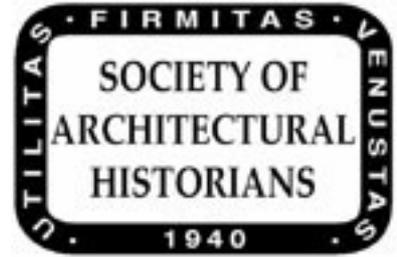




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Harmonic Proportion and Palladio's *Quattro Libri*

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In 1949, Wittkower proposed that musical harmonic ratios were a principle underlying Palladio's designs illustrated in the ground plans of Book II of the Quattro libri. This theory, expounded in Part IV of Wittkower's Architectural Principles in the Age of Humanism, has been widely accepted, despite the fact that his research was based on detailed analysis of only 8 of the 44 plans in Book II.

In the present study, a systematic, quantitative analysis of all the plans in Book II of the Quattro libri is carried out to discover to what extent musical harmonic ratios were an important principle behind Palladio's ground plans. Our results show that Palladio did indeed have a definite preference for numbers which can be related in ratios corresponding to the standard musical intervals. However, he does not make any consistent attempt to render his designs completely harmonic. Only about two-thirds of all the dimensions in the Book II plans are numbers which can be incorporated into musical ratios. Palladio often made no attempt to make his published measurements accord with musical harmonies where this could have been done by minor alterations, such as insignificant changes in wall thicknesses.

The actual buildings, too, show a preference for dimensions which can be related by harmonic ratios, although not quite to the extent of the plans published in the Quattro libri. A few, most notably the Villa Barbaro at Maser, are significantly more "harmonic" in the published versions than in reality. In view of Daniele Barbaro's well-known interest in harmonic proportion, it is significant that all the completely harmonic designs postdate Palladio's collaboration with Barbaro on the Vitruvius edition and the Villa at Maser. Most of the patrons of those designs closely based upon musical harmonies appear to have shared an interest in musical or architectural theory.

While Palladio almost certainly used musical theory in some later designs, his dependence on musical harmonic proportion was by no means as great as Wittkower implied. Elsewhere, his preference for harmonic dimensions probably resulted either from his use of certain favorite room shapes, or from the practical advantages of using simple, easily divisible numbers.

I. INTRODUCTION

The legacy of Rudolf Wittkower

IN 1949 Rudolf Wittkower's influential book *Architectural Principles in the Age of Humanism* was first published.¹ The book contains four seminal essays, the last of which is entitled "The Problem of Harmonic Proportion in Architecture."² In this essay Wittkower discusses the Renaissance interest in musical or "harmonic" proportion and its reflections in architecture of the period. The importance of the essay in rediscovering a concept which was evidently quite widely known in the Renaissance has been generally acknowledged. Wittkower reminds us, for instance, that according to his biographer Manetti, Brunelleschi studied the musical proportions of the ancients; and that Vitruvius mentions music as one of the essential parts of an architect's training.³ In his impressively erudite and wide-ranging study, Wittkower surveys Renaissance investigations into universal harmonies, based on the Pythagorean-Platonic theory of numbers. These appear in the work of philosophers, mathematicians, and writers on music, architecture, painting, and sculpture. According to Wittkower, this widespread interest demonstrates that "the Renaissance analogy of audible and visual proportions was no mere theoretical speculation; it testifies to the solemn belief in the harmonic, mathematical structure of all creation."⁴

Not surprisingly, Wittkower turns to the treatises of Alberti and Palladio, the most intellectual and systematic of Renaissance architectural books, to illustrate the importance of harmonic proportion to architects of the time. Actual buildings are little discussed in this essay, although Wittkower claims to have found

1. This article arises from research begun in connection with a book review which one of us published to commemorate the 400th anniversary of Palladio's death (D. Howard, "Four Centuries of Literature on Palladio," *JSAH*, xxxix, 1980, 224–241).

2. R. Wittkower, *Architectural Principles in the Age of Humanism*, Warburg Institute, London, 1949, Part IV, 89–135.

3. Wittkower, *Architectural Principles*, 103.

4. Wittkower, *Architectural Principles*, 117.

“at least a deliberate insistence on the ratios of small integers not only in Alberti’s façade of S. Maria Novella but also in other Renaissance buildings.”⁵ He points to Alberti’s exposition of the role of musical theory in his *De Re Aedificatoria* as an accessible source of these ideas for Renaissance architects.⁶ However he chooses to focus the essay on Palladio’s *Quattro libri dell’architettura*, published in Venice in 1570, realizing the importance of the treatise “as a means of expounding his conceptions, not only of planning but also of proportion.”⁷ He deduces that Palladio’s representations of his own villas and palaces in Book II of the *Quattro libri* were not merely autobiographical records but were generalized models, based on his own work because of a lack of genuine antique examples of domestic buildings. “If this is a right deduction,” continues Wittkower, “the hypothesis seems justified that Palladio wanted his inscribed measurements to convey ratios of a general character and of universal importance beyond the scope of individual buildings.”⁸

In order to test this hypothesis Wittkower examined “some of Palladio’s plans.”⁹ In fact his discussion is largely limited to detailed examinations of only 8 of the 44 measured plans illustrated in Book II.¹⁰ Although his hypothesis was tested on such a small sample, it has become very widely accepted. The elegant and far more intelligible summary of Wittkower’s ideas in Ackerman’s stimulating short book *Palladio* has conveyed to the general reader the notion that the Greek system of musical harmony was an underlying philosophy which governed Palladio’s principles of design.¹¹

Wittkower’s essay as a whole leaves one in no doubt that in intellectual circles in the Veneto, those in which Palladio was raised, Pythagorean and Platonic theories of musical harmonies were known and understood. The ideas of the Franciscan philosopher, Fra Francesco Zorzi, a monk at San Francesco della Vigna in Venice and author of a treatise on universal harmonies *De harmonia mundi totius* published in 1525, are a notable illustration.¹² Palladio’s friend and patron, Daniele Barbaro, too, showed his knowledge of such concepts in his commentary on Vitruvius in his edition of 1556, illustrated by Palladio himself.¹³ In addi-

5. Wittkower, *Architectural Principles*, 107 and footnote 2.

6. Wittkower, *Architectural Principles*, 97 and 100–103.

7. Wittkower, *Architectural Principles*, 128.

8. Wittkower, *Architectural Principles*, 128.

9. Wittkower, *Architectural Principles*, 129.

10. These are the Villas Godi, Malcontenta, Emo and Thiene (at Cigogna), the Palazzo Porto, and the Villas Pisani (at Bagnolo), Sarego (at Miega) and Barbaro (at Maser) (Wittkower, *Architectural Principles*, 129–136).

11. J. S. Ackerman, *Palladio*, Harmondsworth, 1966, 162ff.

12. F. Zorzi, *De harmonia mundi totius*, Venice, 1525; and Wittkower, *Architectural Principles*, 90–94 and Appendix I in which Zorzi’s famous memorandum on Sansovino’s design for San Francesco della Vigna is published in English translation.

13. D. Barbaro, *I dieci libri di Vitruvius tradutti e commentati da Mons. Barbaro eletto di Aquileggia*, Venice, 1556; and Wittkower, *Architectural Principles*, 120–123.

tion, the architect’s close friend Silvio Belli was the author of a treatise on proportion published in 1573.¹⁴

However, doubts must surely be raised in the minds of anyone who examines Book II of Palladio’s *Quattro libri* at all attentively. If the plans chosen by Wittkower do seem to demonstrate Palladio’s dependence on simple or “musical” harmonies, many others contain awkward dimensions which cannot be incorporated into any simple theory of harmonic proportion. Doubts about Palladio’s strict dependence on these tenets have already been expressed by later scholars, for example by Eugenio Battisti in an article of 1973 and by Howard Burns in the catalogue of the London Palladio exhibition in 1975.¹⁵ It was this current mood of scepticism which led us to investigate more carefully the validity of Wittkower’s hypothesis.

We are fully aware of the limitations of the analysis which follows as an approach to Palladio’s work as a whole. If the discussion seems remote from the beautiful and very tangible reality of the buildings themselves, this is because, following Wittkower, we have concentrated our investigation on the data provided by the plans in Book II of the *Quattro libri*. In the case of an architect as sophisticated and gifted as Palladio, it is obviously artificial to select one particular aspect of his art for such close scrutiny. Nevertheless, we believe that it is important to subject Wittkower’s theory to a more rigorous analysis. We shall demonstrate how it is possible to give a systematic and objective picture of Palladio’s ideas on proportion, which will offer some insight into his processes of design.

The article will be divided into five sections. The first section will introduce our principal source of information, Palladio’s *Quattro libri dell’architettura*. The second section will deal with our procedures for a systematic analysis of Wittkower’s hypothesis and the results of this enquiry. The third section will investigate the degree of conscious idealization in the Book II plans. The fourth section will approach the subject from a slightly different angle, by analyzing the proportions of individual rooms, independently of their relationship to other rooms in the same plan. The final section will summarize the conclusions resulting from these various lines of investigation.

The special nature of Palladio’s Quattro Libri

The intriguing feature of Book II of Palladio’s *Quattro libri*, as Wittkower was aware, is that the depictions in this book (in

14. S. Belli, *Della proportione et proportionalità communi passioni del quanto*, Venice, 1573; and Wittkower, *Architectural Principles*, 123–124.

15. H. Burns, L. Fairbairn and B. Boucher, *Andrea Palladio 1508–1580, The Portico and the Farmyard*, catalogue of Arts Council Exhibition, London, 1975, 224ff.; and E. Battisti, “Un tentativo di analisi strutturale del Palladio tramite le teorie musicali del Cinquecento e l’impiego di figure rettoriche,” *Bollettino del Centro Internazionale di Studi di Architettura ‘Andrea Palladio’*, xv, 1973, 211–230.

which Palladio's own designs are illustrated) do not always correspond with what was actually built.¹⁶ Although Palladio specifically names the patron and the location in most cases "so that anyone who wishes can see how they succeed in reality" (Book II, p. 4), it is well known that there are frequent discrepancies between the published plans and the buildings themselves. These differences can be attributed to a variety of causes: printers' errors, Palladio's own updating of early plans, even adjustments to improve the visual effect on the printed page. External constraints such as restricted sites are frequently discounted, so that the illustrations may serve as generalized models to guide other architects or patrons. It is now accepted by most scholars that Palladio's representations of his own designs in the *Quattro libri* do not show the buildings as they ought to have been built. This was first convincingly demonstrated by Zorzi in 1961, in an article pointing out that the preparatory drawings lead up to the buildings as executed, rather than to the printed versions.¹⁷ In other words, the designs were subjected to a conscious process of adaptation for the purpose of publication.

The fact that the *Quattro libri* had an obviously didactic function is important. Since Palladio was evidently prepared to make adjustments to the schemes for the sake of the internal unity and consistency of the book, then it seems reasonable to expect, as Wittkower did, that the published versions were idealized; that is, that they show what he would have built in an ideal situation, unconstrained by the realities of the site, economic limitations and so on. Thus, if Palladio really gave harmonic proportion the pre-eminence which Wittkower claims he did, then these principles should be embodied in the plans of his own buildings in Book II of the *Quattro libri*.

The three other books of the treatise, Book I on the Orders, Book III on Public Buildings, and Book IV on Temples, did not give Palladio so much scope to put his own principles of design into practice. His theory of the orders in Book I is a clarification of Vitruvian doctrine, and makes little mention of room proportions, as we shall see. The buildings which illustrate the last two books are chiefly examples from antiquity; and although Palladio idealized or reconstructed them to a certain extent, he was obviously restricted by the existence of models not designed by himself, however much he may have admired them. We shall therefore confine our examination to the evidence in Book II.

Palladio's own stated views on proportion

Palladio himself says remarkably little about proportion in the text of the *Quattro libri*. The tone of the writing is consistently

16. Page references to Palladio's treatise in the text are taken from the edition *I quattro libri dell'architettura di Andrea Palladio*, published in Venice in 1570 and now available in modern facsimile editions.

17. G. G. Zorzi, "I disegni delle opere palladiane pubblicate ne 'I quattro libri' e il loro significato rispetto alle opere eseguite," *Bollettino del CISA*, III, 1961, 12-17. A brief survey of the literature on the *Quattro libri* is given in Howard, "Four centuries," 226-230.

succinct, down-to-earth, and practical. The only reference to such an overall harmonic proportional system is a brief sentence in Book I, Chapter XXI (p. 52):

The most beautiful and well proportioned shapes of rooms, and those which succeed best, are seven: that is, either round, or square, or with the length equal to the diagonal of the square on the width, or a square and a third, or a square and a half, or a square and two thirds, or two squares.

