Chapter 1

Numerals and their kin

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In this paper, I focus on the meaning of numerals, i.e., lexical items that express a relationship with numbers. I discuss different functions that cardinal numerals can have including their quantifying and arithmetical use and compare various attempts to capture the relationship between those functions. Furthermore, I examine several types of complex numerical expressions such as ordinals, collective and taxonomic numerals and multipliers in Slavic in comparison to other languages. Finally, I bring novel data concerning Slavic label numerals, i.e., expressions used to identify entities via association with a number and propose a morpho-semantic analysis of such expressions. The core of the proposal is that the arithmetical meaning is the numerals' underlying semantic core from which both the quantifying and the label meaning are derived.

Keywords: numerals, cardinals, numerical expressions, labels

1 Introduction

Numerals are funny words. On the one hand, it seems they share some core component that justifies grouping them together as if they formed a single category, a practice common in both descriptive and theoretic approaches to language. On the other hand, though, morphologically and syntactically they form a very heterogeneous class of expressions with different items often exhibiting distinct properties. This is the case not only when one compares various subclasses of numerals, e.g., cardinals, ordinals and multiplicatives, but also often within the subclass of cardinals different items have adjectival, nominal, or both nominal

¹This intuition dates back at least to the Neo-Latin grammarian tradition, which distinguished between three subclasses within the category *nomen*, i.e., *nomen substantivum*, *nomen adjectivum* and *nomen numerale*.

and adjectival features while in some languages they seem to behave as verbs (Donohue 2005; Ionin & Matushansky 2018).

In generative linguistics, a lot of work has been focused on establishing the syntactic status of numerals with different approaches differing in whether they are lexical or functional categories or whether they are heads or maximal projections. These questions have been studied thoroughly with respect to Slavic numerals with their well-known idiosyncratic properties (e.g., Babby 1987; Pesetsky 1982; Franks 1994; Rutkowski 2002; Klockmann 2012).

On the other hand, since the early days of analytic philosophy and formal semantics a lot of attention has been dedicated to the meaning of numerals. At least from Frege (1884), it has been recognized that capturing the semantics of cardinals is far from trivial. Intuitively, it seems straightforward that they are linguistic expressions that somehow describe a numeric quantity. However, beyond this vague characterization it is much less obvious how to state exactly what they actually are.

In this paper, I will discuss the main formal approaches to the meaning of cardinal numerals. In doing so, I will also focus on examining two issues that only recently have attracted due attention in the semantic literature. Specifically, I will investigate different uses of cardinals as well as various derivationally complex quantifying expressions in Slavic. The two sets of data indicate that cardinals are typally flexibile and that numerals in general are semantically complex expressions. I will argue that a proper approach to the meaning of numerals should describe a compositional mechanism deriving different semantic flavors from the underlying arithmetical meaning.

The paper is outlined as follows. In §2, I will discuss various functions of cardinal numerals. §3 will provide an overview of different semantic approaches to the quantifying function of cardinals, as in *five cats*. Next, §4 will explore the relationship between the quantifying meaning and the ability of cardinals to refer to abstract arithmetical concepts. In §5, I will review the literature on the morphosemantics of complex numerical expressions in Slavic which presents evidence calling for a compositional treatment. §6 is a case study in which I will explore the so-far neglected label use of numerals. Finally, §7 concludes the paper.

2 Cardinals and their various flavors

Let us start with cardinal numerals. One of the challenges in accounting for their meaning is that they can be used in very different ways (Bultinck 2005; Geurts 2006). Consider, for instance, (1). It turns out that a simple word such as English

five is in fact quite tricky. Of course, it can be used to quantify over entities denoted by the modified NP when it appears prenominally as in (1a) but that is surely not the only function it can have. For instance, when it combines with a measure word as in (1b), it designates a portion of a substance rather than a plurality of objects. It can also appear in a predicative context such as (1c) or as part of a measure phrase like in (1d). And that is not it since it can also be used to refer to an abstract arithmetical concept, see (1e), or to label an entity as in (1f).

- (1) a. Five cats meowed.
 - b. Five liters of milk got spilled.
 - c. These are five cats.
 - d. That cat is five years old.
 - e. Five is a Fibonacci number.
 - f. Player number five twisted her ankle.

Below, I will briefly discuss how the measure, predicative and arithmetical function of cardinal numerals relate to their most deeply studied quantifying use.² A thorough investigation into properties of the label function will be undertaken in §6.

2.1 Quantifying vs. measure use

For many years, the mainstream research has been primarily focused on the quantifying use of cardinals exemplified in (1a). This seems justified since counting objects is perhaps the first thing that comes to mind when we think about expressions such as *five*. However, when a numeral appears in a measure phrase such as (1b), it seems to be involved in measuring within a certain dimension, e.g., volume, rather than in counting objects.

Intuitively, the difference between counting and measuring is that the former is about specifying how many discrete objects of a certain kind there are, whereas the latter determines some quantity in relevant measure units. Admittedly, some proposals attempt to reduce measuring to a particular type of counting based on the individuation in terms of quantity (Lyons 1977; Matushansky & Zwarts 2017). Consequently, measuring would simply be counting units determined by measure words. An opposing approach treats counting as a form of measuring, i.e., measuring a quantity of a plural individual in terms of natural or object units

²For an exhaustive discussion of numeral NPs interpreted as measure phrases in Slavic, see Matushansky & Ionin (this volume).

(Krifka 1989; 1995). Yet, there is also the third account which views counting and measuring as two independent operations (Rothstein 2017).

The contrast between the two functions becomes clear when one considers classifier constructions involving container nouns, see (2). Such expressions are ambiguous between a measure (content) and counting (container) interpretation (e.g., Landman 2004; Grimshaw 2007; Partee & Borschev 2012). On the measure reading of (2a), Tomek flushed wine to the quantity of two bottles. On the counting reading, on the other hand, he did something quite spectacular since two actual bottles containing wine went down. Moreover, derivational morphology seems to be sensitive to the distinction since the suffix *-ful* selects only for the measure sense of a container noun, compare (2b)–(2c) (Rothstein 2011).

- (2) a. Tomek flushed two bottles of wine through the toilet.
 - b. Romek added two { glasses / glassfuls } of wine to the soup.
 - c. Marek brought two { glasses / #glassfuls } of wine for our guests.

Another argument for distinguishing counting and measuring as two distinct operations comes from distinct restrictions on the domain of quantification (Wagiel 2018). Though both measuring and counting quantify over entities that need to be disjoint (Landman 2016), only the latter requires objects individuated as coherent integrated wholes. To realize the contrast, imagine someone has spilled some wine on the table in such a way that there are two separate blobs a and b whose volume is exactly one and a half milliliters each. In such a scenario, (3a) is true despite the fact that one of the three milliliters must be split between a portion of a and a portion of b. On the other hand, (3b) is simply false. This contrast shows that measuring, unlike counting, ignores individuation in terms of integrity.

- (3) Scenario: There are exactly two 1.5 ml blobs of wine on the table.
 - a. There are three milliliters of wine on the table.
 - b. There are three objects on the table.

TRUE

FALSE

Though monotonic systems of measurement track certain part-whole relations (Schwarzschild 2002), they do not seem to be sensitive to the spatial arrangement of parts making up a whole. This suggests that despite sharing a common core counting and measuring are two distinct things.

2.2 Quantifying vs. predicative use

It is well known that cardinals used as quantifiers give rise to scalar implicatures. For instance, *five cats* in (4a) allows for an *at least* reading, as witnessed by the

compatibility with the *in fact* clause. Similarly, (4b) is typically interpreted in a way that Tomek must take at least three cards.

- (4) a. Five cats live in the barn; in fact, six cats live there.
 - b. Tomek must take three cards.

However, numeral NPs in predicate position systematically lack a lower-bounded interpretation (Partee 1986; Landman 2003; Geurts 2006). For instance, (5a) can only get the *exactly* reading. This is further corroborated by the infelicity of sentences such as (5b).

- (5) a. These are five cats.
 - b. # The guests are three filmmakers; in fact, they are four filmmakers.

Furthermore, it has been observed that also bare cardinals can appear as predicates in predicate position, see (6a)–(6b) (Rothstein 2017). This resembles the behavior of adjectives rather than determiners.

- (6) a. The apostles are twelve.
 - b. The planets are eight.

Similarly to numeral NPs, bare cardinals in predicate position lack an *at least* reading. Thus both (6a) and (6b) are true if there are exactly twelve apostles and exactly eight planets, respectively. Furthermore, sentences such as (7) are infelicitous.

(7) # My reasons for saying this are four; in fact they are five.

The contrasts discussed above indicate that meaning of the predicative use of cardinals differs from their semantics in the quantifying function.

2.3 Quantifying vs. arithmetical use

It is not always the case that cardinals designate a plurality or measure. In (1e), five refers to an abstract number concept rather than to a collection of entities. In this use, cardinals have different properties than in their quantifying function (Bultinck 2005; Rothstein 2017; Wagiel & Caha 2020). Specifically, number concepts can have special properties such as being prime, see (8a), and they can occur in mathematical statements such as (8b) and dedicated grammatical construction as in (8c). When compared, the dimension of comparison is based on their relative ordering, see (8d).

- (8) a. Five is prime.
 - b. Ten divided by five equals two.
 - c. Hanna can count up to five.
 - d. Five is bigger than four.

