male individuals only (Kim 2009; Lučić 2015; Khrizman 2020). Interestingly, none of the gender-sensitive numerals can be used to refer to a number concept.

5.7 Indefinite numerals

Another intriguing class of numerical expressions concerns indefinite numerals such as English *several* (Kayne 2007). Similar quantifiers appear in all branches of Slavic, e.g., compare Czech *několik*, Russian *neskol'ko*, Bulgarian *nyakolko* and BCS *nekoliko* (all 'several'). Particularly interesting data come from Polish which distinguishes between two different expressions of this type, specifically *kilka* ('several, a few') and *ileś (tam)* ('some, some amount'). While the former is only compatible with count NPs and designates a cardinality between 3 and 9, the latter also combines with mass NPs and can indicate any quantity (Wągiel 2020b).

In terms of morphosyntax and many aspects of semantics, *several* and its Slavic equivalents pattern with cardinals. However, one crucial respect in which the two differ is that indefinite numerals do not fit contexts calling for number concepts, see (48a) and (49) (Schwarzschild 2002; Wągiel 2020b). Despite the fact that one can easily imagine the intended meaning and paraphrase it using the existential quantifier, as in (48b), this is not what one gets in (48a) and (49).

(48) a. # Four plus several is less than ten.
b. There is a number *n* such that four plus *n* is less than ten.

(49) # Cztery plus { kilka / ileś (tam) } to mniej niż dziesięć. Polish
four plus several some this less than ten
Intended: 'Four plus several/some is less than ten.'

Polish indefinite numerals have been analyzed as complex expressions with an incorporated measure function and choice function. The difference between *kilka* and *ileś (tam)* is due to a different type of set the choice function selects a number from and a different kind of built-in measure function (Wągiel 2020b).

5.8 Multipliers

In many languages there is a heavily understudied class of numerical expressions sometimes referred to as multipliers, e.g., English *double*. Though Germanic and Romance languages have borrowed their multipliers from Latin, in Slavic they are morphologically transparent and one can easily notice that they are derived from numerical roots by various affixes, e.g., compare Polish *podwójny*, Russian *dvojnoj* and Czech *dvojitý* (all 'double').

Interestingly, multipliers display non-trivial quantificational behavior. Specifically, they do not quantify over whole entities or events, as cardinals would, but rather over parts thereof (Wągiel 2018; 2020a). For instance, the phrase in (50a) denotes a set of singular objects having a complex internal structure. Each of those objects is a crown but crucially it also has two parts that could be considered as having the property of being a crown themselves, e.g., entities such as the ancient Egyptian Pschent or the papal tiara. Similarly, (50b) refers to a murdering event with two victims. But this means that there were two parts of that event each of which could be described as a murder in its own right.

(50) a. dvostruka kruna BCS
two.MULT.F crown.N
'double crown'

b. dvostruko ubistvo
two.MULT.N murder.N
'double murder'

The behavior of multipliers has been argued to stem from the phenomenon of subatomic quantification (Wągiel 2018). According to the proposal, natural language is sensitive to the arrangement of parts within a singular entity. Certain expressions, e.g., proportional partitives and multipliers, can access such subatomic part-whole structures and quantify over parts conceptualized as contiguous objects within a whole.

5.9 Event numerals

Yet another class of cross-linguistically frequent numerical expressions consists of event numerals also known as multiplicatives. They include adverbial expressions such as English *twice* and *two times* which are primarily used to quantify over events but can be also employed in comparative and equative constructions. Event numerals are attested in all Slavic languages with BCS *dvaput* and Czech *dvakrát* (both 'two times, twice') being representative examples.

Intuitively, event numerals seem to be a type of a broader class of adverbs of quantification including frequency adverbs such English *often* and French *souvent* ('often') (Abeillé et al. 2004, Doetjes 2007). However, close examination reveals interesting differences between event numerals and frequency adverbs one of which regards degree modification (Dočekal & Wągiel 2018). For instance, the sentence with the event numeral in (51a) is ambiguous between the quantified-event reading and the quantified-degree reading. On the former interpretation there were two events of increasing the demand and the total value of the increase is unknown whereas on the latter interpretation there was a twofold increase in the demand irrespective of the number of events on which it increased. Interestingly, (51b) lacks the quantified-degree reading.

(51) a. Poptávka po dotacích vzrostla dvakrát. Czech
demand after subsidies increased.PFV twice
'The demand for subsidies increased (by) two times.' ✓EVENT, ✓DEG

b. Poptávka po dotacích rostla často.
demand after subsidies increased.IPFV often
'The demand for subsidies increased often.' ✓EVENT, #DEGREE

In addition, while event numerals are perfectly fine in comparatives and equatives, frequency adjectives are infelicitous in such constructions, as witnessed by the contrast in (52).