These correspond to the ratios of $1/1$, $1/\sqrt{2}$, $3/4$, $2/3$, $3/5$, $1/2$. All of these ratios are consonant with so-called musical harmonies, with the exception of $1/\sqrt{2}$ which corresponds approximately to a ratio of $5/7$ (1/1.414). Four of them, namely $1/\sqrt{2}$, $2/3$, $3/5$, and $1/2$, could have been simply derived from Book VI of Vitruvius's *Ten Books on Architecture*, in the chapter concerning the "Proportions of the Principal Rooms."¹⁸

Much of Book I, which deals with the Orders, is obviously concerned with the heights or widths of columns, bases, entablatures and so on. However, the proportions of the orders are not an important element in Wittkower's theory, even when they provide the basic module for other dimensions. Other recommendations on proportion in Palladio's treatise are made for the sake of comfort and commodity, for example the recommended sizes for doors, windows, or steps. In the brief texts which accompany the illustrations in Book II (those which are of interest to us here), proportions are little discussed. Palladio seems to be more concerned with the functions of the different parts of each building and the nature of the site. The comments about room dimensions usually refer to their heights, which are more difficult to read from the illustrations. The few specific references to ground dimensions will be discussed in a later section of this article. It is interesting to observe that except in his reconstructions of antique houses, where he follows Vitruvius's guidelines, Palladio makes little or no mention of the dimensions of one room relative to another. Wittkower's hypothesis is that not only individual room dimensions but also the dimensions of all parts of the building are related to each other by musical ratios.

The intuitive feeling of the reader of the *Quattro libri* is that common sense and an innate sense of design were more important to Palladio in determining the proportions of his buildings than any abstract harmonic theory. The purpose of our analysis is to discover whether this supposition is borne out by the evidence of the plans themselves or whether there is an underlying set of principles such as Wittkower's hypothesis of harmonic proportion which guided his intuition.

Proportion in the genesis of Palladio's designs

Any Renaissance architect using the basic Vitruvian discipline of the orders had to consider proportion at a very early stage in each design. However, the grammar of the orders mainly affects the

18. Vitruvius, *The Ten Books on Architecture*, trans. M. H. Morgan, New York, 1960, Book VI, Chapter III, pp. 177-179.

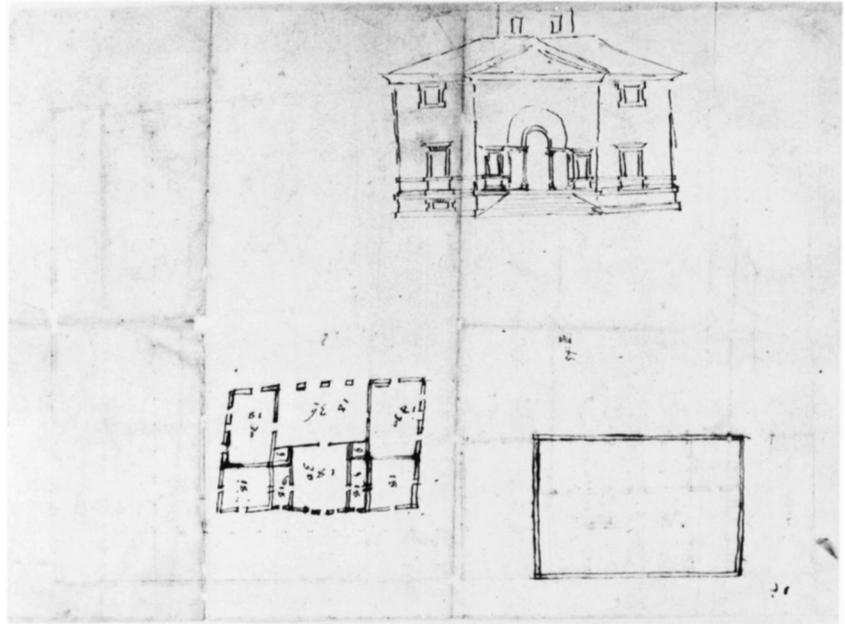


Fig. 1. Preparatory drawing for the Villa Poiana, RIBA XVI, 4v. (By kind permission of The British Architectural Library, RIBA, London)

vertical articulation of a building. That Palladio, more than any other Renaissance architect, was also deeply preoccupied with proportion in the ground plans of his buildings, right from the start of each design, is clear from his preparatory drawings.

Even hasty freehand sketches are frequently marked with carefully worked out dimensions. For example, in the sketch of the Villa Poiana (RIBA XVI 4v) (Fig. 1), probably dating from the late 1540s, the whole plan is based on simple multiples of three, and the proportions of the rooms accord with his own recommendations. The main rooms are $30 \times 18'$ or $5:3$, the medium-sized rooms $18'$ square, and the small rooms $18 \times 9'$ or $2:1$. The loggia is $36 \times 18'$ or $2:1$, and the stairwells are $9 \times 9'$. This very simple scheme takes no account of wall thicknesses, and had to be adjusted for practical reasons, since the staircases needed more space. However, it is clear that even before such practical modifications were introduced, the basic harmony of the scheme as a whole and the proportions of individual rooms were worked out.

A sheet of sketches such as RIBA XI, 22v (Fig. 2), datable to the 1550s, shows Palladio experimenting freely with plans of town houses. In these quick freehand jottings he tries out different ways of disposing suites of large, medium-sized, and small rooms around central halls. Many questions have to be resolved—the placing of doors and windows, the position of the stairs, and how to achieve a basic symmetry in the plan and elevation. Here a musical analogy is tempting, for Palladio works like a composer exploring variations on a single theme. Just as a composer of music has to ensure that there are the right number of beats in each bar and that the harmonies are completely worked out, so Palladio has to be certain that the various sizes of rooms “add up” correctly and that each room has satisfying, harmonious

proportions. Thus we find that, even in these preliminary jottings, many of the plans are marked with dimensions, some amended as his ideas develop. On the same sheet are two sections, of an entablature and a cornice respectively. In simple addition sums at the side of the page Palladio adds up the heights of the different parts of the two sections, and calculates the overall length of one of the ground plans.

Examples such as these suggest that, from the first scribbles onward, the precise numerical dimensions of Palladio's designs were a conscious element. It would not therefore be surprising to discover yet more carefully worked out and systematic proportions in the relatively unconstrained context of the *Quattro libri*.

II. THE ANALYSIS OF WITTKOWER'S THEORY OF HARMONIC PROPORTION IN PALLADIO

Mathematicizing Wittkower's Theory

Wittkower gives no single set of rules to determine which ratios of dimensions are of particular significance. In the course of Part IV of his *Architectural Principles*, a wide variety of numerological and harmonic schemes are mentioned. Thus, the powers of small whole numbers, such as 2, 4, 8, 16, 32, 64, or 3, 9, 27, 81, 243, as well as the golden section ($\frac{1}{2}[\sqrt{5} + 1] = 1.618:1$) and arithmetic, geometric, and harmonic means are discussed. By far the greatest weight is given, however, to systems based upon the ratios of small numbers corresponding to musical harmonies. In particular, much of Wittkower's case rests on the fact that many of the rarer harmonic ratios of the musical scale can be found in Palladio's designs. Our aim is therefore to devise a set of rules, by which, if a particular set of numbers appears on a design, we can assess whether they can be incorporated into a set of harmonic ratios or not.

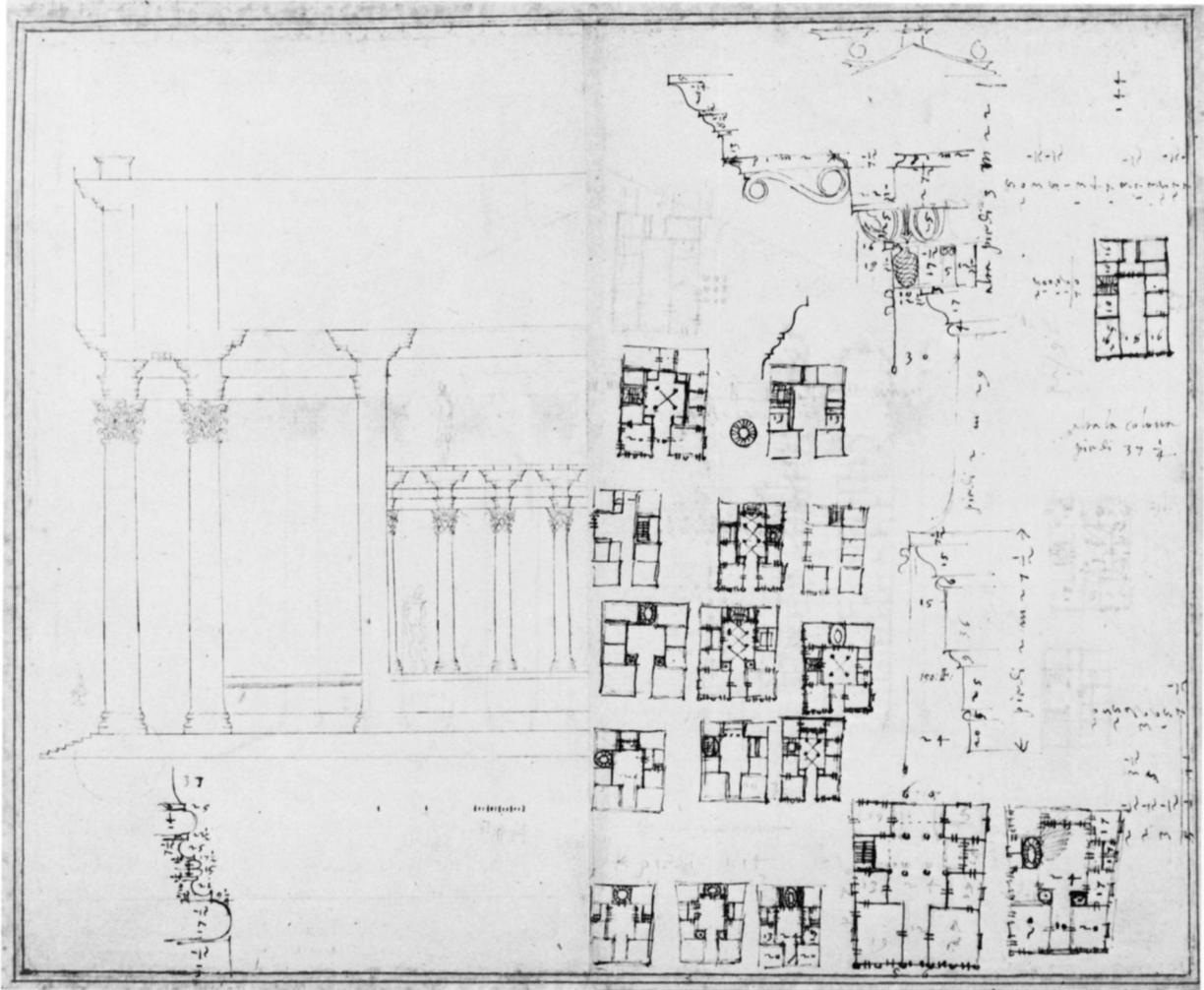


Fig. 2. Sketches of town houses, RIBA XI, 22v. (By kind permission of The British Architectural Library, RIBA, London.)

We take advantage of the fact that most of the numbers appearing on the plans are whole numbers (in fact, 87% of all non-repeated numbers). Those which are not can be converted into whole numbers by multiplying by 2, 3, 4, or 6 in all cases. The significance of these factors will be apparent in a moment.

We shall consider two musical scales. The one given most attention by Wittkower is what would now be called the JUST scale. In this case, the scale CDEFGABC corresponds to a ratio of frequencies of vibration from C of:

C	C-D	C-E	C-F	C-G	C-A	C-B	C-C	
1	9/8	5/4	4/3	3/2	5/3	15/8	2	JUST (1)

What Renaissance musical theorists noticed was that there are a number of additional ratios implied by these intervals. Thus we can deduce by division of one interval by another that the following ratios of small numbers are also implied:

C-D	D-E	E-F	F-G	G-A	A-B	B-C	
9/8	10/9	16/15	9/8	10/9	9/8	16/15	(2)

The other scale similar to this is known as the PYTHAGOREAN scale, in which the corresponding intervals are taken to be:

C	C-D	C-E	C-F	C-G	C-A	C-B	C-C	
1	9/8	81/64	4/3	3/2	27/16	243/128	2	PYTHAGOREAN (3)

It will be noticed that the Pythagorean scale is entirely based on ratios of powers of 2 and 3. For example, $81/64 = 3^4/2^6$. In this scale the "small numbers" involved in the ratios are the same as the power series 2, 4, 8, 16 and 3, 9, 27, 81. We will incorporate all the intervals present in both the Just and Pythagorean scales in our analysis.

The only additional rule we need is that to increase an interval by an octave, we simply multiply by 2. Thus, from C to G is a ratio of 3/2 in frequency and from C to the G, an octave higher, is $3/2 \times 2 = 3$. Similarly, to decrease an interval by an octave, we divide by 2.