This behavior contrasts with cardinals in their quantifying function, which lack the properties mentioned above. (9a) is odd since being prime is not a property that can be attributed to a collection of things. Similarly, expressions denoting a plurality of entities are illicit in constructions calling for a numeric value such as (9b)–(9c). Finally, (9d) has different truth conditions than (8d), e.g., it would not be true if one compared five cherries with four watermelons.

- (9) a. # Five things are prime.
 - b. # Ten things divided by five things equals two things.
 - c. # Hanna can count up to five things.
 - d. # Five things are bigger than four things.

Furthermore, cardinals referring to number concepts are incompatible with numeral modifiers such as *more than* and *at least*, see (10a). Finally, the arithmetical function does not give rise to scalar implicatures and always get an *exactly* reading, as witnessed in (10b).

- (10) a. # More than five is prime.
 - b. # Two multiplied by five equals ten, if not more.

As we have seen, cardinal numerals can be used in various ways each of which has different semantic characteristics. In the next two sections, I will review major approaches to the meaning of cardinals and compare how they account for at least some aspects of the flexibility discussed above.

3 Theories of cardinals

Given the variety of uses discussed in the previous section, any quest for 'the' meaning of cardinal numerals is probably a misunderstanding. Rather, a theory of cardinals should be able to capture the relationship between meanings of cardinals in their various functions. And yet, the mainstream research has been mainly focused on the quantifying use often ignoring how it relates to other uses (with Bylinina & Nouwen's 2020 systematic inquiry being a notable exception).

3.1 Cardinals as determiners

Let us start with the earliest and most prominent formal account of the meaning of cardinal numerals, namely the standard Generalized Quantifiers (GQ) approach (Barwise & Cooper 1981; though it can be traced back via Montague to Frege). The intuition behind this analysis is that a cardinal such as English *five* is in fact a determiner similar to *some*, *most* or *all*, as suggested by its occurrence in prenominal position as in (11).

(11) { Some / Most / All / Five } cats meowed.

In the GQ theory, a determiner expresses a particular relation holding between two sets (type $\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$), i.e., a set denoted by the NP and a set denoted by the VP. For instance, in (11) *some* yields the truth value True if the intersection between the set of cats and the set of entities that meowed is non-empty, see (12a). Similarly, on the GQ analysis *five* returns True if the cardinality of the set of cats and the set of entities that meowed equals 5, see (12b).

(12) a.
$$[some] = \lambda P \lambda Q [P \cap Q \neq \emptyset]$$

b. $[five] = \lambda P \lambda Q [|P \cap Q| = 5]$

A nice thing about the GQ approach is that it aims to develop a unified semantics of all quantified DPs. Furthermore, it can be enriched with a type-shifting mechanism which allows for systematic mapping between different semantic types (Partee 1986). Consequently, the theory can account for predicative uses of numeral NPs such as (1c) by lowering the type of a general quantifier ($\langle \langle e, t \rangle, t \rangle$) to the type of a predicate ($\langle e, t \rangle$).

However, for some time it has been realized that the GQ approach is most probably not a good way of capturing what cardinals are (e.g., Krifka 1999; Landman 2003). In the next sections, I will briefly discuss problematic data and alternative approaches.

3.2 Cardinals as predicates

There are several problems with the GQ approach to cardinals. First of all, there is a well-known asymmetry between DPs with numerals and DPs with regular determiners in predicate position. For instance, DPs headed by *every* cannot be used predicatively, see (13a). Similarly, examples such as (13b) are infelicitous on a non-partitive interpretation. Given the felicity of sentences such as (1c) and others discussed in §2.2, this fact is puzzling (e.g., Landman 2003).

- (13) a. # Hanna is every filmmaker at the party.
 - b. # The guests are { most / some } filmmakers.

Furthermore, as witnessed in (14a) bare determiners also cannot be used predicatively. This contrasts with examples such as (6), in which cardinals pattern with adjectives, see (14b). Since there is no standard type-shifting rule allowing for mapping the type of determiners ($\langle \langle e, t \rangle, \langle \langle e, t \rangle, t \rangle \rangle$) onto the type of predicates ($\langle e, t \rangle$), the predicative use of bare cardinals is unaccounted for. But even if such a rule were postulated, it would still fail to explain the contrast between cardinals and determiners such *most* and *all*.

- (14) a. # My reasons for saying this are { most / all / every }.
 - b. My reasons for saying this are { serious / personal / five }.

The last problematic data point to be discussed here concerns the fact that cardinals can appear along with bona fide determiners within a single DP (e.g., Rothstein 2017). From the GQ perspective, the compatibility of *five* with the definite article in (15a) and with *every* in (15b) is unexpected and raises serious questions regarding the semantic contribution of each of the elements.

- (15) a. The five cats that I saw meowed.
 - b. Every two students got to share a hotel room.

The evidence discussed above motivated developing an alternative approach, which treats cardinals as predicates (Landman 2003; 2004). Within such a framework, cardinals get an interpretation very similar to that of intersective adjectives, specifically they express a cardinality property. For this approach to work, it is required to distinguish between two types of entities within the domain of individuals, namely atoms, i.e., singular entities, and pluralities (Link 1983). While atoms are minimal elements of a nominal denotation, pluralities are formed from atoms via a pluralization operation * which closes the predicate under sum, i.e., takes a set of atomic entities and returns that set extended with all the pluralities that can be formed by summing the atoms. In addition, the two types of individuals are associated with each other via a part-whole relation defined in terms of mereological parthood, e.g., an atomic entity a is part of a plurality $a \oplus b$.

According to the semantics provided in (16a), *five* denotes a set of pluralities each of which contains 5 atomic entities. Similarly to intersective adjectives, cardinals combine with NPs they modify via Predicate Modification (Heim & Kratzer

³For an introduction to mereological theories of plurality, see, e.g., Champollion & Krifka (2016).

1998). Therefore, when *five* is composed with *cats*, see (16b), the resulting phrase denotes the intersection of a set of pluralities of cats and a set of plural individuals that consist of five atomic entities, i.e., a set of sums of five cats.

(16) a. [five] =
$$\lambda x [\#(x) = 5]$$

b. [five cats] = $\lambda x [*CAT(x) \land \#(x) = 5]$

The approach discussed above gives a straightforward explanation for the contrasts between cardinals and determiners such *every* and *most* in predicate position as well as the well-formedness and felicity of constructions such as those in (15). However, it faces some challenges as well.

3.3 Cardinals as predicate modifiers

A problem with treating cardinals as cardinal properties is that cross-linguistically the use of bare cardinals in predicate position is very restricted (Ionin & Matushansky 2018: 33–34). For instance, in Russian (17) is ungrammatical. Similarly, Dutch (18) can only mean that the children are two years old and not that they are two in number. And even in English the predicative use of bare cardinals is heavily restrained, as witnessed by the fact that examples such as (19) are highly degraded. If cardinals are assigned type $\langle e, t \rangle$, then this is rather startling.

- (17) * Deti byli { dva / dvoe }.

 children were two two.gndr

 Intended: 'The children were two in number.' (Russian)
- (18) # De kinderen zijn twee.

 the children are two
 Intended: 'The children are two in number.' (Dutch)
- (19) ?? The chairs in this room are twelve.

Another issue concerns the fact that what counts as 'one' seems to be highly dependent on the meaning of the NP the cardinal combines with. Arguably, atoms should be defined relative to a particular property rather than in absolute terms. To see why, let us consider partitive constructions such as those in (20) (see Chierchia 2010; Wagiel 2018). Example (20a) is weird since the numeral NP designates triples of body parts and cannot be understood as referring to three pluralities of boys. Yet, in (20b) quantification over subgroups of boys is possible. Importantly, neither parts of the boys nor parts of the group denote entities that are atomic in

any absolute sense. The first is true of portions of matter whose parts are also parts of the boys. Similarly, the latter designates subgroups which themselves can also consist of subgroups of boys. This suggests that the cardinal determines what counts as 'one' relative to the denotation of the modified NP (and possibly some contextual hints, see Rothstein 2010) and it is not obvious how this fact could be captured if cardinals were interpreted simply as intersective predicates.

- (20) a. # Three parts of the boys were sleeping.
 - b. Three parts of the group were sleeping.

A prominent alternative to treating cardinals as cardinal properties is to analyze them as predicate modifiers (type $\langle \langle e,t \rangle, \langle e,t \rangle \rangle$) equipped with a pluralization operation * and a measure function #(P) (Krifka 1989; 1995). In (21a), * closes the predicate under sum, i.e., takes a set of atomic entities and returns that set extended with all the pluralities one can form by summing atoms. On the other hand, the operation # returns for a property P a measure function that yields pluralities of five individuals having that property. In other words, *five* maps pluralities of entities onto the natural number 5.⁴ When the cardinal combines with an NP, after the predicate slot is saturated we obtain a set of relevant plural individuals (type $\langle e,t \rangle$). For instance, *five cats* denotes a set of pluralities of cats each of which consists of five cats, see (21b). Thus, what counts as 'one' in this system is relativized to the denotation of the NP.