(52) Petr je { dvakrát / #často } vyšší než Marie. Czech
Petr is twice often taller than Marie
'Petr is two times taller than Marie.'

Unlike all other types of numerical expressions discussed in this section, event numerals can be used in arithmetical environments provided that they express multiplication. However, it is puzzling that not all event numerals can do that. For instance, Polish has two sets of event numerals but only one can be felicitously used in a context such as (53) which suggests that the element *razy* ('times') might be a homonymous form for a multiplicative morpheme used to quantify over events and an expression of an arithmetical relation.

(53) { Dwa razy / #Dwukrotnie } trzy równa się sześć. Polish
two times twice three equas six
'Two times three equals six.'

Though event numerals seem key to understanding quantification in the domains of events and degrees, it is surprising that so far they have not received nearly as much attention as cardinals.

5.10 Frequency numerals

Finally, in many languages there are expressions I will refer to as frequency numerals, i.e., multiplicative adjectives equivalent to English *two-time*. Slavic is no exception here and Polish *dwukrotny*, Czech *dvojnásobný* and Russian *dvukratnyj* (all 'two-time') are representative examples of the class in question.

At first sight, it seems that frequency numerals such as *two-time* are expressions of the same type as frequency adjectives like *occasional* and *frequent* (Zimmermann 2003, Schäfer 2007, Gehrke & McNally 2015). Notice, however, that while *occasional* can have the adverbial reading, expressions such as *two-time* pattern with *frequent* in that they lack such an interpretation. For instance, while (54a) means that occasionally a sailor strolled by, both (54b) and (54c) cannot be understood that way.

(54) a. An occasional sailor strolled by. ✓ADVERBIAL
b. A frequent sailor strolled by. #ADVERBIAL
c. A two-time senator strolled by. #ADVERBIAL

Furthermore, frequency numerals fail to combine with sortal nouns and seem to require role nouns denoting socially salient capacities such as positions with a term and award recipients that can be acquired repetitively. That is because frequency numerals quantify over conventionalized events of acquiring a property, e.g., via a ceremony, compare (55a) and (55b).

(55) a. Jan to dwukrotny { mistrz / #człowiek }. Polish
Jan this two-time champion man
'Jan is a two-time champion.'
b. Jan został { mistrzem / #człowiekiem } dwa razy.
Jan became champion.INS man.INS two times
'Jan became a champion twice.'

Having examined various complex numerical expressions, let us now summarize the discussed data show us.

5.11 Summary

In this section, I have reviewed various types of numerical expressions in Slavic and beyond. In particular, I have discussed a wide range of morphologically complex forms and examined how they correspond to certain non-trivial semantic effects. The reviewed dataset is far larger and more diverse than typically analyzed in mainstream semantic theories of numerals, see §3. And yet, there is an intuition that a general theory of numerals should cover all of the examined cases. Therefore, I believe the relevance of the above-discussed Slavic data for such a theory is twofold.

First, morphological transparency of the discussed forms clearly shows that all complex numerical expressions share a common component, i.e., a numerical root designating a certain number. That number is somehow employed in counting but what exactly is counted may vary depending on a type of numeral, e.g., whole objects in the case of cardinals, parts in the case of multipliers and subkinds in the case of taxonomic numerals. Therefore, a theory of numerals should aim at providing a general mechanism that will allow us to capture not only the behavior of cardinals but also other numerical expressions discussed here.

Second, despite sharing an element designating a number none of the complex numerical expressions can be used to refer to abstract arithmetical concepts, i.e., the arithmetical function is never available except for basic cardinals.[20] This fact turns out to be problematic for the approaches deriving the arithmetical meaning from the quantifying one, see §4.2. The reason is that they shift a counting device semantics of a cardinal into a proper name of a number concept. But if so, why cannot that shift be also applied in the case of other types of counting expressions, e.g., a taxonomic, an aggregate or an indefinite numeral, or *both*? Semantically, those expressions do not seem to be radically different from cardinals (they all count something) to justify why shifting their meaning to a name of a number concept is impossible. If, on the other hand, the arithmetical meaning is basic, one expects to derive various quantificational flavors from it. Therefore, it would not be surprising that only basic cardinals can have the arithmetical function and other types of numerical expressions would lack it.

[20] And perhaps event numerals in contexts expressing multiplication.

Abbreviations

CL	classifier	AGGR	aggregate numeral
COP	copula	GNDR	gender-sensitive numeral
F	feminine	MULT	multiplier
N	neuter	NBR	arithmetical numeral
IPFV	imperfective	TAX	taxonomic numeral
PFV	perfective		
GEN	genitive		
INS	instrumental		
REFL	reflexive pronoun		
TOP	topic		

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To be added.

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