A further interesting point is that by multiplying the ratios above, we can generate all the ratios appropriate to the twelve notes of the duodecaphonic scale:

C C^z D D^z E F F^z G G^z A A^z B C

In this way, we can generate much more complicated harmonic ratios than those discussed by Wittkower.

Obviously it is cumbersome to have to plod through all these numbers one by one, and it will soon become apparent to the reader that in fact there exists a set of "harmonic whole numbers," such that *any two of them*, when reduced by division by the appropriate power of 2, will correspond to the harmonic ratios listed above. The "harmonic whole numbers" up to 100 are:

TABLE I

List of *harmonic whole numbers*, i.e., those which in any combination can be related to musical ratios.

1	2	3	4	5	6	8	9	10	12	15	16	18	20
24	25	27	30	32	36	40	45	48	50	54			
60	64	72	75	80	81	90	96	100					

The way in which this list was constructed was as follows. Each small number in the ratios listed in scales (1), (2), and (3) above can be factorized into prime numbers. For example, $15/8 = (5 \times 3)/(2 \times 2 \times 2)$. The reader will readily convince himself that all the numbers appearing in the harmonic ratios in the series (1), (2), and (3) can be generated by products of the numbers, 2, 3, and 5. The series of harmonic numbers listed above simply consists of all the numbers between 1 and 100 which can be generated by multiplying twos, threes, and fives together in different ways.

Let us look at some of the types of ratios which are generated. Some ratios clearly correspond to the Just or Pythagorean scales. For example, $54/48 = 36/32 = 18/16 = 9/8$. Others correspond to harmonic ratios once they are reduced within the range of a single octave. For instance, $60/8$ is two octaves above the interval $15/8$, because $60/8 = 2 \times 2 \times 15/8$. Equally, some ratios have to be "moved up" octaves to convert them into the ratios listed in (1), (2), and (3). For example, $10/16 = 5/8$, on being raised an octave becomes $2 \times 5/8 = 5/4$ which is present in the harmonic scale, i.e., just as $5/4$ corresponds to the interval C-E, $5/8$ corresponds to the interval from C to the E below C. More complex ratios can always be related to products of ratios appearing in the scales. Even a number like $81/80$ is no more than $9/8 \times 9/10$.

We have confirmed that *all* ratios mentioned by Wittkower in support of his hypothesis are included in our scheme. Indeed, our scheme includes many more ratios than those considered by Wittkower, but they are all derived from simple harmonic ratios. In this way we have devised an even more generous set of criteria than Wittkower's, for the purpose of detecting harmonic proportions in Palladio's plans in Book II of the *Quattro libri*, if indeed they are to be found there.

Thus, if *all* the buildings in Palladio's Book II were constructed according to strict harmonic principles, we could predict that *only the "harmonic whole numbers" listed above (in Table I) would appear on the plans*. Notice that we are able to include the

fractional numbers as well, because they can be "moved up an octave" by multiplying by 2, by an octave and a fifth by multiplying by 3, by two octaves by multiplying by 4, or by two octaves and a fifth by multiplying by 6. This prediction is the basis for the various investigations which we have made of the buildings in Book II.

Treatment of the Data in the Quattro Libri

In order to make a valid comparison between Wittkower's hypothesis and the data contained in Book II of Palladio's *Quattro libri*, we must set up well-defined rules *a priori* and treat all the data in a systematic way.

Wittkower treats only eight buildings in detail in his analysis, and does not apply any rigorous criteria as to which building should be selected or which dimensions should be considered. For example, in the Villa Emo, he finds that he can incorporate essentially every dimension given by Palladio within his harmonic sequence, including those of the wings.¹⁹ In the Villa Godi, by contrast, the scheme works well for the main buildings, the dimensions of which are those discussed in his analysis. However, he ignores the outbuildings which could not be accommodated in the scheme.²⁰

We have therefore chosen two approaches which span the range of reasonable hypotheses implicit in Wittkower's practice.

- 1) Test all dimensions appearing on Palladio's plans, irrespective of what they refer to. In this approach we include outbuildings, porticoes, distances between columns, etc.
- (2) Test only the dimensions of the main living rooms of the building, i.e., the main section of a palace or villa, excluding porticoes, gardens, loggias, courtyards, stairs, corridors, stables, etc., but including the *atria* of the antique reconstructions since these are conceived as open rooms.

For each of these two cases we note the numbers appearing on all the plans in Book II of the *Quattro libri*, including both Palladio's own palaces and villas, whether executed or not, and his reconstructions of antique domestic architecture. (Table A2 in the Appendix).

In comparing these data with the theory, we will adhere strictly to the hypothesis that only harmonic numbers should be considered as significant, i.e., those listed in Table I. When non-harmonic numbers are present on a plan, Wittkower on one occasion tries to incorporate them into his scheme by treating them as averages of harmonic numbers. This appears in a footnote to his discussion of the Villa Maser, where he treats the only non-harmonic measurements, 14 and 46 feet, as arithmetic means of $(12+16)$ and $(32+60)$ respectively.²¹ This procedure obviously

19. Wittkower, *Architectural Principles*, 114.

20. Wittkower, *Architectural Principles*, 112-113.

21. Wittkower, *Architectural Principles in the Age of Humanism*, 3rd rev. ed., London, 1962, 136, note 1. The extended note does not appear in the original 1949 edition.

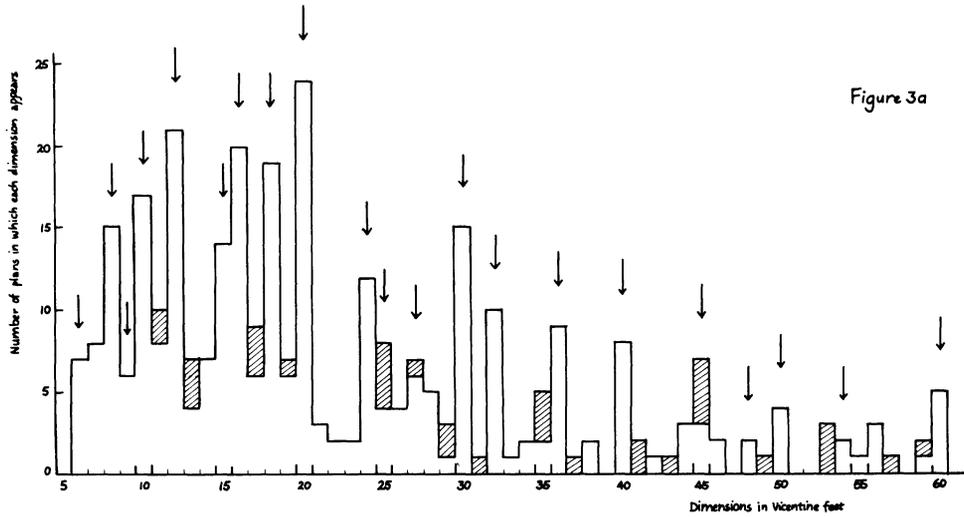


Figure 3a

Fig. 3a. Histogram showing the distribution of all numbers over 5 in the plans in Book II of Palladio's *Quattro libri dell'architettura*. Each number is counted only once in each plan in which it appears.

Notes

Additional numbers greater than 60: 62 63 66 68 72 73 74 76 78 85 99 100 102 115 120 130 214

Fractions, multiplied out to make whole numbers: 61 65 69 (twice) 75 83 (twice) 85 89 93 105 109 135 250 307 445
 "Harmonic" numbers are underlined.

The shaded portions of the blocks in the histogram indicate fractions multiplied out to make whole numbers. "Harmonic" numbers are indicated by arrows.

invalidates the whole concept of harmonic proportions, since any non-harmonic whole number can be generated from the average of two harmonic numbers, thus rendering the theory meaningless.

We have also to consider which numbers are worth including in the analysis. Since all numbers up to 6 are automatically harmonic numbers, the small numbers provide little evidence either way for the hypothesis. We have therefore considered only those numbers appearing on the plans which are greater than 5, i.e., 5 1/2 upwards.

We have followed Wittkower in considering only horizontal dimensions. However, it should not be forgotten that the relationship between the height and the length or breadth of a room or a building was a very important element in Palladio's process of design. He certainly envisaged his designs in three dimensions, as one can see even in the *Quattro libri* illustrations, with their clearly related elevations and plans. But the height measurements are not always indicated on the elevations, and moreover, they are usually external rather than internal dimensions. It would thus be impossible to include heights in the analysis in any complete or systematic way. Furthermore, we would argue that any scheme which is harmonic in three dimensions is automatically also harmonic in two of them. We have therefore decided to confine the analysis to the measurements indicated on the ground plans in Book II.

We have to consider how to deal with numbers which appear more than once on a single plan. Should they be counted once or more than once? We argue here that unit weight is the correct statistic. If the building is designed according to strictly harmonic principles, all the different dimensions in the one building will be related so that harmonic ratios exist between them. In other words, what determines the proportion of a building is a *sequence* of numbers, regardless of how many times each one of them occurs. All Palladio's buildings possess a high degree of

symmetry, and it is often this symmetry which determines whether or not a particular number appears more than once. We therefore characterize each building by a simple series of numbers, those appearing on the plans in Book II of the *Quattro libri*, considering each different number *once only* in each plan.

It should by now be clear that the analysis refers only to published plans and considers only horizontal measurements. These are the same sources of information as those analyzed by Wittkower. All the numbers to be considered here are dimensions in Vicentine feet, the unit of measurement used by Palladio. (The length adopted by Palladio for the Vicentine foot was about 34.7 cm.)

The Results of the Analysis

The table in Appendix 2 lists all the numbers appearing in the plans of Book II of the *Quattro libri*, with those referring to the main living rooms being indicated in bold type.

The first test is to plot the histogram which shows how often different numbers appear on the plans (Figure 3a). In this test each number listed on Table A2 is counted once. The fractions appearing in Table A2 have been included in the diagram by multiplying them by 2, 3, 4, or 6, in order to make them into whole numbers; these are indicated by hatched boxes. The "harmonic" numbers, that is, those listed in Table 1 (on p. 121), are indicated by arrows.

It is immediately obvious that there is a very marked tendency for harmonic numbers to appear on the plans. Without exception all the prominent peaks on the histogram correspond to harmonic numbers. Of the 365 numbers appearing in Table A2, 239 are harmonic and 126 are not, a harmonic percentage of 65.5%, or almost two-thirds. It does not need a statistical test to show that Palladio has a clear preference for the harmonic numbers in Table A2.

Nonetheless, virtually all other numbers are present as well,

Figure 3b

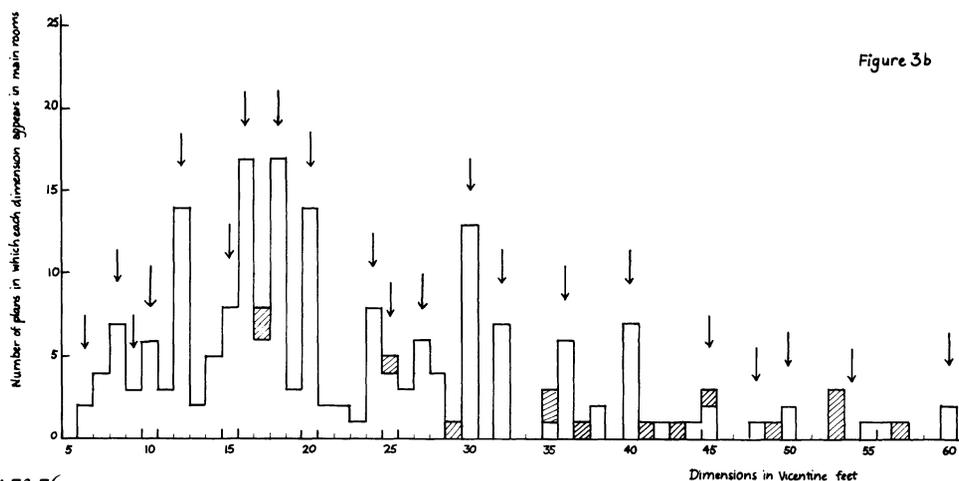


Fig. 3b. Histogram showing the distribution of all numbers over 5 which are dimensions of *main living rooms* in the plans in Book II of the *Quattro libri*. Each number is counted only once in each plan in which it appears.

Notes

Additional numbers greater than 60: 72 76

Fractions, multiplied out to make whole numbers: 65 69 (twice) 75 83 (twice) 85 89 93 105 109 135 250

“Harmonic” numbers are underlined.

The shaded portions of the blocks in the histograms indicate fractions multiplied out to make whole numbers. “Harmonic” numbers are indicated by arrows.

although with much lower frequency than the harmonic numbers. It is interesting that seven, a number not found in the harmonic series, appears as often as six.