(21) a. [five] =
$$\lambda P_{\langle e, t \rangle} \lambda x_e [^*P(x) \land \#(P)(x) = 5]$$

b. [five cats] = $\lambda x_e [^*CAT(x) \land \#(CAT)(x) = 5]$

Another possibility is to interpret cardinals as predicate modifiers providing the cardinality of a partition of a plural individual (Ionin & Matushansky 2006; 2018). A partition of a plurality x is a cover of x, i.e., a set of entities such that the sum of all those entities forms x; in addition, it is a cover whose cells do not overlap, i.e., do not share any parts (see Gillon 1987; Schwarzschild 1996). In (22a), S is a partition Π of an individual x and the cardinality of S equals 5. The cardinal combines with the NP via standard Function Application and the result is of type $\langle e, t \rangle$. For instance, *five cats* gets the meaning in (22b), i.e., it denotes a set of pluralities divisible into 5 non-overlapping entities each of which has a property

⁴This is a slight simplification since in fact Krifka (1989) postulates a special operation NU for measuring pluralities in terms of 'natural units' they consist of, whereas Krifka (1995) proposes ou for measuring the number of 'object units' realizing a particular kind.

of being a cat.5

(22) a. [five] =
$$\lambda P_{\langle e, t \rangle} \lambda x_e \exists S_{\langle e, t \rangle} [\Pi(S)(x) \land |S| = 5 \land \forall s \in S[P(s)]]$$

b. [five cats] = $\lambda x_e \exists S_{\langle e, t \rangle} [\Pi(S)(x) \land |S| = 5 \land \forall s \in S[[\text{cat}](s)]]$

Notice, however, that both approaches discussed above require the denotation of NPs the cardinal combines with to be singular, i.e., to consist only of atoms. It is postulated that the source of plurality is the numeral and the plural marking on the noun is, e.g., due to agreement with no semantic interpretation. Supporting evidence comes from expressions such as those in (23) where the plural is not associated with a plurality (Krifka 1989; Bylinina & Nouwen 2018) as well as from languages such as Finnish and Turkish which display the singular/plural distinction but in which unlike in, say, English cardinals systematically require singular NPs, see (24).

- (23) a. zero students b. 1.0 students
- (24) üç { çocuk / *çocuklar } three child children 'three children' (Turkish)

Finally, analyzing cardinals as predicate modifiers makes it easier to account for restrictions on bare cardinals in predicate position, see (18)–(19), by appealing to the well-described peculiarities of the copula which would allow for a type-shift only under particular circumstances (Ionin & Matushansky 2018: 33–34).

3.4 Cardinals as degree quantifiers

The last approach to be discussed in this sections builds on two observations regarding the behavior of cardinals. The first observation is that sentences with existential modals such as those in (25) are ambiguous (Kennedy 2013; 2015). On the strong reading, (25a) means that Hanna is allowed to eat five cookies and she is not allowed to eat more. The strong reading is probably the most natural interpretation of (25a). There is, however, also a weak reading which merely states that eating five cookies by Hanna is allowed without saying anything about eating other quantities of cookies. This interpretation becomes more prominent in questions, see (25b).

⁵The system is devised this way in order to provide a unified analysis of both simple and complex numerals such as *five hundred*. For a more detailed discussion of the approach, see Matushansky & Ionin (this volume).

- (25) a. Hanna is allowed to eat five cookies.
 - b. Is Hanna allowed to eat five cookies?

The second observation is that numerals can take scope independently of the rest of the NP. The evidence comes from decimals in sentences such as (26a) (Kennedy & Stanley 2009). Crucially, it does not entail the existence of families with 2.1 cats. This contrasts with (26b) which is infelicitous due to such an entailment.

- (26) a. The average Polish family has 2.1 cats.
 - b. # A normal Polish family has 2.1 cats.

The data discussed above suggest that cardinal numerals are in fact quantifiers. We have already seen that the analysis of *five* as a determiner is most probably incorrect and obviously treating it as a generalized quantifier over individuals (type $\langle \langle e, t \rangle, t \rangle$) would not make much sense. However, an interesting idea is to interpret cardinals as quantifiers over degrees, i.e., expressions of type $\langle \langle d, t \rangle, t \rangle$ (Kennedy 2013; 2015). Degrees (type *d*) are objects similar to individuals (type *e*) with the crucial difference that the domain of degrees D_d , unlike the domain of individuals D_e , is ordered.⁶ Hence, on the degree quantifier analysis *five* denotes a set of degree properties whose maximal value equals 5, see (27a). A sentence with a numeral NP would be then interpreted as in (27b), e.g., the maximal number of entities that Hanna had and that are cats is 5.

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(27) a. [five] = \lambda D_{\langle d, t \rangle}[\text{MAX}(D) = 5]
b. [Hanna had five cats] = = \text{MAX}\{n \mid \exists x[\text{HAD}(x)(\text{HANNA}) \land \text{*CAT}(x) \land \#(x) = n)\} = 5
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The analysis of cardinals as degree quantifiers provides a promising perspective to explain interactions between modals and both unmodified and modified cardinals such as *more than five* and *at least five* (Nouwen 2010). In the next section, I will discuss the arithmetical function of cardinals and how it relates to the theories described so far.

4 Relating cardinalities and number concepts

The theories of cardinal numerals discussed so far focus on the quantifying function. However, as we have already seen in (1e) and §2.3, cardinals can also be used to refer to abstract numeric values.

⁶For more discussion on degrees and an introduction to degree semantics, see, e.g., Morzycki (2016: Ch. 3).

4.1 Cardinals as names of numbers

On the arithmetical function, cardinals seem to behave as proper names of number concepts (type n) or, alternatively, degrees (type d). From this perspective, the arithmetical meaning of *five* is simply the number 5, see (28a). Expressions such as *prime* would then denote properties of numbers (type $\langle n, t \rangle$). For instance the extension of (28b) would be the set $\{2, 3, 5, 7, 11, 13, \ldots\}$. On the other hand, expressions used to formulate mathematical statements such as *plus*, *times*, *divided by* and *equals* seem to describe relations between number concepts. For instance, (28c) is interpreted in terms of addition of two numeric values.

(28) a. [five] = 5
b. [prime] =
$$\lambda n_n[PRIME(n)]$$

c. [plus] = $\lambda n_n \lambda m_n[n+m]$

The key question is what the relationship between names of number concepts, i.e., the arithmetical use, and meanings of cardinals used in the quantifying function is. In the next sections, I will discuss two possible approaches to how to relate the two meanings.

4.2 Deriving number concepts from cardinalities

The mainstream view is to take the quantifying meaning as basic, be it the predicate, predicate modifier or degree quantifier analysis, and to derive the arithmetical meaning from it. Let us discuss three variants of that view.

As we have already seen in §3.2, on the predicate analysis cardinals are interpreted as denoting a cardinal property, see (29a). Rothstein (2017) assumes this semantics to be basic.⁸ The arithmetical meaning is then derived via a special type-shifting operation $^{\cap}$ which when applied to a cardinal property yields a proper name, see (29b).

(29) a.
$$[\text{five}]_{\langle e,t\rangle} = \lambda x_e [\#(x) = 5]$$

b. $[\text{five}]_n = {}^{\cap} [\text{five}]_{\langle e,t\rangle} = 5$

⁷Throughout the paper, I will assume there is no difference between the two notions and following the widespread convention in the literature on the arithmetical use of cardinals I will simply use the label n.

⁸In fact, this is a slight simplification since the theory distinguishes typally between lower numerals and 'lexical powers', i.e., multiplicands such as *hundred* in *five hundred*.

The proposal is seemingly motivated by an analogy with adjectival and nominal uses of expressions such as *white*. It builds on Chierchia's (1985) theory of predication according to which every property has an entity correlate of type π derived by $^{\cap}$. For instance, the predicate *white* can be turned into the name of the property of being white. Similarly, $^{\cap}$ turns *five* into the name of the property of being five in number.

However, as argued convincingly by Ionin & Matushansky (2018: 31–33) this analogy does not hold since arithmetical environments such as those discussed in §2.3 require the number 5 itself as an argument rather than the property of being five in number. Consequently, in order to derive the arithmetical function Ionin & Matushansky (2018: 26–27) propose a null suffix Nomnum which is a nominalizer that when applied to a predicate modifier, turns it into a numeric singular term. More specifically, for any cardinal numeral it yields the corresponding cardinality. An argument for such an account comes from the fact that cardinal numerals as names of numbers display full morpho-syntactic uniformity suggesting they are derived expressions. On the other hand, cardinals in the quantifying function often show variation with respect to ϕ -features, e.g., grammatical gender.

Another mode of relating the arithmetical and quantifying meaning is proposed by Kennedy (2015). The idea is to derive the first from the latter by the standard type-shifting operations BE and IOTA defined in (30a) and (30b), respectively (Partee 1986). BE takes a quantifier and returns a property. On the other hand, IOTA yields the unique entity of which the relevant property is true.

(30) a. BE =
$$\lambda \mathcal{P}_{\langle \langle e, t, \rangle, t \rangle} \lambda x_e [\mathcal{P}(\lambda y_e[y=x])]$$

b. IOTA = $\lambda P_{\langle e, t \rangle} \iota x[P(x)]$

Generalizing the type-shifts defined above so that they can also apply to quantifiers over degrees and properties of degrees (or numbers), respectively, allows to derive the arithmetical meaning by applying BE and IOTA consecutively to the degree quantifier semantics. Specifically, when BE applies to a set of degree properties whose maximal value equals 5, see (27a), the result in (31a) is the singleton set {5}. The subsequent application of IOTA yields the number 5, as shown in (31b).

(31) a.
$$BE([five]) = \lambda n_n [n = 5]$$

b. $IOTA(BE([five])) = IOTA(\lambda n_n [n = 5]) = 5$

A nice feature of such an approach is that it allows to relate the quantifying and arithmetical functions using standard independently motivated type-shifting machinery. In the next section, I will discuss accounts that derive cardinalities from the arithmetical meaning.