Figure 3b shows the same histogram for all the numbers in bold type in Table A2, i.e., those referring to the main living rooms of the buildings. The same notation as in Figure 1 is used. The same phenomena can be observed in this histogram. All the prominent peaks are associated with harmonic numbers, the percentage of harmonic numbers increasing slightly to 69.9% (153 out of 219). We can therefore conclude that it does not matter greatly, so far as the overall statistics are concerned, whether we consider *all* the dimensions over five feet, or only those of the main living rooms, excluding courtyards, loggias, stables and other outbuildings.

Of course, even if Palladio had selected his dimensions purely at random, there is quite a strong chance that he would have chosen harmonic numbers. For example, if one has to select a number between 11 and 20 inclusive, five of them are harmonic (12, 15, 16, 18, and 20), and five are not (11, 13, 14, 17, and 19). Thus there is a 50% probability that a harmonic number will be chosen. In this interval there are 138 numbers in Table A2. If the numbers were selected at random, we would expect 69 of them to be harmonic on average. The fact that we find 98 of them, i.e., 71% rather than the expected 50%, is an indication of how “harmonic” the buildings are on average. In a similar way, we can work out what percentage of numbers would be expected to be harmonic overall if the numbers were selected at random. To do this we find the total number of digits in the ranges 6 to 10, 11 to 20, 21, to 30, 31 to 40, 41 to 50, 50 to 100, and greater than 100, and work out the probability of a number chosen at random being harmonic in each range. The result is that we should expect about 45% of all the numbers, if chosen at random, to be harmonic. This should be compared with the actual percentage of about 65–70% of harmonic numbers in the Book II plans.

The next test is to find out whether all the buildings have

similar proportions of harmonic numbers, or whether some are more “harmonic” than others. We do this for each building by calculating the percentage of harmonic numbers among all the numbers appearing on the plan, i.e., all the numbers listed in Table A2. Thus, for example, for the Palazzo Chiericati (Book II, p. 6), we assign a percentage of 67 ($= 6/9 \times 100$), when all dimensions are considered, counting each number only once as before. For each building we then repeat the analysis using only the dimensions of the main rooms (i.e., the figures in bold type on Table A2). The Palazzo Chiericati now becomes 100% ($= 4/4 \times 100$).

In Table A3 we list all the buildings in Book II with the fractions and percentages of harmonic dimensions for each plan. The distributions of these percentages are shown in Figures 4a and b. It can be seen that both distributions are rather broad. There are some buildings with less than the mean value expected of about 45%, but there is a very obvious trend towards larger percentages of harmonic numbers than would be expected at random. In fact, there are many plans in which all, or almost all, of the dimensions are harmonic, and which are thus worthy of particular attention.

We can deduce, however, from the smooth nature of the histograms of Figures 4a and b that the overall preponderance of harmonic numbers cannot be entirely attributed to a small number of completely harmonic buildings. It can be seen that both distributions are shifted to values greater than 45%, and that the main peak is centered about 60–70%. If the whole effect were to be attributed only to a small number of harmonic buildings, the main peak would be centered on about 45%, with another very pronounced peak at percentages greater than 80%.

There is a slight trend towards higher percentages of harmonic buildings when only main rooms are considered. In Figure 5 we have plotted the values of the two percentages for each building. It can be seen that there are a few examples which reach 100% when only main rooms are considered (one of them, the Villa

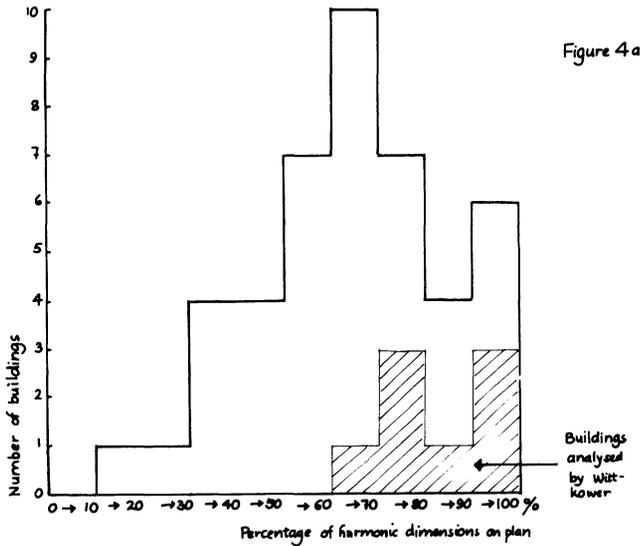


Figure 4a

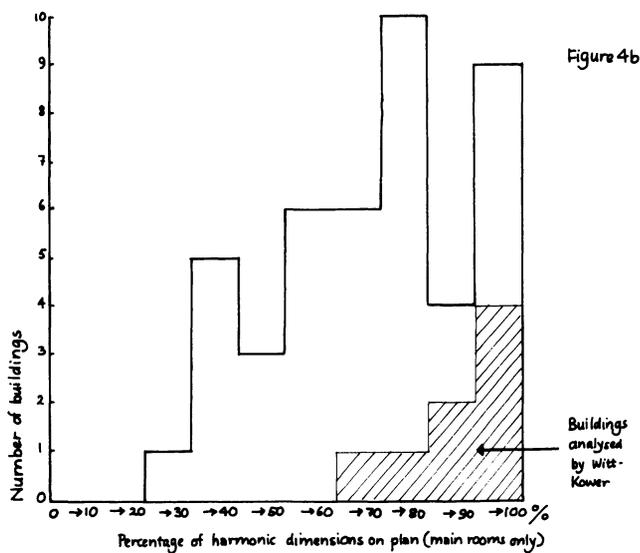


Figure 4b

Fig. 4a. Histogram showing the percentages of harmonic numbers in the plans of Book II of Palladio's *Quattro libri dell'architettura*. Each number is counted only once in each plan in which it appears. 4b. Histogram showing the percentages of harmonic numbers in the Book II plans, considering *main rooms only*.

Repetita, having only one room dimension marked!). However, in general, as many become more harmonic as become less so, with only a small tendency to higher percentages of harmonic numbers in the main rooms than in the buildings as a whole. Figure 5 confirms that it does not matter greatly whether all dimensions, or only those of main rooms, are considered.

It is interesting to note where the eight buildings selected for special mention by Wittkower are located on the histograms. These are indicated by hatched boxes on Figure 4. Remarkably, Wittkower selected for analysis only a small fraction of the buildings which could properly be considered harmonic. This is partly due to the fact that our technique for detecting degrees of har-

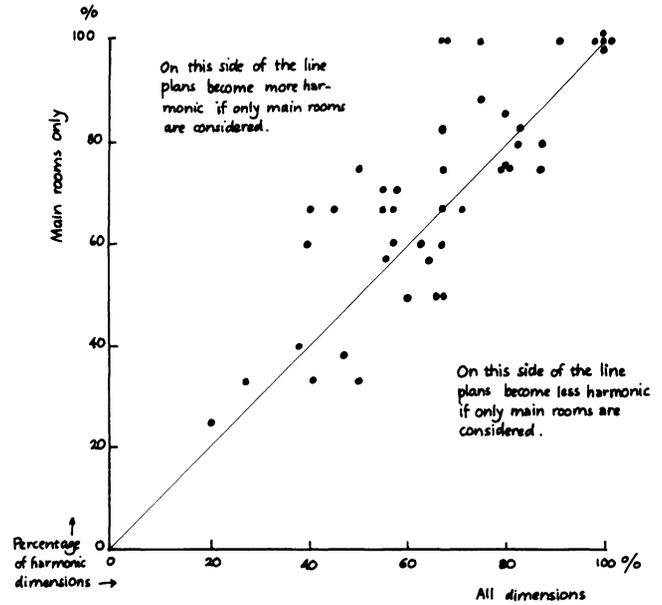


Fig. 5. Graph showing the shift in percentages of harmonic numbers if only main rooms are considered.

mony, in the sense of musically related ratios, is much more complete and systematic than his approach.

The next step is to find out whether there are any general trends associated with the particularly harmonic buildings. We have tested for a number of possible correlations. Specifically, we have investigated date, building type, location, and patron.

(i) Date

The dates of the projects have mostly been derived from Puppi's monograph, unless these have been convincingly revised by later research. The sources of the dates adopted here are given in Appendix A 1. It should be stressed that there is still no definitive documentary evidence for the dates of many of Palladio's villas and palaces, and much controversy surrounds this problem. Where no precise date is known, an estimated date is used. As a guide to the reliability of the dates adopted we have classified them as either A (accurate to within about three years, i.e., $\pm 1\frac{1}{2}$), B (accurate to within about ten years, i.e., ± 5), or C (possibly not accurate to within ten years). In each case we have used the date of the start of the building work if known, since Palladio seems to have revised some of his designs over long periods before the buildings themselves were begun.

It should be remembered that the blocks for the illustrations in the *Quattro libri* were presumably all prepared in the years preceding the publication of the treatise in 1570. What we are attempting to analyze is the importance of the date of the *final design*, but it should not be forgotten that the executed design may not correspond at all closely with the published version.

In Figures 6a and b we show the correlation diagrams for the date of the design against the percentage of harmonic numbers,

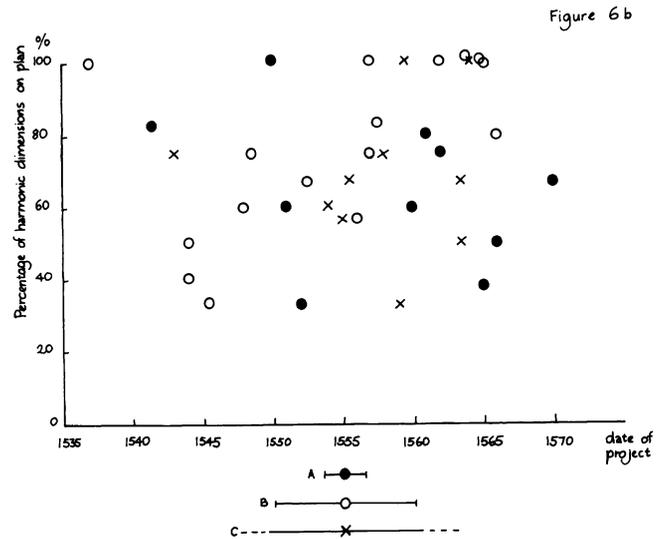
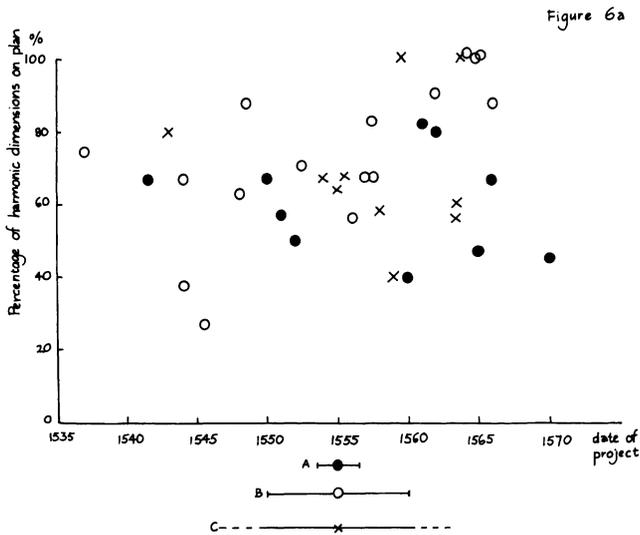


Fig. 6a. The relationship between the date of each project and the degree of harmony in its published plan. 6b. The relationship between the date of each project and the degree of harmony in its published plan, considering *main rooms only*.

for all dimensions and for main rooms only, using different symbols for dates which are classified A, B, and C. In the first case (Figure 6a) there is a trend towards larger harmonic percentages in the later buildings. It can be seen that most of those with percentages greater than 80% are later than 1550. That this is a marginally significant correlation can be demonstrated by a non-parametric Spearman rank correlation test.²² This test shows that there is less than one chance in ten of this distribution happening at random (or that the distribution of points seen in Figure 6a could arise from a random sample of randomly distributed points). This is not a high level of significance, especially considering the uncertainty of some of the data, but it nevertheless deserves to be taken seriously. On the other hand, if only main rooms are considered (Figure 6b), no correlation can be discerned.

We conclude that the tendency to design main living rooms with harmonic proportions is to be found in Palladio's designs from all periods, at least in the form in which they are presented in the *Quattro libri*. On the other hand, to judge from the evidence of the printed plans, the tendency to incorporate all parts of a building, however minor, into a single harmonic system, increased significantly during Palladio's career. There is certainly no evidence for Ackerman's assertion that "Palladio seems to have been especially attracted to musical proportions early in his career."²³

(ii) Building type

It is apparent that there are examples with a high proportion of

harmonic numbers among all types of buildings illustrated in Book II. These include palaces, villas, and projects for villas and palaces which never materialized. In no one group of Palladio's own buildings is there a particularly marked tendency towards harmonic proportions, even in the unexecuted designs where Palladio would have had the opportunity to select strictly harmonic numbers had he wished to do so. Since most of the unrealized projects are included in a section showing how to cope with the exigencies of particular sites, this is perhaps not surprising, for Palladio did not consider even these designs as purely ideal.

The one surprising feature concerns Palladio's reconstructions of antique domestic architecture. These include his design for the Convento della Carità in Venice (Book II, p. 30), which he published in this series since it was a deliberate reconstruction of an antique house. The "private house of the Greeks" cannot be considered, since the plan in the *Quattro libri* has no dimensions indicated. Despite the fact that Palladio's text comments extensively on the proportions of these schemes, only one of the projects analyzed, the "Atrio Testugginato" (Book II, pp. 34 and 35), is more than 67% harmonic. Thus, although he followed several of Vitruvius's recommendations, choosing ratios like 2:3, 3:5, 1:2 which are in fact harmonic, he did not achieve any kind of perfect overall harmony in his antique models.

(iii) Location

A map showing where each of the projects is located in the Veneto suggests no obvious concentration of "harmonic" buildings in any particular region.

An alternative geographical distribution can be obtained by considering the home towns of the patrons. Of the 25 projects with at least two thirds of all their dimensions harmonic, eight

22. See, for example, S. Siegel, *Non-parametric Statistics for the Behavioral Sciences*, Tokyo etc., 1956, 202.

23. Ackerman, *Palladio*, 167.

were commissioned by Venetian patrons (out of a total of 13 for Venetians in Book II), three by Veronese patrons (out of four), and 12 by Vicentines (out of 20). The two remaining plans are of a reconstruction of an antique scheme. Thus again we find no obvious evidence of geographical concentration of interest in harmonic proportion in any one area.

(iv) *Patrons*

Let us consider again the "top 25" projects in Book II, i.e., those in which at least two thirds of all their measurements are harmonic numbers. (Tables 2a and b). (Notice that this is more than half of the 44 plans which can be analyzed.)

Two of these, as mentioned, are plans of a reconstructed antique model, the Atrio Testugginato. Five are early projects, substantially altered or updated for publication. Three are very incomplete in reality and bear little resemblance to the published plans. Four were not executed at all. Five have been partly or completely destroyed, but were probably never completed. One

TABLE 2a

List of the 25 projects in which at least two-thirds of the dimensions appearing in the Quattro libri plans are harmonic numbers.

100%	Villa Emo, Fanzolo, p. 55. Villa Thiene, Cigogna, p. 62. Villa Sarego, Santa Sofia, p. 67. Palazzo Angarano, Vicenza, p. 75. Palazzo della Torre, Verona, p. 76.
91%	Villa Sarego, Miega, p. 68.
88%	Villa Poiana, Poiana Maggiore, p. 58. Villa Trissino, Meledo, p. 60.
83%	Villa Barbaro, Maser, p. 51.
82%	Villa Mocenigo, Brenta, p. "66".
80%	Palazzo Porto, Vicenza, p. 8. Atrio Testugginato, p. 34. Atrio Testugginato (detail), p. 35. Villa Mocenigo, Marocco, p. 54.
75%	Villa Godi, Lonedo, p. 65. Venetian palace, p. 72.
71%	Villa Foscari (Malcontenta), p. 50.
67%	Palazzo Chiericati, Vicenza, p. 6. Villa Rotonda, Vicenza, p. 19. Villa Pisani, Bagnolo, p. 47. Villa Badoer, Fratta Polesine, p. 48. Villa Saraceno, Finale, p. 56. Villa Ragona, Ghizzole, p. 57. Villa Repeta, Campiglia, p. 61. Palazzo Garzadore, Vicenza, p. 77.
	<i>Other plans which become 100% harmonic when only main rooms are considered</i>
	Palazzo Chiericati, Vicenza, p. 6. Villa Repeta, Campiglia p. 61. (only one relevant dimension!) Villa Godi, Lonedo, p. 65. Villa Sarego, p. 68.

progressed little in Palladio's lifetime and was finally completed in the late 17th century. In all of these cases one could expect a fairly free rendering in the *Quattro libri* and hence a certain degree of idealization.

What is remarkable is the character of the five villas which remain in the "top 25," once the schemes listed above are removed. These are the Villas Emo, Badoer, Barbaro, Malcontenta, and Rotonda (the first four commissioned by Venetians and the last by a Vicentine). All of these were more or less completed in Palladio's lifetime and show a rather close correspondence in layout between the executed buildings and published plans. This suggests that Palladio himself considered them as exemplary de-

TABLE 2b

Classification of the "top 25" designs in Book II of the Quattro libri, i.e., those in which at least two out of three of all the dimensions appearing on the plans are harmonic numbers.

Not executed

Un-named palace in Venice, p. 72.
Palazzo Angarano, Vicenza, p. 75.
Palazzo della Torre, Verona, p. 76.
Palazzo Garzadore, Vicenza, p. 77.

Early projects, much altered in Quattro libri versions

Palazzo Porto, Vicenza, p. 8.
Villa Pisani, Bagnolo, p. 47.
Villa Saraceno, Finale, p. 56.
Villa Poiana, Poiana Maggiore, p. 58.
Villa Godi, Lonedo, p. 65.

Incomplete, destroyed or partly destroyed

Villa Mocenigo, Marocco, p. 54.
Villa Ragona, Ghizzole, p. 57.
Villa Repeta, Campiglia, p. 61.
Villa Sarego, Miega, p. 68.
Villa Mocenigo, Brenta, p. "66".*

Very incomplete. Quattro libri plans largely ideal

Villa Trissino, Meledo, p. 60.
Villa Thiene, Cigogna, p. 62.
Villa Sarego, Sta. Sofia, p. 67.

Mainly executed after Palladio's death

Palazzo Chiericati, Vicenza, p. 6.

Ideal (antique reconstruction)

Atrio Testugginato, p. 34.
Atrio Testugginato (detail), p. 35.

Mature projects with executed layouts

close to Quattro libri versions

Villa Almerico (Rotonda), Vicenza, p. 19.
Villa Badoer, Fratta Polesine, p. 48.
Villa Foscari (Malcontenta), Mira, p. 50.
Villa Barbaro, Maser, p. 51.
Villa Emo, Fanzolo, p. 55.

* New information on the former existence of this villa is given in Lewis, cat. nos. 56, 57, 72, and 73.

signs, seeing little need to revise them. It can hardly be fortuitous that these are probably the most famous and best loved of all Palladio's villas and palaces.

We have already remarked upon the tendency for Palladio to incorporate more harmonic dimensions in his plans later in his career. Clearly the almost totally harmonic buildings from his later period deserve special attention, in order to see whether their patrons shared certain interests or characteristics.

It is not surprising that one of the most strikingly harmonic buildings should be the Villa Barbaro at Maser (Book II, p. 51), in which all but two dimensions are harmonic, and if main rooms only are considered, all but one are harmonic. This is the building which Wittkower analyzed in most detail, showing that it embodies a wide range of harmonic ratios, including the more unusual ones present in the just scale.²⁴ Since it is well known that the patron, Daniele Barbaro, was deeply interested in theories of harmonic proportion, the presence of so many musical ratios in his villa is unlikely to be fortuitous.²⁵ The commission is datable to 1557–1558, which predates all the projects in Book II which are over 90% harmonic overall (see Table 2a). It seems very plausible that Palladio made a conscious effort to design more harmonic buildings after his collaboration with Barbaro on the Vitruvius edition and the Villa at Maser.

Among the almost totally harmonic buildings are the two villas designed for the Sarego or Serego family of Verona, neither of which was completed. In the villa at Miega, designed for Annibale Sarego in about 1562 (Book II, p. 68), ten out of eleven dimensions are harmonic and can therefore be inter-related by musical ratios. The only non-harmonic number on the plan is an intercolumniation of 6½ feet, a dimension dependent on the width of the columns. The design for the villa at Santa Sofia for his brother Marc'Antonio Sarego (Book II, p. 67) has six harmonic dimensions out of six. The unexecuted palace project for another Veronese patron, their brother-in-law Giambattista della Torre (Book II, p. 76), which closely resembles the Sarego villa at Miega, is also 100% harmonic. The supposition that such perfectly harmonic designs are unlikely to have happened accidentally, seems to be confirmed by the fact that the Sarego family were well-known for the learned musical reunions held at their

home in Verona.²⁶ The della Torre were among the other prominent Veronese families who held musical gatherings, called *ridotti* or *orti* depending on whether they were held inside or in the open. Indeed music was one of the major cultural interests among intellectual circles in both Verona and Vicenza in the 16th century, and was actively encouraged by small *accademie* in both cities.

It may also be significant that the two Vicentine patrons of totally harmonic schemes, Giacomo Angarano and Odoardo Thiene (Book II, pp. 62 and 75), were both known to hold so-called “heretic,” or radical Lutheran religious views.²⁷ The first two books of the *Quattro libri* were dedicated to Giacomo Angarano, under whose auspices Palladio says he began the work of writing the treatise (Book I, p. 3). Angarano was therefore almost certainly interested in architectural theory and may have communicated this interest to other members of the Vicentine Lutheran circle.

Our conclusion from this analysis is that the tendency to choose harmonic numbers cannot be wholly attributed to any single cause. Rather, in the *Quattro libri* Palladio's general preference for harmonic numbers appears in any type of domestic building from any stage in his career, although the plans which are most harmonic overall tend to be later projects. Available evidence suggests that the patrons of the most harmonic schemes probably shared an interest in musical or architectural theory. Indeed, it seems that the proclivities of the patrons contributed significantly to the degree of harmony present in the schemes which they commissioned. This is suggested by the fact that even late in Palladio's career when he was designing projects for theoretically-minded patrons in which all the dimensions could be interrelated by musical ratios, he was also still producing other designs which displayed relatively little regard for overall musical harmony.

We still have to try to explain why Palladio tends to prefer what we have called “harmonic” numbers, even if he does not use these measurements exclusively. Before attempting this, however, it is important to investigate other aspects of Palladio's choice of dimensions. These provide further evidence on the question of whether or not Palladio was consciously striving to include harmonic proportions in the plans of the *Quattro libri* whenever feasible.

24. Wittkower, *Architectural Principles*, 1949 ed., 118–119.

25. On Daniele Barbaro and his intellectual ideas see, for example, P. J. Laven, *Daniele Barbaro: Patriarch-Elect of Aquileia*, unpublished Ph.D. thesis, University of London, 1957; E. Forssman, “Palladio e Daniele Barbaro,” *Bollettino del CISA*, VIII (ii), 1966, 68–81; and B. Boucher, “The last will of Daniele Barbaro,” *Journal of the Warburg and Courtauld Institutes*, XLII, 1979, 277–282. The existence of a mediaeval castle on the site at Maser, which was incorporated into Palladio's villa, may explain the presence of the one non-harmonic dimension (14 feet) in the main block. See Don U. Basso, *La villa e il tempietto dei Barbaro a Maser di Andrea Palladio*, Montebelluna, 1976, 6–29; and D. Lewis, *The Drawings of Andrea Palladio*, exhibition catalogue, National Gallery of Art, Washington, D.C., 1981, 157.

26. V. Bolcano, “L'ambiente musicale a Vicenza e a Verona ai tempi del Palladio,” in *Palladio e Verona*, exhibition catalogue, Verona, 1980, 18–30. For further information on Palladio's projects for the Sarego and della Torre families, see the relevant catalogue entries in *Palladio e Verona*, 232–237 and 244–250. We are grateful to Geoffrey Newman for his advice regarding this line of enquiry.

27. L. Puppi, *Andrea Palladio*, Milan, 1973, II, 273; Burns et al., II, 83–84; and M. F. Tiepolo et al., *Testimonianze veneziane di interesse palladiane*, exhibition catalogue, Archivio di Stato di Venezia, 1980, 40. The Villa Repeta (Book II, p. 61) which is 67% harmonic was also designed for a “heretical” patron.

III. THE QUESTION OF IDEALIZATION OF PROPORTIONS IN THE *QUATTRO LIBRI*

The Problem of Wall Thicknesses

According to Wittkower's concept of harmonic proportion in architecture, each part of a perfectly harmonic building should be proportional to every other part and to the building as a whole. This implies that a harmonic relationship should exist between the external dimensions and the sizes of the internal rooms.

It will immediately be clear to the reader that the architect cannot easily achieve such consistency. For example, if three rooms side by side have widths of 16', 8', and 24' respectively, the overall width of the building will be 48' only if the wall thicknesses are discounted altogether. If, on the other hand the walls are given thicknesses of, say, 1' for the internal walls and 1½' for the external walls, the overall width of the building will be 53'. The resulting external measurement clearly has no harmonic relationship to the internal dimensions.

In other cases, taking account of the wall thicknesses can actually help to relate all the dimensions harmonically. For instance, take the common Palladian case of a large internal room 30' square. If this hall is flanked by three smaller rooms measuring 8', 12', and 8' respectively, the allowance of 1' for each partition wall enables the width of each smaller room to bear a harmonic relationship not only to the other smaller rooms but also to the large hall alongside.