4.3 Deriving cardinalities from number concepts

Though the view that the quantifying function of cardinals is the basic one seems to have become mainstream, there are a number of approaches that build on a contrary intuition, namely that underlyingly cardinal numerals are simply names of number concepts (e.g., Scha 1981; Krifka 1989; Scontras 2014; Wagiel 2018). According to that alternative view, cardinals in prenominal position are in fact syntactically and semantically complex expressions derived from the arithmetical meaning via various shifts.

An early theory of cardinals postulating that they are derived from number concepts was developed by Scha (1981). This system distinguishes between numbers, numerals and numerical determiners. The first simply denote number concepts. On the other hand, the NUMERAL head shifts an integer (type n) to a cardinal predicate while the numerical D head transforms such a cardinal predicate into a determiner with a GQ-style semantics, see (32).

(32)
$$\left[D_{\langle \langle e,t \rangle, \langle \langle e,t \rangle, t \rangle} \right] \left[NUMERAL_{\langle e,t \rangle} \left[NUMBER_n \ five \right] \right]$$

A related idea is the core of the system developed by Hackl (2000) who makes use of a covert quantificational operator MANY in order to turn a number concept into a determiner, see (33a). On this approach, the cardinal on its quantifying use is accompanied with an additional null element that ensures the shift, as depicted in (33b). The result combines with the denotation of the NP to form a generalized quantifier.

(33) a.
$$[\![\mathsf{MANY}]\!] = \lambda n_n \lambda P_{\langle e, t \rangle} \lambda Q_{\langle e, t \rangle} \exists x_e [\#(x) = n \land P(x) \land Q(x)]$$

b. $[\![\mathsf{five\ cats}]\!] = [\![\![\ [\ 5\ \mathsf{MANY}\]\ \mathsf{cats}\]\!]]$

Another proposal on how to derive the quantifying meaning from a name of a number concept dates back to Krifka (1995). On this view, all cardinals are assumed to underlyingly designate number concepts but they also involve an additional classifier element that depending on a language can be either overt or covert. There are several variants of this approach but what they all share is that such a classifier element takes an integer and returns a counting device equipped with an appropriate measure function, see (34a) (Scontras 2014; Wagiel 2018). For instance, in the derivation of the phrase *five cats* the name of a number concept

⁹The exact denotation of the numerical determiner depends on whether the whole sentence gets a collective, distributive or cover reading. In other words, there are three distinct Ds each of which deriving a different interpretation.

is first turned into a quantifying predicate modifier which then combines with the noun to yield the meaning in (34b).

(34) a.
$$[\![cL]\!] = \lambda n_n \lambda P_{\langle e, t \rangle} \lambda x_e [\![*P(x) \land \#(P)(x) = n]\!]$$

b. $[\![five cats]\!] = [\![[\![5 CL]\!] cats]\!] = \lambda x_e [\![*CAT(x) \land \#(x) = 5]\!]$

In yet another variant of the discussed approach, the classifier semantics turns an arithmetical concept into a predicate (Sudo 2016). The classifier takes a numeric value and yields a cardinal property (type $\langle e,t\rangle$) rather than a predicate modifier. In addition, it can introduce a special presupposition concerning the nature of referents of the modified noun. The resulting expression then combines with the NP via Predicate Modification.

From the theoretical perspective both types of the relationship between the arithmetical and quantifying function, i.e., the derivation from the quantifying meaning to the arithmetical meaning and vice versa, seem plausible. Thus, the question which one is correct appears to be an empirical issue. In §4.2, we have already seen that certain aspects of cardinals' morphology have been argued to support the quantifying-function-is-basic view. In the next section, I will discuss a different type of evidence that can be useful in determining the direction of the derivation.

4.4 Morphological evidence

It has been observed that many languages distinguish formally between so-called counting, i.e., arithmetical, and attributive, i.e., quantifying, numerals (Hurford 1998; 2001). For instance, in Mandarin the form $\grave{e}r$ 'two' is used in arithmetical environments whereas $li\check{a}ng$ 'two' is an attributive cardinal appearing in the quantifying use. Similarly, the Maltese numeral tnejn 'two' indicates the arithmetical function while $\grave{z}ew\grave{g}$ 'two' has the quantifying meaning. Though the distinction in question is typically restricted to a subset of numerals in a language and there are a number of patterns of how the two forms can differ, it is a relatively widespread phenomenon across languages.

Interestingly, the two families of theories discussed above make different predictions regarding the morphological make-up of cardinal numerals cross-linguistically. On the assumption that morphology expresses meaning, if the arithmetical function is derived from the quantifying function, see §4.2, then we would expect that across languages a pattern with arithmetical numerals being morphologically more complex than quantifying cardinals should be relatively widespread. This is because an additional affix is expected to introduce a shift

from a counting device to a number concept. On the other hand, if the quantifying function is derived from the arithmetical one, see §4.3, then we would expect an inverse pattern to be common. Namely, quantifying cardinals should include an additional morpheme encoding a shift from number concepts to quantifying modifiers.

It turns out that the cross-linguistically widespread asymmetry attested, e.g., in many obligatory classifier languages, is of the latter type (Wagiel to appear; Wagiel & Caha 2020). For instance, bare cardinals in Japanese cannot modify NPs and require an additional element, traditionally referred to as a classifier, see (35a). However, such an element is illicit in an arithmetical environment like (35b) despite the fact that *ko* being a general classifier could be used to indicate any type of inanimate entity.

```
(35) a. {*go-no / go-ko-no} ringo
five-gen five-clf-gen apple
'five apples'
b. juu waru {go-wa / #go-ko-wa} ni-da.
ten divided five-top five-clf-top two-cop
'Ten divided by five equals two.' (Japanese)
```

Therefore, it seems that the approaches that derive the arithmetical semantics from the quantifying one make incorrect predictions with respect to meaning/form correspondences in cardinals cross-linguistically. However, there is a twist here since a pattern with more complex arithmetical numerals is also attested, though scarce. For instance, in German the arithmetical form for 1 consists of an additional morpheme compared to its quantifying equivalent, see (36).

```
(36) a. { ein / *ein-s } Mädchen
one one-NBR girl
'one girl'
b. Zehn geteilt durch { *ein / ein-s } ist gleich zehn.
ten divided by one one-NBR is equal ten
'Ten divided by one equals ten.' (German)
```

The pattern attested in German is puzzling and seemingly indicates that both types of relationship between the two functions in question are possible. Never-

¹⁰Typically, classifiers also introduce certain restrictions on the referents of the NP modified by the cardinal, e.g., being a round object, a flat object, a plant, a big animal etc. This aspect of their meaning can be captured by accommodating various presuppositions concerning the nature of the denoted individuals (Sudo 2016).

theless, it can be accounted for by postulating a more complex morpho-semantic structure of numerals along with a spell-out mechanism based on late insertion (Wagiel & Caha 2020). On this analysis, though the arithmetical meaning is more basic, it is still derived from an even more primitive concept of a number scale. The quantifying meaning is then obtained by turning a number concept into a quantificational modifier. The pattern in (36) can be then explained by postulating that *ein* denotes the concept of a number scale, the suffix -s forges the name of the numerical value and that there is yet another phonologically null morpheme -Ø that shifts the number scale into a counting device. Crucially, not only does such an account capture cross-linguistic variation but also it explains why the German pattern is rare. On the other hand, no alternative explanation of the typological facts that would build on the quantifying meaning as the primitive component has been proposed so far.

To conclude, the patterns discussed above suggest that the arithmetical meaning of cardinals is the basic one and that other uses can be derived from it. In the next section, I will discuss complex numerical expressions in Slavic and suggest how this type of evidence can provide further hints on which of the two ways of relating number concepts and cardinalities discussed so far is on the right track.

5 Complex numerical expressions

While the mainstream semantic research focuses mostly on cardinal numerals, there is a growing body of formal literature acknowledging that cardinals are in fact only a subset of various numerical expressions. This fact has been well known in Slavic descriptive traditions. Since Slavic languages have a rich morphology, it is not surprising that one can find abundant formal complexity also in numerals. In this section, I will briefly review recent developments in the study of different types of derivationally and semantically complex numerical expressions. Though the focus of the section is on Slavic data, some parallels with other languages will be drawn.

5.1 Ordinals

The cross-linguistically most widespread type of derived numerical expressions are ordinal numerals, i.e., forms such as English *fifth* and Russian *pjatyj* 'fifth'. Intuitively, ordinals represent position or rank in a sequential order. If a language distinguishes morphologically a class of ordinals, they are always derived by an

additional affix (Stolz & Veselinova 2013).11

Perhaps somewhat surprisingly, the semantics of ordinals has been typically linked to superlatives. Cross-linguistically, the two types of expressions often share identical or related morphemes (Hurford 1987; Veselinova 1998)¹² and have the same syntax (Barbiers 2007). Furthermore, the acquistion of ordinals seems to be influenced by superlative morphology (Meyer et al. 2018). Even more importantly, ordinals, just like superlatives, give rise to an ambiguity between an absolute and comparative interpretation (Bhatt 2006; Sharvit 2010). On the absolute reading of (37a), John gave Mary a telescope that is older than all other telescopes. On a comparative reading, however, he might have given her a relatively new telescope than happens to be older than the telescopes other people gave her, i.e., John's gift is compared to other gifts rather that to telescopes in general. Similarly, (37b) can mean either that Mary got a telescope that was older than other telescopes, or that John's telescope was the first Mary has received.