In order to discover Palladio's own method of dealing with this problem, we calculated the wall-thickness on all the plans of Palladio's buildings in Book II which give the necessary dimensions to allow this. (In 19 of the plans there was not enough information to deduce any of the wall thicknesses.) It should be noted that there are no plans from which it is possible to calculate the widths of outside walls, since the external dimensions are not given in the *Quattro libri*. The results of this enquiry (Table 3) show that Palladio had no standard policy for determining wall thicknesses. The widths calculated from the *Quattro libri* plans vary between 0 and 2½'. Apart from those discounted altogether, these thicknesses vary within roughly the same limits (approximately 1-2½') as those of his actual buildings.

In two of the three instances in which Palladio completely discounts the wall thicknesses, this does not help to achieve a completely harmonic result:

Villa Pisani, Bagnolo	18 + 24 = 42
Villa Mocenigo, Marocco	10 + 16 = 26

although in the third case the reverse is true:

Villa Valmarana, Lisiera	15 + 12 = 27.
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In other words, there is only *one* case, that of the Villa Valmarana, in which Palladio can be said to have ignored the wall thickness for the sake of achieving a more harmonic result in the printed version. As it happens, another wall thickness on the same plan can also be measured, but in this case there is no harmonic relationship between the three dimensions. Here Pal-

TABLE 3

Calculation of wall thicknesses in Book II of the Quattro libri
 "t" represents the wall thickness.

+	Palazzo Antonini, Udine, p. 5.	32 = 8+8+12½+2t	t = 17/8'
×	Palazzo Porto, Vicenza, p. 8.	30 = 9+9+10+2t	t = 1'
	Palazzo Valmarana, Vicenza, p. 16	35 = 10+10+10+2t	t = 2½'
	Palazzo Barbarano, Vicenza, p. 22.	41½ = 24+16+t	t = 1½'
		25 = 7+7+8½+2t	t = 1¼'
=	Villa Pisani, Bagnolo, p. 47.	42 = 18+24+t	t = 0'
	Villa Badoer, Fratta, p. 48.	34 = 8+8+16+2t	t = 1'
	Villa Foscari (Malcontenta), p. 50.	46½ = 16+16+12+2t	t = 1¼'?
	Villa Pisani, Montagnana, p. 52.	28 = 8½+8½+8½+2t	t = 1¼'
+	Villa Cornaro, Piombino, p. 53.	27¼ = 10+16+t	t = 1¼'
=	Villa Mocenigo, Marocco, p. 54.	26 = 10+16+t	t = 0'
×	Villa Poiana, Poiana Magg., p. 58.	36 = 18+8+8+2t	t = 1'
*	Villa Valmarana, Lisiera, p. 59.	27 = 15+12+t	t = 0'
		32½ = 17+15+t	t = ½'
	Villa Thiene, Quinto, p. 64.	41½ = 20+20+t	t = 1½'
×	Villa Sarego, Miega, p. 68.	40 = 9+9+20+2t	t = 1'
×	Palazzo Trissino, Vicenza, p. 74.	40 = 15+15+8+2t	t = 1'
×	Palazzo Angarano, Vicenza, p. 75.	36 = 20+15+t	t = 1'
+	Schemes which could have been easily made more harmonic by a minor change in wall thickness.		
×	Successful and harmonic solutions.		
*	Harmonic but impossible solutions.		
=	Impossible and non-harmonic solutions.		

ladio makes a small allowance of $\frac{1}{2}'$ for the wall thickness, giving the sequence:

$$17 + 15(+\frac{1}{2}) = 32\frac{1}{2}$$

Thus it is clear that this is not a completely harmonic building in any case, so that any conscious manipulation of the wall thicknesses for the sake of achieving strict musical harmonies seems unlikely.

In five of the Book II plans, the inclusion of wall thicknesses of $1'$ in each case definitely helps to relate the dimensions harmonically:

Palazzo Porto, Vicenza	$9 + 9 + 10(+1+1) = 30$
Villa Poiana, Poiana Maggiore	$18 + 8 + 8(+1+1) = 36$
Villa Sarego, Miega	$9 + 9 + 20(+1+1) = 40$
Palazzo Trissino, Vicenza	$15 + 15 + 8(+1+1) = 40$
Palazzo della Torre, Verona	$20 + 15(+1) = 36$

(The last two of these plans show unexecuted projects.)

On the other hand we can identify a number of instances where Palladio could have achieved harmonic proportion in the room dimensions by making insignificant changes to the wall thicknesses. For example, if " t " is the wall thickness, in the Palazzo Antonini in Udine (Book II, p. 5), $8 + 8 + 12\frac{1}{4} + 2t = 32$, making $t = 1\frac{7}{8}'$. This series could equally well have been $8 + 8 + 12\frac{1}{2} + 2t = 32$, by making $t = 1\frac{3}{4}'$. In addition the central hall could have been $12\frac{1}{2} \times 12\frac{1}{2}$ which is also harmonic. Similarly, in the Villa Cornaro (Book II, p. 53), $10 + 16 + t = 27\frac{1}{4}$, making $t = 1\frac{1}{4}'$ whereas this series could have been $10 + 16 + t = 27$ by making $t = 1'$.

It appears that the more unusual wall thicknesses do not necessarily "improve" the harmonic proportions. For example, in the plan of the Palazzo Valmarana at Vicenza (Book II, p. 16), three rooms side by side, each $10'$ wide, flank a loggia $35'$ wide, indicating walls as thick as $2\frac{1}{2}'$. A more normal thickness of $1'$ for each dividing wall would have given an overall width of $32'$, which is harmonic. In this case, however, the width of the loggia was evidently determined by the six columns, each $2\frac{1}{2}'$ wide, on the opposite side of the loggia, and the related intercolumniations. In other words, taking three or four dimensions in isolation, as we must do in order to calculate the implied wall thickness, we artificially ignore the many other requirements which Palladio had to satisfy in each design as a whole.

From the above evidence we can conclude that Palladio did *not* consistently manipulate the wall thicknesses of the *Quattro libri* plans in order to improve the overall proportions of the resulting schemes, although in a few cases implied wall thicknesses of $1'$ appear to have had this effect.

Are the published plans more "harmonic" than the actual buildings?

The assumption that the *Quattro libri* versions of Palladio's own buildings were idealized models is central to Wittkower's argu-

ment. In order to assess the degree to which the proportions of the published plans were in fact idealized, a comparison can be made between the *Quattro libri* plans and the buildings themselves.

Only a few accurate modern surveys of the buildings have so far been published. However we are fortunate that this enquiry can be more completely carried out by a comparison between Palladio's depictions of his own buildings in the *Quattro libri* and Bertotti Scamozzi's plans in his book *Le fabbriche e i disegni di Andrea Palladio*, published in Vicenza in four volumes between 1776 and 1783.²⁸ Bertotti Scamozzi's plans are "corrected" versions, supposedly giving the exact dimensions of the buildings themselves, measured in Vicentine feet like those of Palladio, although Bertotti used a slightly longer unit for the foot than Palladio.

The results of a survey of the Villa Malcontenta, carried out under the direction of Erik Forssman and published in 1973, have shown that Bertotti's measurements are all accurate to within two inches in this case and usually agree exactly with the modern survey.²⁹

The evidence of two of the *Corpus Palladianum* monographs, those on the Villas Emo and Badoer, in which the dimensions given by Bertotti Scamozzi are compared with those of the actual buildings and those of the *Quattro libri*, confirms that Bertotti's measurements are reasonably reliable. The figures published in the *Corpus* are all converted into metres, but without taking account of the fact that Palladio and Bertotti used different values for the Vicentine foot. Whereas the *Corpus Palladianum* gives the Vicentine foot a length of 34.7 cm, equivalent to the length used by Palladio himself, Bertotti Scamozzi used a value of about 35.7 cm, corresponding closely to the official length marked on a tablet in the Loggia of San Vincenzo in Vicenza in 1583.³⁰ Once this discrepancy is accounted for, it appears that Bertotti's results are generally correct. Only a few differ from the modern measurements by more than an inch or two, and this should not be enough to affect the overall picture very seriously.

The evidence of Bertotti Scamozzi must be used with caution, not only because of occasional slightly inaccurate measurements,

28. A graphic comparison between the dimensions and proportions of all Palladio's buildings as shown in the *Quattro libri* and as depicted by Bertotti Scamozzi was carried out in the 19th century by L. Chapuy, A. Corréad, and A. Lenoir, *Les Oeuvres complètes d'André Palladio*, Paris, 1842.

29. E. Forssman, *Visible Harmony: Palladio's Villa Foscari at Malcontenta*, trs. G. Irons, Stockholm, 1973, 34–35.

30. Burns et al., *Andrea Palladio*, 209. Modern surveys of the Villas Rotonda, Emo and Badoer, the Palazzo Porto and the Convento della Carità, are all published in the relevant *Corpus Palladianum* monographs. The dimensions given by Palladio, Bertotti Scamozzi, and the modern surveys are compared in two of the volumes: G. Bordignon Favero, *La Villa Emo di Fanzolo*, Vicenza, 1970, 31–32, and L. Puppi, *The Villa Badoer at Fratta Polesine*, University Park and London, 1975, 97.

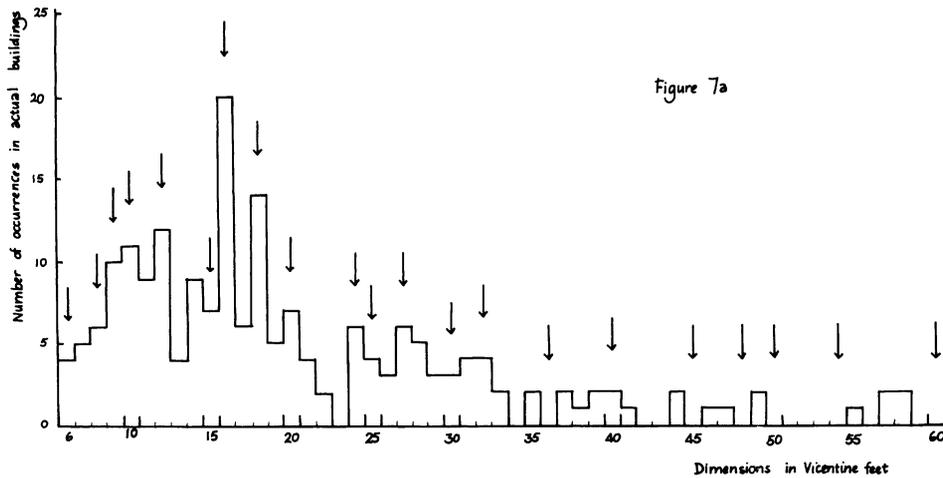


Figure 7a

Fig. 7a. Histogram showing the distribution of the actual ground dimensions of those of Palladio's buildings which are illustrated in Book II of the *Quattro libri*.

Notes

Measurements are taken from O. Bertotti Scamozzi, *Le fabbriche e i disegni di Andrea Palladio*, here converted to the same unit for the Vicentine foot as that used by Palladio himself, and rounded off to the nearest foot. "Harmonic" numbers are indicated by arrows.

Additional numbers greater than 60: 77 78 (twice) 105 109 114 140

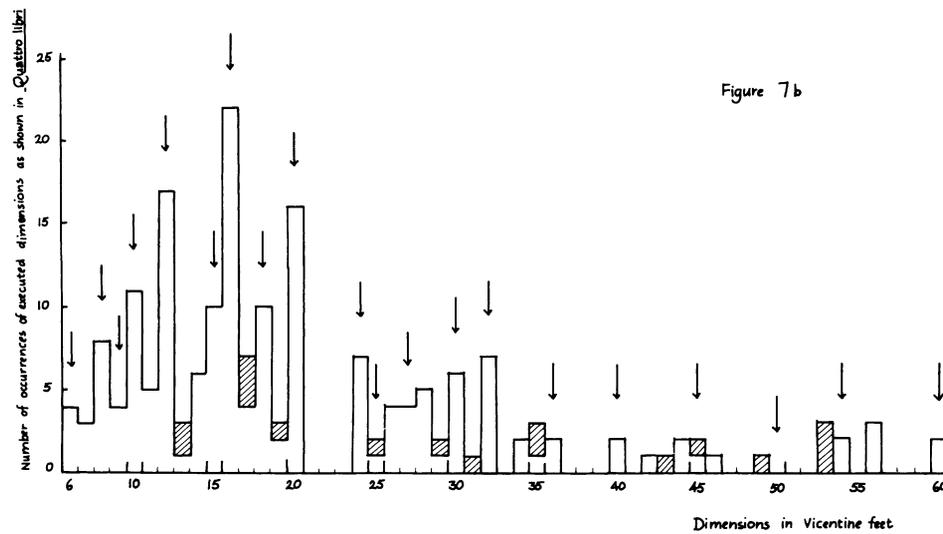


Figure 7b

Fig. 7b. Histogram showing the distribution of the corresponding dimensions as recorded by Palladio in the *Quattro libri* plans.

Notes

Additional numbers greater than 60: 74 (twice) 78 102 130
Fractions multiplied out to make whole numbers: 65 69 93 109 307
The shaded portions of the blocks in the histogram indicate fractions multiplied out to make whole numbers. "Harmonic" numbers are indicated by arrows.

but also since he does not clearly distinguish the executed portions of each scheme from the rest. Where a building or part of a building was not in fact constructed, Bertotti Scamozzi simply uses the *Quattro libri* dimensions, correcting only those that are clearly impossible or incorrect. Nevertheless, with careful handling, Bertotti Scamozzi's evidence is a useful tool.

There is certainly no reason to believe that Bertotti Scamozzi himself consciously tried to increase the degree of musical harmony in Palladio's plans in his own versions. Although he claimed to have rediscovered the existence of such harmonic ratios in the *Quattro libri*, he himself did not pursue the idea and probably did not fully understand the theory himself.³¹

In order to make the comparison between the *Quattro libri* plans and the actual buildings we have converted all dimensions to the same unit of measurement. For the sake of consistency with the rest of the article we have adopted Palladio's unit for the Vicentine foot (one foot equals 34.7 cm). We have therefore multiplied each of Bertotti's measurements by a factor of 1.029 ($\frac{35.7}{34.7}$), and compared each of the resulting dimensions with the equivalent measurements given in the plans in Book II of the *Quattro libri*. Dimensions which Bertotti obviously took directly from the *Quattro libri* have not been converted, and have been omitted from the initial comparison.

The results of this investigation are summarized in Figures 7a-d. In Figure 7a all Bertotti Scamozzi's actual measurements, converted to Palladian feet and rounded off to the nearest foot, are plotted as a histogram, counting each different figure only

31. See Howard, "Four Centuries," 235.

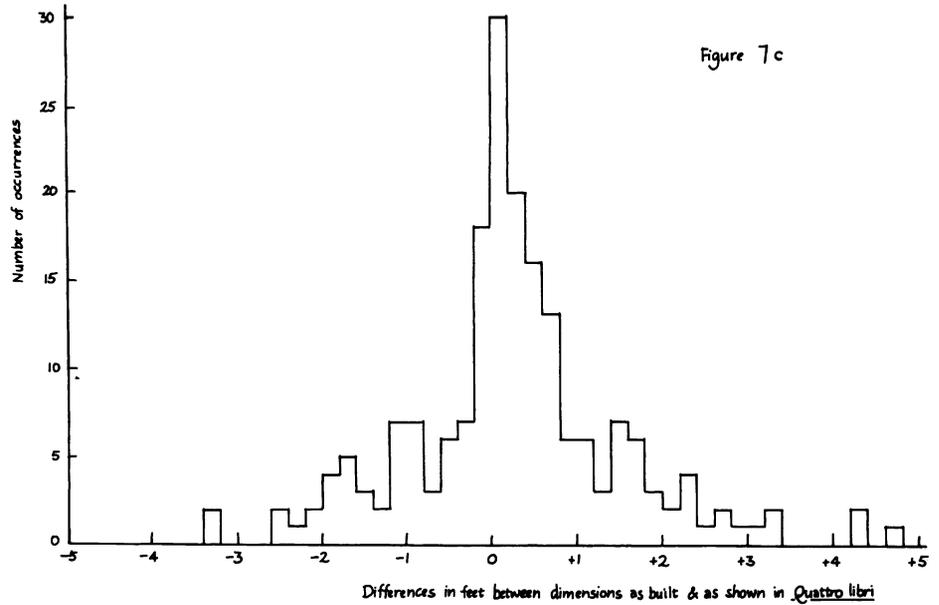


Fig. 7c. Histogram showing the differences between the dimensions of the actual buildings (measurements by Bertotti Scamozzi, here converted to Palladian feet) and the equivalent dimensions as recorded in the *Quattro libri*.

Notes

Differences greater than ± 5 feet:
 -10.60 -5.85 -5.03 +6.29
 +7.21 +7.33 +8.90 +10.20
 +53.79

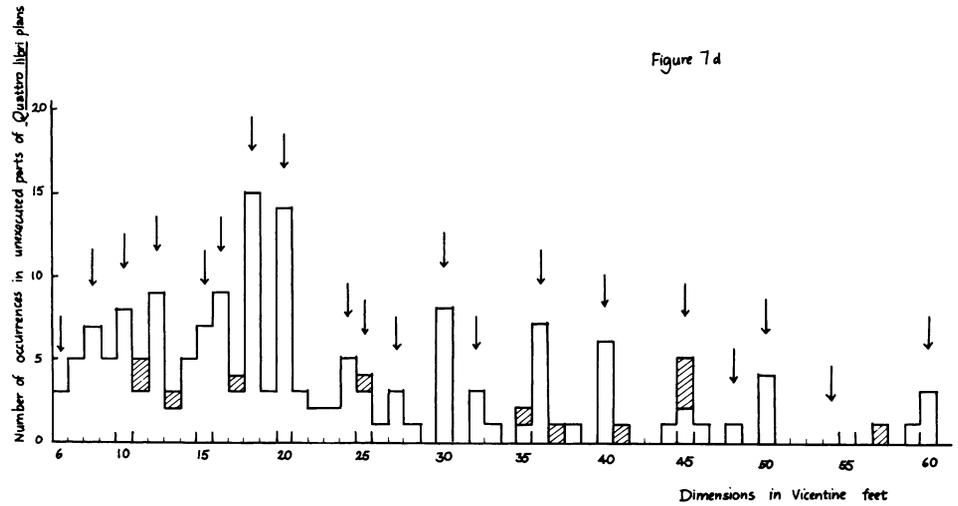


Fig. 7d. Histogram showing the distribution of measurements appearing on the unexecuted portions of the plans in Book II of the *Quattro libri*.

Notes

Additional numbers greater than 60: 62 66 72 73 75 76 85 99 100
 115 120 214

Fractions multiplied out to make whole numbers: 61 69 75 83 (twice) 85 89 105 135 250 445

Harmonic numbers are underlined.

Other conventions as in Figure 7b.

once if the same figure occurs more than once in a single building. In Figure 7b Palladio's own measurements given in the *Quattro libri* for all the corresponding dimensions are plotted on a similar graph for comparison, omitting all those which were not checked by Bertotti Scamozzi. Figure 7a should therefore give an indication of whether or not the executed portions of the actual buildings correspond to harmonic numbers, and 7b the extent to which these dimensions were "improved" by Palladio in the *Quattro libri*.

It will be immediately apparent that the two distributions are remarkably similar. All the most prominent peaks on both graphs, i.e., 10, 12, 16, 18, 20, and 24, are "harmonic" numbers. A few numbers which cannot be incorporated into any musical sequence, such as 11, 14, and 31, do occur more frequently in Figure 7a, but this is hardly surprising since these are all close to

harmonic numbers which Palladio used frequently (10, 12, 15, 16, and 30). To express our results quantitatively, 68% of the dimensions in Figure 7b are harmonic numbers, which is the same as found in the analysis of Section II when all dimensions of all buildings are considered. On the other hand, in the buildings as constructed only 57% of the dimensions are harmonic numbers, i.e., less than the percentage for the plans of the *Quattro libri*, but significantly greater than the value of 45% expected if the dimensions were selected at random. This demonstrates that Palladio's buildings themselves exhibit a degree of musical harmony far greater than that which would be expected by chance.

Another consequence of this result is that the buildings as constructed must follow rather closely the plans in the *Quattro libri*, since a typical error of even a foot would be sufficient to eliminate the prominent peaks at harmonic numbers. In order to

assess with greater accuracy how closely the *Quattro libri* records the measurements of the buildings themselves, we have plotted the difference between Bertotti's measurements of the actual buildings, converted to Palladian feet as before and correct to two decimal places, and the same dimensions as given in Palladio's treatise. The results are shown in Figure 7c. This graph gives a surprisingly decisive result, for it indicates that a clear majority (62%) of the dimensions recorded in the *Quattro libri* are accurate to within a foot, and many of these to within ± 2 inches! As would be expected there are a number of measurements which have been intentionally changed in the treatise, but only nine measurements differ from those of the executed buildings by more than five feet. Thus, if the *Quattro libri* demonstrates that Palladio had a preference for numbers which can be interrelated by musical harmonies, the same is true of the buildings themselves. Apart from conscious changes, or numbers which were incorrectly printed, the *Quattro libri* reproduces remarkably faithfully the measurements of the executed portions of the ground plans illustrated in the treatise.

It is also interesting to discover whether Palladio attempted a greater degree of idealization when he was unconstrained by the existence of executed buildings. We therefore plotted all the dimensions of buildings or parts of buildings which were never erected and those of Palladio's reconstructions of antique domestic architecture. The results of this enquiry are shown in Figure 7d. Once again, a very similar distribution is obtained, and the peaks in the graph are almost the same as those in Figure 7b, including 8, 10, 12, 16, 18, 20, 24, and 30. Other harmonic numbers, such as 36, 40, 45, and 50 are used more frequently than in Figure 7b, but at the same time there are a number of large and relatively unconstrained dimensions which appear to have been selected with little regard for musical harmony. The percentage of dimensions which are harmonic numbers in Figure 7d is 69%, almost exactly the same as the percentage for those buildings which were erected and for the treatise as a whole. In short, Palladio's ideal or unexecuted schemes were not more "harmonic" than the executed portions of his own buildings as depicted in the *Quattro libri* plans.

Finally it is worth considering briefly some of the individual projects, in order to see how far some of the most harmonic *Quattro libri* schemes achieved this degree of harmony in the actual buildings. Of the five perfectly harmonic schemes in Book II (see Table 2a) only three were executed, even in part. Of these, two can hardly be compared with what was built. Nothing was built of the Villa Thiene at Cicogna other than part of one of the wings, while at the Villa Sarego at Santa Sofia the existence of an older building on the site meant that only the courtyard façade of Palladio's project could be constructed; the interior, as Bertotti Scamozzi observed, is so totally different from the *Quattro libri* plan that he simply reproduced Palladio's published ground plan

unaltered.³² The Villa Emo is the only perfectly harmonic scheme in Book II which bears any comparison with the real thing. In this case all the dimensions of the central block are very close indeed to the published measurements, as has been observed by Ackerman.³³ This portion of the building can therefore be said to exhibit perfect musical harmony, both in the *Quattro libri* and in reality. By contrast, the side wings of the Villa Emo do not correspond precisely with the *Quattro libri* plan. The arcade is slightly narrower (13.38 Palladian feet instead of 15), and the rooms behind a little wider (21.09 rather than 20 feet), while the arrangement of the service rooms in these wings is different.

Another scheme which is almost exactly harmonic in the *Quattro libri* version is the Villa Sarego at Miega.³⁴ Of this villa about half the central block was completed, with a different layout for the staircase area. The building was in poor condition even in Bertotti Scamozzi's time, and was completely rebuilt in the early 20th century. However, the measurements carried out by Bertotti Scamozzi confirm that, in its executed form as in the *Quattro libri*, the villa was almost exactly harmonic in every ground dimension.

The Villa Malcontenta, as Forssman revealed in his survey, mirrors the *Quattro libri* very closely.³⁵ All the actual dimensions agree with the treatise to within about half a foot, except for the back rooms which are 10 feet wide instead of 12. Since 10 and 12 are both "harmonic" numbers, this does not affect the degree of harmony of the villa. However, this precision is rarely so consistently apparent within a single building. In the case of the Villa Barbaro at Maser, for example, while seven measurements agree almost exactly with those in Palladio's published plan, others may vary by as much as 10 feet. Seven dimensions which are harmonic numbers in the Book II plan are not harmonic in the actual villa, indicating that Palladio apparently made a conscious attempt to idealize the proportions of this ground plan in the *Quattro libri*. In the published plan of the Villa Rotonda, three dimensions are almost identical with those of the actual building, while three are "improved" in the treatise. Conversely, one harmonic measurement in this villa, 25 feet, is recorded in the treatise as 26 feet, a figure which cannot be related to the other dimensions by any musical ratio. Other villas which are more "harmonic" in the published plans are the Villa Badoer (three dimensions "improved") and the Villa Valmarana at Lisiera (five dimensions "improved"). However, this was not a procedure which Palladio carried out consistently. None of the other published plans exhibits more than two dimensions which are "harmonic" in the treatise but not in reality.

32. O. Bertotti Scamozzi, *Le fabbriche e i disegni di Andrea Palladio*, Vicenza, 1776–83, Book III, 48. See also our footnote to Appendix A1.