- (37) a. John gave Mary the oldest telescope.
 - b. John gave Mary the first telescope.

However, it has been observed that ordinals, unlike superlatives, do not give rise to so-called upstairs *de dicto* readings in sentences with intensional operators such as *want* (Bylinina et al. 2015). For instance, (38a) can have a particular comparative interpretation which makes it true in the scenario below. Yet, that reading is unavailable for (38b).

- (38) SCENARIO: There are many trains throughout the day. John wants to take a train. Any of the trains between 3 pm and 4 pm is fine for him. Similarly, Bill and Steve want to take a train, and they are fine as far as the departure time is between 5 pm and 6 pm and between 7 pm and 8 pm, respectively. These people do not know anything about one another.
 - a. John wants to take the earliest train.

TRUE

b. John wants to take the first train.

FALSE

As a result, Bylinina et al. (2015) argue that superlatives and ordinals differ with respect to where they are interpreted. According to that proposal, while superlatives involve movement, ordinals are always interpreted *in situ*.

 $^{^{11}}$ Unless an ordinal is suppletive, e.g., one \sim first in English.

¹²Notice that this is also the case in some Slavic languages. For instance, Polish *pierw-szy* and Ukrainian *per-šyj*, both 'first', contain the comparative markers *-szy* and *-šyj* which also appear in superlatives, e.g., *naj-star-szy* and *naj-star-šyj*, both 'the oldest', respectively.

5.2 Collective numerals

One of the key topics discussed in the literature on plurality regards distributive and collective interpretations of sentences including plural DPs such as (39) (e.g., Scha 1981; Link 1983; Landman 1989; 2000; Schwarzschild 1996). For instance, (39b) is ambiguous between a reading on which a total of three letters have been written, i.e., a distributive interpretation, and a reading on which only one letter has been written, i.e., a collective interpretation. The source of the ambiguity has often been located inside the VP (e.g., Hoeksema 1983; Link 1984; Schwarzschild 1991; see also Lasersohn 1995: Ch. 7 for an overview).

- (39) a. The boys wrote a letter.
 - b. Three boys wrote a letter.

Interestingly, Slavic languages have special numerical expressions which force a collective interpretation of the whole sentence. For instance, consider the Czech sentences in (40) (Dočekal 2012; 2013).¹³ While (40a) patterns with (39b) in that it is ambiguous between a distributive and a collective reading, the sentence in (40b) rules out the distributive interpretation, and thus the sole possible reading is that only one letter has been written.

- (40) a. Tři chlapci napsali dopis.
 ✓ COLL, ✓ DISTR three boys wrote letter.ACC
 'Three boys wrote a letter.'
 - b. Trojice chlapců napsala dopis. ✓ COLL, # DISTR three.COLL boys.GEN wrote letter.ACC
 'A group of three boys wrote a letter.' (Czech)

Since the difference between the two sentences boils down to the alternation between the basic cardinal numeral *tři* 'three' and the morphologically complex derived form *trojice* 'group of three', the suffix *-ice* has been proposed to be a morphological reflex of a semantic operation forcing a collective interpretation, specifically the group-forming operator ↑ proposed by Landman (1989).¹⁴

Importantly, collective numerals cannot be used to refer to number concepts, as witnessed in (41). This fact is surprising if the arithmetic meaning were derived

¹³In the literature, expressions such as *trojice* 'group of three' have been referred to as collective or group numerals as well as denumeral group nouns (due to their nominal-like behavior).

¹⁴The affix -oj- appears only in morphologically complex numerical expressions derived from the roots for 2 and 3 and is usually assumed to have a purely structural function, i.e., it marks non-cardinal stems.

(Czech)

from the quantifying one, see §4.2. Despite the collective meaning component described above, NPs involving collective numerals denote pluralities and one would expect the same shift that is posited to turn the quantificational meaning of a cardinal into the name of a number concept should be applicable also in this case, contrary to facts.

- (41) a. # Trojice je prvočíslo. three.COLL is prime.number Intended: 'Three is prime.'
 - b. #Šestice děleno trojicí je dvojice. six.coll divided three.coll.ins is two.coll Intended: 'Six divided by three is two.'

Collective numerals have been identified also in other Slavic languages. In particular, Polish *trójka* 'group of three' patterns with (40b) (Wagiel 2015) and similar examples include Slovak *trojica*, Bosnian/Croatian/Montenegrin/Serbian (BCMS) *trojka* and Russian *trojka*, all 'group of three'.¹⁵ The existence of the discussed phenomenon in Slavic is unexpected since it is typically postulated to hold crosslinguistically that if a language has a scopal quantifier, it is a quantifier forcing a distributive interpretation (Gil 1992).¹⁶ Hence, the Slavic data discussed above compel to revise that generalization. In any case, a systematic theory-driven cross-linguistic research of collective numerals has not been carried out so far.

5.3 Both

An interesting feature of many languages including Slavic is that alongside a regular cardinal numeral corresponding to English *two* there is also another form corresponding to English *both*. In an early GQ account, *both* has been analyzed as a quantificational determiner with the same meaning as *the two* (Barwise & Cooper 1981). However, it has been observed that the two expressions are not equivalent (Ladusaw 1982). For instance, *both* is incompatible with collective predicates, see (42a), and cannot appear inside partitives, see (42b). As witnessed in (43), the same behavior is also attested in Slavic (Łazorczyk & Pancheva 2009).

(42) a. { The two / #Both } students are a happy couple.

¹⁵Russian shows a strong tendency to lexicalize collective numerals, e.g., *trojka* is also a word for a carriage drawn by a team of three horses abreast.

¹⁶In Hebrew, there are derived expressions that at first blush resemble Slavic collective numerals, e.g., *šlišiya* 'threesome of'. However, they have been reported to differ in that they do not force a collective interpretation (Gil 1992).

- b. One of { the two / #both } children sneezed.
- (43) a. # Obe ženščiny prišli vmeste. both woman.GEN came together Intended: 'Both women came together.'
 - Kto iz etix { dvoix / *oboix } sdelal to, čto who from these.GEN two.GEN both.GEN did this.ACC what xotel otec?
 wanted father
 'Which of these two did what his father wanted?'

The data presented above led to proposals postulating that *both*, unlike cardinals, involves an additional distributive component (e.g., Ladusaw 1982; Landman 1989).¹⁷ Interestingly, however, there is yet another respect in which the two differ. Specifically, *both* and its counterparts in other languages cannot be used to refer to a number concept in an arithmetical environment, see (44).

- (44) a. # Oba to liczba parzysta.
 both that number even
 Intended: 'Both is an even number.'
 - b. # Dziesięć dzielone przez oba równa się pięć.
 ten divided by both.ACC equals REFL five
 Intended: 'Ten divided by both equals five.' (Polish)

Similarly to collective numerals, the behavior of *both* and its Slavic equivalents presented in (44) poses a serious challenge for approaches deriving the arithmetical function of numerals from the allegedly basic quantifying meaning.

5.4 Taxonomic numerals

A unique property of some Slavic languages is that they have taxonomic numerals. Likewise collective numerals, they are derived by a special suffix which triggers a non-trivial semantic effect. Unlike cardinals, such expressions do not quantify over object-level entities, i.e., tokens of a type, but rather over subkinds of a particular kind corresponding to the meaning of the modified noun.¹⁸ For instance, let us consider the taxonomic numeral *dvoji* 'two kinds of' in Czech

 $^{^{17}}$ For the recent research on bona fide distributive numerals, see, e.g., Gil (2013), Cable (2014) and Wohlmuth (2019).

¹⁸For an introduction to the kind/object distinction, see, e.g., Krifka et al. (1995).

(Dočekal 2012; 2013; Grimm & Dočekal to appear). The phrase in (45a) does not refer to two musical instruments but rather to two types of violin, e.g., a set of classical violins and a set of electric violins. Likewise, (45b) denotes two kinds of the beverage, e.g., white and red wine, irrespective of its amount.

(45) a. dvojí housle
two.tax violin
'two kinds of violin'
b. dvojí víno
two.tax wine
'two kinds of wine'

(Czech)

Unlike cardinals, taxonomic numerals specify the domain of quantification to be within the realm of kinds exclusively. Therefore, the suffix -i has been proposed to introduce a quantificational operation dedicated to counting subkinds, specifically the κu (for 'kind unit') operation proposed by Krifka (1995).

Though taxonomic numerals seem to be best preserved in Czech, Polish *dwojaki* and *dwoisty*, Bulgarian *dvojak* and Russian *dvojakij*, all 'twofold', are arguably expressions of a similar type. However, there appear to be certain differences with respect to their distribution and behavior that have not been described sufficiently so far. Crucially, none of the taxonomic numerals can be used to refer to an abstract number concept.

5.5 Aggregate numerals

Another intriguing type of Slavic derivationally complex numerical expressions is sometimes referred to as aggregate numerals, e.g., forms such as Czech *dvoje* 'two collections' (Dočekal 2012; 2013) as well as BCMS *dvoji* (Lučić 2015). Likewise collective numerals, they are derived by a special suffix which triggers a non-trivial semantic effect and lack the arithmetical meaning. While cardinals simply count atomic individuals in the denotation of the modified NP, aggregate numerals are used to quantify over collections of entities that typically form a particular spatial configuration. For instance, (46a) refers to two aggregates of cards, e.g., two decks of cards. Similarly, (46b) denotes two pairs of shoes.

(46) a. dvoje karty
two.AGGR cards
'two sets of cards'

b. dvoje boty
two.AGGR shoes
'two pairs of shoes' (Czech)

Aggregate numerals are not limited to Slavic. For instance, similar expressions seem to have existed in Latin, e.g., $tr\bar{e}s$ 'three' $\sim tr\bar{i}n\bar{i}$ 'three collections' (Ojeda 1997), and Estonian uses pluralized numerals for this purpose, e.g., kaks 'two' $\sim kahed$ 'two collections' (Norris 2018). However, it is the Slavic data what has been taken as evidence for the relevance of mereotopology in natural language, specifically the significance of topological relations holding between atomic members of a plurality within a part-whole structure. ¹⁹ In particular, the suffix -e in phrases such as (46) is sensitive to how parts making up a whole are arranged, and thus it has been proposed to introduce a semantic operation dedicated to counting clusters, i.e., topologically structured spatial groupings of entities (Grimm & Dočekal to appear).

5.6 Gender-sensitive numerals

All Slavic languages mark grammatical gender on low numerals whereas some of them, e.g., Bulgarian, Polish and Slovak, distinguish between virile (personal masculine) and non-virile forms also in higher numerals. (Cinque & Krapova 2007; Pancheva 2018; Wagiel to appear). However, what is probably even more interesting is the existence of complex expressions which I will refer here to as gender-sensitive numerals.²⁰ Such expressions introduce non-trivial restrictions with respect to the natural gender of individuals making up a plurality.

Let us consider Polish gender-sensitive numerals such as *troje* 'three (at least one male and at least one female)' (Wagiel 2014; 2015). Unlike virile and non-virile cardinals in (47a) and (47b), respectively, they do not show grammatical gender agreement with modified NPs. Instead, they are neuter, as demonstrated in (47c). And yet they introduce a gender-related effect. While (47a) denotes a plurality of either males or students whose natural gender is not known to the speaker and (47b) refers to three females, (47a) is interpreted as designating a mixed-gender group.

(47) a. tych trzech studentów these.ACC.v two.v students.GEN.v 'these three students (male or indeterminate)'

¹⁹For an introduction to mereotopology, see Grimm (2012: Ch. 4), Wagiel (2018: Ch. 6).

²⁰In traditional Slavic linguistics such forms are typically termed 'collective numerals'. In order to avoid confusion with expressions discussed in §5.2, I will not use that term.

- b. te trzy studentki
 these.ACC.NV three.NV students.NOM.NV
 'these three female students'
- c. to troje studentów
 this.NOM.N three.GNDR.N students.GEN.V
 'these three students (at least one male and at least one female)'
 (Polish)

Based on the evidence discussed above, numerals such as Polish *troje* have been proposed to introduce a presupposition that a plurality of individuals referred to by the whole phrase is heterogeneous and consists of both male and female individuals. Though similar expressions seem to be only attested in BCMS, e.g., *dvoje*, 'two (one male and one female)', there are also other types of gender-sensitive numerals in Slavic. For instance, BCMS *dvojica* and Russian *dvoe*, both 'two (male)', presuppose a homogeneous group consisting of male individuals only (Kim 2009; Lučić 2015; Khrizman 2020). Interestingly, none of the gender-sensitive numerals can be used to refer to a number concept.

5.7 Indefinite numerals

Another intriguing class of numerical expressions concerns indefinite numerals such as English *several* (Kayne 2007). Similar quantifiers appear in all branches of Slavic, e.g., compare Czech *několik*, Russian *neskol'ko*, Bulgarian *nyakolko* and BCMS *nekoliko*, all 'several'. Particularly interesting data come from Polish which distinguishes between two different expressions of this type, specifically *kilka* 'several, a few' and *ileś* (*tam*) 'some, some amount'. While the former is only compatible with count NPs and designates a cardinality between 3 and 9, the latter also combines with mass NPs and can indicate any quantity (Wagiel 2020b).

In terms of morphosyntax and many aspects of semantics, *several* and its Slavic equivalents pattern with cardinals. However, one crucial respect in which the two differ is that indefinite numerals do not fit contexts calling for number concepts, see (48a) and (49) (Schwarzschild 2002; Wagiel 2020b). Despite the fact that one can easily imagine the intended meaning and paraphrase it using the existential quantifier, as in (48b), this is not what one gets in (48a) and (49).

- (48) a. # Four plus several is less than ten.
 - b. There is a number n such that four plus n is less than ten.

(49) # Cztery plus { kilka / ileś (tam) } to mniej niż dziesięć. four plus several some this less than ten Intended: 'Four plus several/some is less than ten.' (Polish)

Polish indefinite numerals have been analyzed as complex expressions with an incorporated measure function and choice function. The difference between *kilka* and *ileś* (*tam*) is due to a different type of set the choice function selects a number from and a different kind of built-in measure function (Wagiel 2020b).

5.8 Multipliers

In many languages there is a heavily understudied class of numerical expressions sometimes referred to as multipliers, e.g., English *double*. Though Germanic and Romance languages have borrowed their multipliers from Latin, in Slavic they are morphologically transparent and one can easily notice that they are derived from numerical roots by various affixes, e.g., compare Polish *podwójny*, Russian *dvojnoj* and Czech *dvojitý*, all 'double'.

Interestingly, multipliers display non-trivial quantificational behavior. Specifically, they do not quantify over whole entities or events, as cardinals would, but rather over parts thereof (Wagiel 2018; 2020a). For instance, the phrase in (50a) denotes a set of singular objects having a complex internal structure. Each of those objects is a crown but crucially it also has two parts that could be considered as having the property of being a crown themselves, e.g., entities such as the ancient Egyptian Pschent or the papal tiara. Similarly, (50b) refers to a murdering event with two victims. But this means that there were two parts of that event each of which could be described as a murder in its own right.

- (50) a. dvostruka kruna two.mult.f crown.n 'double crown'
 - b. dvostruko ubistvo two.MULT.N murder.N 'double murder'

(BCMS)

The behavior of multipliers has been argued to stem from the phenomenon of subatomic quantification (Wagiel 2018). According to the proposal, natural language is sensitive to the arrangement of parts within a singular entity. Certain expressions, e.g., proportional partitives and multipliers, can access such subatomic part-whole structures and quantify over parts conceptualized as contiguous objects within a whole.

5.9 Event numerals

Yet another class of cross-linguistically frequent though surprisingly understudied numerical expressions consists of event numerals also known as multiplicatives (but see Landman 2006; Donazzan 2013; Kayne 2015). They include adverbial expressions such as English *twice* and *two times* which are primarily used to quantify over events but can be also employed in comparative and equative constructions. Event numerals are attested in all Slavic languages with BCMS *dvaput* and Czech *dvakrát*, both 'two times, twice', being representative examples.

Intuitively, event numerals seem to be a type of a broader class of adverbs of quantification including frequency adverbs such English often and French souvent 'often' (Abeillé et al. 2004; Doetjes 2007). However, close examination reveals interesting differences between event numerals and frequency adverbs one of which regards degree modification (Dočekal & Wągiel 2018). For instance, the sentence with the event numeral in (51a) is ambiguous between the quantified-event reading and the quantified-degree reading. On the former interpretation there were two events of increasing the demand and the total value of the increase is unknown whereas on the latter interpretation there was a twofold increase in the demand irrespective of the number of events on which it increased. Interestingly, (51b) lacks the quantified-degree reading.

- (51) a. Poptávka po dotacích vzrostla dvakrát. ✓ EVENT, ✓ DEG demand after subsidies.Loc increased.PFV twice 'The demand for subsidies increased (by) two times.'
 - b. Poptávka po dotacích rostla často. ✓ EVENT, #DEG demand after subsidies.LOC increased.IPFV often
 'The demand for subsidies increased often.' (Czech)

In addition, while event numerals are perfectly fine in comparatives and equatives, frequency adjectives are infelicitous in such constructions, as witnessed by the contrast in (52).

(52) Petr je { dvakrát / #často } vyšší než Marie.

Petr is twice often taller than Marie

'Petr is two times taller than Marie.' (Czech)

Unlike all other types of numerical expressions discussed in this section, event numerals can be used in arithmetical environments provided that they express multiplication. However, it is puzzling that not all event numerals can do that. For instance, Polish has two sets of event numerals but only one can be felicitously used in a context such as (53) which suggests that the element *razy* 'times' might be a homonymous form for a multiplicative morpheme used to quantify over events and an expression of an arithmetical relation.

```
(53) { Dwa razy / #Dwukrotnie } trzy równa się sześć.
two times twice three equals REFL six
'Two times three equals six.' (Polish)
```

Though event numerals seem key to understanding quantification in the domains of events and degrees, it is surprising that so far they have not received nearly as much attention as cardinals.

5.10 Frequency numerals

Finally, in many languages there are expressions I will refer to as frequency numerals, i.e., multiplicative adjectives equivalent to English *two-time*. Slavic is no exception here and Polish *dwukrotny*, Czech *dvojnásobný* and Russian *dvukratnyj*, all 'two-time', are representative examples of the class in question.

At first sight, it seems that frequency numerals such as *two-time* are expressions of the same type as frequency adjectives like *occasional* and *frequent* (Zimmermann 2003; Schäfer 2007; Gehrke & McNally 2015). Notice, however, that while *occasional* can have the adverbial reading, expressions such as *two-time* pattern with *frequent* in that they lack such an interpretation. For instance, while (54a) means that occasionally a sailor strolled by, both (54b) and (54c) cannot be understood that way.

(54) a. An occasional sailor strolled by.
b. A frequent sailor strolled by.
c. A two-time senator strolled by.
#ADVERBIAL
#ADVERBIAL

Furthermore, frequency numerals fail to combine with sortal nouns and seem to require role nouns denoting socially salient capacities such as positions with a term and award recipients that can be acquired repetitively. That is because frequency numerals quantify over conventionalized events of acquiring a property, e.g., via a ceremony, compare (55a) and (55b).

(55) a. Jan to dwukrotny { mistrz / #człowiek }. Jan this two-time champion man 'Jan is a two-time champion.'

b. Jan został { mistrzem / #człowiekiem } dwa razy.Jan became champion.ins man.ins two times'Jan became a champion twice.'(Polish)

Having examined various complex numerical expressions, let us now summarize what the discussed data show us.

5.11 Summary

In this section, I reviewed various types of numerical expressions in Slavic and beyond. In particular, I discussed a wide range of morphologically complex forms and examined how they correspond to certain non-trivial semantic effects. The reviewed dataset is far larger and more diverse than typically analyzed in mainstream semantic theories of numerals, see §3. And yet, there is an intuition that a general theory of numerals should cover all of the examined cases. Therefore, I believe the relevance of the above-discussed Slavic data for such a theory is twofold.

First, morphological transparency of the discussed forms clearly shows that all complex numerical expressions share a common component, i.e., a numerical root designating a certain number. That number is somehow employed in counting but what exactly is counted may vary depending on a type of numeral, e.g., whole objects in the case of cardinals, parts in the case of multipliers and subkinds in the case of taxonomic numerals. Therefore, a theory of numerals should aim at providing a general mechanism that will allow us to capture not only the behavior of cardinals but also other numerical expressions discussed here.

Second, despite sharing an element designating a number none of the complex numerical expressions can be used to refer to abstract arithmetical concepts, i.e., the arithmetical function is never available except for basic cardinals.²² This fact turns out to be problematic for the approaches deriving the arithmetical meaning from the quantifying one, see §4.2. The reason is that they shift a counting device semantics of a cardinal into a proper name of a number concept. But if so, why cannot that shift be also applied in the case of other types of counting expressions, e.g., a taxonomic, an aggregate or an indefinite numeral, or *both*? Semantically, these expressions do not seem to be radically different from cardinals (they all count something) to justify why shifting their meaning to a name of a number concept is impossible. If, on the other hand, the arithmetical meaning is basic, one expects to derive various quantificational flavors from it. Therefore, it would

²¹Unless, of course, suppletion is involved.

²²And perhaps event numerals in contexts expressing multiplication.

not be surprising that only basic cardinals can have the arithmetical function and other types of numerical expressions would lack it.

In the next section, I will attempt to show how the two conclusions reached above can guide formulating a particular proposal of yet another type of numerical expressions, namely label numerals.

6 Case study: Numerals as labels

In this section, I will return to the label use of numerals, as already introduced in (1f) in §2, repeated here as (56).

(56) Player number five twisted her ankle.

So far, the label meaning has received little attention in the literature (see Hurford 1987: 167–168; Wiese 2003: 37–42; Bultinck 2005: 119–129). I will demonstrate that not only is this function conceptually different from other functions discussed in §2 but also that the distinction is reflected in grammar. I will argue that the label function is derived from the arithmetical meaning and I will propose a morphosemantic account that allows for deriving both the form and the meaning of dedicated label numerals in Slavic. The proposed analysis will combine standard compositional semantics with a Nanosyntax model of morphology.

6.1 Label uses are distinct

Intuitively, label numerals provide means for identification of an object via its association with a unique number, which allows for the recognition of that object among other entities. This mechanism is sometimes referred to as NOMINAL NUMBER ASSIGNMENT (Wiese 2003: 37–38). A set of numbers would be consistent labels as long as the numerals corresponding to distinct numbers in that set are uniquely used for each distinct object. This can be achieved by a one-to-one mapping between a set of labeled entities and a given set of numbers. Importantly, the actual values of the numbers represented by label numerals are irrelevant since they do not indicate quantity, order or any other type of measurement. Furthermore, the set of labels can change over time, e.g., it can grow as new entities are labeled. The only thing that matters is that at a given time every labeled entity is associated with a unique number.

It is clear that from a conceptual point of view the label function differs from the quantifying and the arithmetical meaning. Importantly, however, the distinction is also encoded in grammar. First, numerals used as labels do not allow for modification by numeral modifiers and prequantifiers, see (57a). Moreover, label constructions do not give rise to scalar implicatures. For instance, in the context of (57b), taking bus number six would not guarantee getting to the university.

- (57) a. # Tram { more than / at least / all } number five has left.
 - b. You must take tram number five to get to the university. #AT LEAST

The data in (57) differ from the quantifying function and pattern with the properties of the arithmetical use of numerals discussed in §2.3. However, these two functions can be distinguished linguistically as well. For instance, English constructions with numerals used as labels differ from other count NPs in that they are ungrammatical with the indefinite article, see (58a). On the other hand, names of number concepts cannot appear without the definite article, as demonstrated in (58b) (for a more detailed discussion on the distribution of English phrases such as *number five*, see Bultinck 2005: 124–129).

- (58) a. Kim took (*a) tram number five.
 - b. * (The) number five is odd.

Above, I argued that the label meaning of numerals is both conceptually and linguistically different from the quantifying and the arithmetical function. In the following two sections, I will discuss further evidence for the relevance of the label function as well as its relationship with the other two discussed meanings.

6.2 Evidence from co-lexicalizations

In many languages, an additional expression can be optionally used alongside the numeral in order to indicate its quantifying, arithmetical or label use. Sometimes, the same lexical item can introduce all of the functions above. For instance, English employs a single noun, namely *number*, to indicate cardinality, to designate a mathematical object and to signal that an entity is identified via association with a relevant integer, see (59).

- (59) a. The cats are five in number.
 - b. The number five is odd.
 - c. Tram number five left.

While English shows full co-lexicalization, i.e., the noun *number* can be used to indicate all three functions, more often lexicons develop a different word for at least one of the functions in question. For instance, Bulgarian exhibits a pattern

with no co-lexication, i.e., each of the discussed functions is signalled by a morphologically distinct expression, specifically *broj* is for quantification, *čislo* is for arithmetic and *nomer* is for the label function, all 'number'.

Though ways in which meanings and forms correspond to each other vary, it seems that they do not vary in an unrestricted manner. The co-lexicalization patterns found in Slavic are summarized in Table 1. Interestingly, the pattern that is not attested in the sample is a case of non-contiguous co-lexicalization where the quantifying and label function are expressed with the same form while the arithmetical function is expressed by a different lexical item. Perhaps surprisingly, this result resembles the well-known *ABA principle (Bobaljik 2012).

LANGUAGE	QUANTIFYING	ARITHMETICAL	LABEL
BCMS	A broj	A broj	A broj
Polish	A liczba	A liczba	B numer
Slovak	A počet	B číslo	B číslo
Bulgarian	A broj	B čislo	C nomer
unattested	A	В	A

Table 1: Co-lexicalization of 'number'

Examining permissible patterns of co-lexicalization is rarely considered evidence in formal semantics. However, I believe that looking at how certain parts of lexicon are structured can be instructive. Arguably, it can reveal that some meanings have more in common than others and in some cases that certain categories, though theoretically possible, are empirically unrealized.²³ Consequently, one can gain valuable hints regarding what those meanings are.

On the assumption that co-lexicalization does in fact reflect the relationship between concepts, the data in Table 1 suggest that while the quantifying and the label meaning are unrelated, they both seem to stem from the arithmetical function. The reason is that if both the quantifying and the label meaning are derived from the arithmetical meaning, the attested co-lexicalization patterns are unsurprising since the unrelated derived functions are not expected to be expressed by a single term that is different from the one expressing the underlying meaning (ABA). On the other hand, if the quantifying meaning were basic, then one would expect to find the ABA pattern instead of the Czech and Slovak ABB.

²³This way of viewing data has been successful in lexicology, e.g., in generalizations about kinship terms (see, e.g., Hage 1997).

Whether a language which has a term for the quantifying and the label function to the exclusion of the arithmetical function can be found is an empirical issue. So far, the Slavic co-lexicalization data suggest a strong relationship between the label and the arithmetical meaning and no relationship between the label and the quantifying meaning. In the next section, I will discuss a class of complex numerical expressions for labeling in Slavic.

6.3 Derived label numerals

An interesting property of Slavic languages is that they have specialized label numerals. Such dedicated expressions are derived with the suffixes *-ka* and *-ica*. Table 2 gives an overview of the label forms for the numerals forresponding to 5 across Slavic.

LANGUAGE	NUMBER	CARDINAL	LABEL
Czech	5	pět	pětka
Polish	5	pięć	piątka
Russian	5	pjat'	pjaterka
Slovenian	5	pet	petka/petica
BCMS	5	pet	petica

Table 2: Cardinal and label 'five' in Slavic

Though there are a number of interesting differences in their productivity and distribution, e.g., Polish derives label forms with -ka up to 999 while Russian has them only for 2–10, 20 and 30, all Slavic label numerals are morphosyntactically nominal expressions used to refer to objects that can be identified via nominal number assignment. Though very often they appear bare, the noun specifying what entity is labeled can optionally precede the label numeral, as in (60). In the absence of the noun this information is typically provided by the context.

(60) (Tramvaj) pět-ka vyjede na novou trať v listopadu. tram five-lbl will.go.out on new.ACC track.ACC in November.LOC 'Tram number five will head out on the new track in November.' (Czech)

Similarly to classifier constructions and other complex numerical expressions, recall §4.4 and §5, Slavic label numerals exhibit uniform behavior in that they cannot be felicitously used in mathematical statements such as (61), This, in turn, sug-

gests that their semantics is more intricate compared to the arithmetical meaning of cardinal numerals.

- (61) a. Dva-krát pět se rovná deset. two-times five REFL equals ten 'Two times five equals ten.'
 - b. # Dvoj-ka krát pět-ka se rovná desít-ka. five-lbl times two-lbl equals ten-lbl Intended: 'Two times five equals ten.'

(Czech)

The data above show that Slavic label numerals are both morphologically and semantically complex. With this in mind, let us now propose an account that will capture their morpho-semantics as well as the relationship between the arithmetical meaning, quantification and labeling.

6.4 Analysis

In order to account for the relationship between the three uses of cardinal numerals discussed in this paper, i.e., their arithmetical, quantifying and label function, I develop a morpho-semantic system combining a standard inventory of formal semantics with the Nanosyntax model of morphology (see Wagiel & Caha 2020). The core of the proposal is that the quantifying and the label function are both derived from the arithmetical meaning which is taken to be basic.

I propose the ingredients in (62) as a set of morpho-semantic components that numeral expressions conveying the relevant functions are formed of. Specifically, I postulate the bottom-most $NumP_n$ (for 'number') layer along with two additional heads, namely CL (for 'classifier') and LBL (for 'label').

```
(62) a. [NumP_n]_n = n
b. [CL]_{\langle n, \langle \langle e, t \rangle, \langle e, t \rangle \rangle} = \lambda n \lambda P \lambda x [*P(x) \wedge \#(P)(x) = n]
c. [LBL]_{\langle n, \langle \langle e, t \rangle, e \rangle} = \lambda n \lambda P \iota x [P(x) \wedge LABEL(n, x)]
```

The meaning of $NuMP_n$ is simply an abstract number concept (type n), see (62a). The subscript indicates that $NuMP_n$ is lexically encoded. Though for the purpose of this paper I assume that it is the basic structural component of each cardinal numeral, I do not claim that it is primitive. It might very well be further decomposed into a number of features.²⁴ On the other hand, the components CL and LBL are interpreted as shifts from n to a more complex type.

²⁴This is indicated by the triangle in Figure 1 and 2. For why one would want to further decompose $NuMP_n$, see Wagiel & Caha (2020).

I will assume here that CL is interpreted as a function from a number concept to a predicate modifier devised for counting individuals (type $\langle n, \langle \langle e, t \rangle, \langle e, t \rangle \rangle \rangle$), see (62b).²⁵ CL takes a number and yields an expression with the built-in pluralization operation * and measure function #(P), see §3.3. In cases such as English *five* or Czech *pět* 'five', CL is already incorporated in the structure of a numeral but in some other languages it is encoded in as additional morpheme, e.g., a classifier.

On the other hand, LBL is a labeling device (type $\langle n, \langle \langle e, t \rangle, e \rangle \rangle$) which selects a number and returns a function from properties to unique individuals such that they have the relevant property and are labeled with the corresponding number by the LABEL operation, as provided in (62c). LBL is typically introduced by a free morpheme such as *number* or a specialized affix like the Czech suffix -ka, as discussed in §6.2 and §6.3, respectively.

As a result of combining the components in (62) by syntax, we obtain the structures in Figure 1 and 2. The tree in Figure 1 represents the structure and meaning of the quantifying function of a numeral corresponding to 5 whereas the tree in Figure 2 depicts the parallel label function.

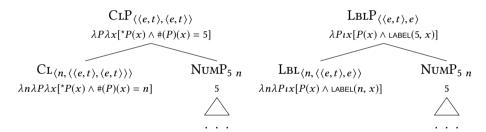


Figure 1: The quantifying function

Figure 2: The label function

When a quantifying numeral, e.g., Czech pět 'five', and a label expression, e.g., Czech číslo pět 'number five', combine with an NP, what we obtain are the denotations such as those in (63). In particular, (63a) denotes a set of pluralities of players such that each plurality in that set consists of five players. On the other hand, (63b) refers to the contextually unique player identified via their association with the number 5.

(63) a.
$$[pět hráčů]_{\langle e,t\rangle} = \lambda x[*player(x) \land \#(player)(x) = 5]$$

b. $[hráč číslo pět]_e = \iota x[player(x) \land label(5, x)]$

²⁵For the purpose of this paper, it is not important what the exact interpretation of C_L is. The proposed system is compatible with other theories of quantifying numerals discussed in §3 as long as it is possible to shift the primitive type n to a type for the quantifying function.

But how to account for the fact that a numeral such as Czech $p\check{e}t$ 'five' can be used to express both the quantifying and the arithmetical meaning? This ambiguity can be captured as a consequence of a core property of late insertion, namely that lexical entries are not tailor-made for one specific use. ²⁶ In Nanosyntax, a lexical entry pairs a well-formed syntactic structure with phonology. For instance, let us assign to the cardinal $p\check{e}t$ the lexical entry in Figure 3. When syntax builds a structure interpreted as the quantifying meaning, as in Figure 1, the entry in Figure 3 can be used to lexicalize it. Consequently, the structure or, more precisely, its top-most phrasal node is pronounced by pjet, as indicated by the big circle in Figure 4.

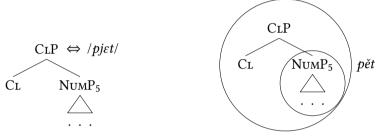


Figure 3: Lexical entry

Figure 4: Lexicalizations

Crucially, however, when syntax builds a structure interpreted as the arithmetical meaning, i.e., the sole NumP₅, it can also be pronounced by Figure 3, as indicated by the small circle in Figure 4. This is due to the Superset Principle, defined in (64) (Starke 2009), which states that a lexical entry can be used to pronounce a particular structure iff it contains that structure as a sub-part (either proper or improper). As a result, the Czech numeral *pět* 'five' is ambiguous between the quantifying and the arithmetical meaning since both circled structures in Figure 4 are contained in the tree in Figure 3.

(64) The Superset Principle (Starke 2009) A lexically stored tree matches a syntactic node iff the lexically stored tree contains the syntactic node.

Let us now consider how the label meaning arises. For this purpose, I will demonstrate how specialized label numerals such as Czech *pětka* 'number five' are derived. Since *pět* 'five' is stored as Figure 3, and thus cannot express the label meaning, the LBL component needs to be spelled out by a separate entry, e.g., as

²⁶For an introduction to late insertion models such as Distributed Morphology and Nanosyntax, see, e.g., Bobaljik (2017) and Baunaz & Lander (2018), respectively.

the suffix -ka in Figure 5. Notice that the semantics of Lbl is not compatible with the quantifying meaning and requires type n as its input, recall (62c). Thanks to the Superset Principle, see (64), $p\check{e}t$ can also pronounce the arithmetical meaning, i.e., the sole NumP₅, as indicated by the small circle in Figure 4. However, in order for this structure to apply at the LblP node, NumP₅ must move from its base-position. In Figure 6, its trace, i.e., the lower copy of the displaced element, is shaded.²⁷ After the NumP₅ is moved, Figure 5 matches the lower LblP, as shown in Figure 7, where the label suffix -ka is inserted at the relevant node.



Figure 5: Czech label suffix

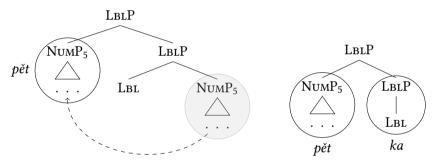


Figure 6: Movement

Figure 7: Spellout

The proposed analysis has several advantages. First of all, it provides a unified approach that captures the relationship between the arithmetical, the quantifying and the label meaning. Second, it explains when and how a numeral can be ambiguous between different meanings. Finally, it gives a detailed account for the meaning/form correspondences both in cardinal and in label numerals.

7 Conclusion

In this paper, I explored the semantics of numerals in Slavic and beyond. I discussed various functions that cardinal numerals can have as well as different approaches to capturing the relationships between these functions. Furthermore, I

²⁷In Nanosyntax, the movement in Figure 6 is driven by the need to lexicalize the phrasal LblP node by the lexical entry in Figure 5. For a detailed explanation of how precisely the movement in Figure 6 is triggered, see Baunaz & Lander (2018) and Caha et al. (2019).

examined many types of complex numerical expressions that only recently attracted attention in the literature. I also provided novel evidence in favor of the view suggesting that the arithmetical meaning is basic while other functions are derived from it. Finally, I explored the so-far understudied label function and proposed a morpho-semantic analysis of Slavic derived label numerals that could be further extended to other types of complex numerical expressions.

Abbreviations

ACC	accusative	LOC	locative
AGGR	aggregate numeral	MULT	multiplier
CLF	classifier	N	neuter
COP	copula	NBR	number-denoting numeral
F	feminine	NV	non-virile
GEN	genitive	PFV	perfective
GNDR	gender-sensitive numeral	REFL	reflexive pronoun
INS	instrumental	TAX	taxonomic numeral
IPFV	imperfective	TOP	topic
LBL	label numeral	V	virile

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