33. Ackerman, *Palladio*, 167–169.

34. See the entry on this villa by P. Marini in *Palladio e Verona*, exhibition catalogue, Verona, 1980, 243–246.

35. Forssman, *Visible Harmony*, 35.

TABLE 4

Palladio's own comments on room shapes in the text of Book II

	<i>stated room proportions</i>	<i>corresponding ratio</i>	<i>is ratio harmonic?</i>	<i>on plan is it true?</i>	<i>dimensions on plan</i>
Palazzo Barbarano, Vicenza, p. 22.	main rooms: a square & $\frac{1}{2}$ medium rooms: square	3:2 1:1	Yes Yes	Yes Yes	24 × 16 16 × 16
Villa Pisani, Bagnolo, p. 47.	main rooms: a square & $\frac{2}{3}$ medium rooms: square & $\frac{1}{2}$	5:3 3:2	Yes Yes	? Yes	? × 18 24 × 16
Villa Pisani, Montagnana, p. 52.	main rooms: a square & $\frac{3}{4}$ medium rooms: square	7:4 1:1	No Yes	Yes Yes	28 × 16 16 × 16
Villa Cornaro, Piombino, p. 53.	wings with niches: width = $\frac{1}{3}$ length main rooms: square & $\frac{3}{4}$ medium rooms: a square	3:1 7:4 1:1	Yes No Yes	not clear not quite Yes 26 $\frac{1}{2}$ × 16 16 × 16
Villa Saraceno, Finale, p. 56.	main rooms: a square & $\frac{5}{8}$	13:8	No	almost	26 $\frac{1}{2}$ × 16
Villa Poiana, Poiana Magg., p. 58.	main rooms: a square & $\frac{2}{3}$	5:3	Yes	Yes	30 × 18
Un-named palace, Venice	entrance hall: square main rooms: a square & $\frac{2}{3}$ medium rooms: a square & $\frac{1}{2}$ small rooms: a square & $\frac{1}{2}$ <i>sala</i> : a square & $\frac{2}{3}$	1:1 5:3 3:2 3:2 5:3	Yes Yes Yes Yes Yes	Yes Yes Yes Yes can't tell	30 × 30 30 × 18 27 × 18 18 × 12
Villa Mocenigo, Brenta	main <i>sala</i> : two squares & $\frac{1}{2}$	5:2	Yes	almost	76 × 30

The antique reconstructions are not included in this table.

In brief, we can conclude that the ground dimensions of the executed buildings are surprisingly close to those of the plans published in the *Quattro libri*, except where conscious changes to the designs have been made. None of the published plans is less “harmonic” than the building it represents; and a few, most notably the Villa Barbaro at Maser, appear to have been consciously “improved” for publication, in order to make the proportions more harmonic.

IV. AN ALTERNATIVE APPROACH

Specific references to individual room dimensions in the text of Book II of the Quattro libri

We have already noted the paucity of Palladio's own comments about the room dimensions which appear in his plans. The horizontal proportions of the rooms are mentioned in the case of only 8 of Palladio's 37 designs for villas and palaces illustrated in Book II of the *Quattro libri*. Proportions are more fully discussed in the remaining seven of the Book II schemes, those which reconstruct antique domestic architecture, but these comments are entirely based on Vitruvius. For example, the *atria* are 3:5, 2:3, or 1: $\sqrt{2}$, and the peristyles 2:3, exactly as Vitruvius recommends.³⁶

The sparseness of Palladio's comments on the horizontal proportions of his own buildings should not, however, be taken as evidence of a lack of interest on his part. As we have already noted, he makes more frequent references to the heights of the rooms in his text. Moreover, one of his main aims in writing the treatise was brevity, as he stressed in the Preface to Book I: “. . . in tutti questi libri fuggirò la lunghezza delle parole.” Since he rightly believed that the concise writing would be one of the book's great assets, he saw no need to duplicate in the text information already given on the plans. He made this clear in his comments on the Palazzo Valmarana, where he concluded:

I don't need to say any more about this building since, as in the others, I have put the measurements of each part on the drawings. (Book II, p. 16).

Palladio's comments on the horizontal proportions of his rooms are summarized in Table 4. From these it will be clear that the layout of the Palazzo Barbarano is fairly typical:

On the right and left (of the entrance hall) there are two rooms a square and a half long, and beyond them two square rooms, and then two small rooms. (Book II, p. 22).

In this common Palladian solution the main living accommodation consists of two suites of rooms on either side of a central hall, each comprising a large rectangular room, usually at the front of the house, a medium-sized room, often square, beyond, and finally a small room, usually rectangular. In the cases where Palladio notes their proportions, the main rooms are either 5:3

36. Vitruvius, *The Ten Books on Architecture*, Book VI, Ch. III, 177–179.

(Villa Pisani at Bagnolo, Villa Poiana, unnamed Venetian palace), 7:4 (Villa Pisani at Montagnana, Villa Cornaro), 3:2 (Palazzo Barbarano), or 13:8 (Villa Saraceno). In all but two of these instances the measurements on the diagrams show identical proportions to the ratios mentioned in the text. In the two exceptions the correspondence is close or very close. In the Villa Cornaro the main rooms, stated to be a square and three quarters, or 7:4, are in fact $26\frac{1}{2} \times 16$ feet, rather than 28×16 . In the Villa Saraceno the main rooms, here said to be a square and five eighths, or 13:8, are also $26\frac{1}{2} \times 16$, in this case just $\frac{1}{2}$ foot too long.

Two important facts emerge from the examination of Palladio's comments. First, he does not make any remarks about the proportions of the perfectly "harmonic" buildings in Book II, that is, those which possess only dimensions to be found among the harmonic numbers in Table 1. Second, on three occasions when he specifically mentions the proportions of the main rooms, these ratios (7:4 twice, 13:8 once) are not equivalent to any musical harmony. The other proportions mentioned (5:3 or 3:2) are simple ratios recommended by Vitruvius and included in his seven preferred shapes of rooms (quoted on p. 118).

There are two possible ways of interpreting these facts:

- (1) *Either* Palladio saw no need to explain obviously harmonic proportions since the numbers on the plans would be self-explanatory, needing only to justify or explain his more unconventional proportions. (If this is a true deduction, then he would have expected his readers to be familiar with the concepts of harmonic proportion.)
- (2) *Or* Palladio himself attached no particular importance to strictly musical harmonies in the designs to which these comments refer.

The villas with their main rooms $26\frac{1}{2} \times 16$ feet are particularly interesting in this context (Villas Badoer, Cornaro, and Saraceno). This ratio is not "harmonic" in Wittkower's sense, although either 25×16 or 27×16 could be incorporated into a scheme of musical harmonies. The ratio $26\frac{1}{2}:16$ is fairly close to the golden section (25.888:16); as we shall see it is even closer to 5:3 (26.667:16) which is both a harmonic ratio and one of his preferred room shapes. Yet Palladio himself chooses to explain it as either 13:8 or 7:4, neither of which is a musical ratio.

What is obvious from a quick inspection of the plans in Book II is that dimensions of 16 by 24–28 feet are seemingly the most popular for the main living rooms of Palladio's villas (Villas Badoer, Malcontenta, Pisani at Montagnana, Cornaro, Mocenigo, Emo, Saraceno, Valmarana, Godi and Sarego at Miega). The main rooms of the Villa Rotonda are 15×26 feet, which is very similar. Palladio must have found this size and shape to be a practical and visually satisfying one for these very important rooms. Small variations of size surely depended on other constraints imposed by each design as a whole. After all, the eye would not readily distinguish differences of a foot or two (al-

though of course, for reasons of symmetry in each individual villa the two main rooms were of identical size). The ratios mentioned by Palladio in his textual comments seem to be a way of subsequently justifying what he felt to be harmonious, if not strictly "harmonic," proportions. Whether or not he felt precise mathematical relationships to be important in his designs, it seems hardly likely that he would have begun to design a villa with the deliberate intention of giving the main rooms dimensions of 13:8 or 7:4!

Palladio's preferred room shapes in the Book II plans

Statistical methods can be used once again, in order to discover how often Palladio uses his own seven recommended shapes of rooms in the plans in Book II of the *Quattro libri*.³⁷ We can perform two tests in the same analysis. First, we can find out how frequently Palladio chooses room dimensions in the ratios which he himself recommends, i.e., 1:1, $\sqrt{2}:1$, 3:2, 4:3, 5:3, and 2:1. Second, we can find out how often both dimensions of a room are "harmonic" numbers. Obviously there is a large overlap between the two tests, since all Palladio's seven recommended ratios, with the exception of $\sqrt{2}:1$, are harmonic in the musical sense. However, it is possible for the dimensions of individual rooms to accord with Palladio's preferred ratios without being harmonic. For example, a room 19×19 feet is not harmonic, in that it cannot exist in musical proportion to the other rooms around, except in the unlikely event of these being composed in multiples of 19 feet. Yet the room itself, being exactly square, corresponds with one of Palladio's preferred shapes.

As before we set up a strict set of rules and include all rooms satisfying these criteria in all plans. Again we consider only main living rooms, and exclude all porticoes, gardens, loggias, courtyards, stairs, corridors, stables, etc., with the *atria* of the antique reconstructions considered as rooms. This is the set of rooms which would be expected to show the most significant effects if Palladio really adhered to his own recommendations. There are a number of rooms in which one or both dimensions are missing, but this should not greatly affect the overall picture.

We have been strict in accepting only those room dimensions which can be unambiguously identified by means of the numbers appearing on the plans. A few others could be guessed by making assumptions about wall thicknesses, but this is a highly unreliable procedure, and we have excluded all such cases from our analysis. There are a few instances in which Palladio's dimensions appear to be incorrect, either because of distortions in the plan or because of misprinted numbers (e.g., $20\frac{1}{2} \times 17\frac{1}{2}$ in the Palazzo Valmarana, p. 16), but for the sake of consistency we have included these in our analysis. We have tabulated all the

37. This approach is the basis for Battisti's analysis and thoughtful discussion (Battisti, "Un tentativo di analisi strutturale . . .," 211ff).

main room dimensions, counting only once those room dimensions which appear several times on the plan, since the repetition is entirely due to the symmetry of the plan (Table A4). The results of this analysis are then plotted as a histogram in which the ratio of room dimensions is plotted along the x-axis. The width of each box is chosen to be 0.05, centered on the values, 1, 1.05, 1.1, 1.15, 1.2. . . . Numbers such as 1.175 are counted in the box 1.125 to 1.175. This interval provides sufficient resolution to discriminate between the principal preferred ratios, and yet has large enough boxes for ratios very close to the exact preferred ratios to be included in them.

The resulting histogram is shown in Figure 8. The boxes in heavy outlines show those ratios which correspond exactly to the principal harmonic ratios in the just scale, indicated by the arrows above. Palladio's own preferred ratios, i.e., 1:1, 4:3, $\sqrt{2}:1$, 3:2, 5:3, and 2:1 are indicated by arrows below the x-axis. It can immediately be seen that there is strong preference for his recommended ratios, for these are by far the most prominent features of the histogram.

The following points are worth noting:

(1) There are relatively few rooms with ratios corresponding to the ratio $\sqrt{2}:1$, the only one of Palladio's preferred room shapes which is not equivalent to a musical ratio. Despite the fact that Palladio draws attention to this ratio in connection with antique houses, it only appears once in this series of plans, as 28:20 in the Atrio Toscano (Book II, p. 25).

Perhaps the most striking case is that of the Villa Ragona (Book II, p. 57). In this plan one room has the dimensions $21\frac{1}{4} \times 15$ feet, which is within 0.2% of the value of $\sqrt{2}:1$. Only on one other occasion does Palladio use quarters in the dimensions of main rooms, which strongly suggest that the figure $21\frac{1}{4}$ was deliberately chosen in order to give dimensions as close as possible to $\sqrt{2}:1$.

(2) Much of the prominent peak corresponding to 5:3 is due to approximate values. Three of these are associated with the ratio $26\frac{1}{2} \times 16$, which is 1.656. This is within 0.6% of the ratio 5:3, or 1.6666. . . , and is the closest integer or half integer which gives a ratio of 5:3 with 16. The exact ratio would be given by $26\frac{2}{3} \times 16$. Only once does a third of a foot (actually 4 inches) appear in Book II, and on that occasion it is also associated with the ratio 5:3, that is, the ratio $83\frac{1}{3} \times 50$ appearing in the detail of the Atrio Testugginato (Book II, p. 35).

Another interesting case is that of the Palazzo Barbarano (p. 22), where the dimensions of the main hall are $41\frac{1}{2} \times 25$, equivalent to 1.66:1, again very close to 5:3. The remarkable feature of this ratio is that it enables Palladio to place smaller rooms of dimensions 16×16 and 24×16 on either side, with a plausible wall thickness of $1\frac{1}{2}$ feet. This may of course be chance, but it does result in a series of five rooms symmetrically disposed, all of which have preferred Palladian ratios. Nevertheless, the sequence is not strictly harmonic in Wittkower's sense, since $41\frac{1}{2}$

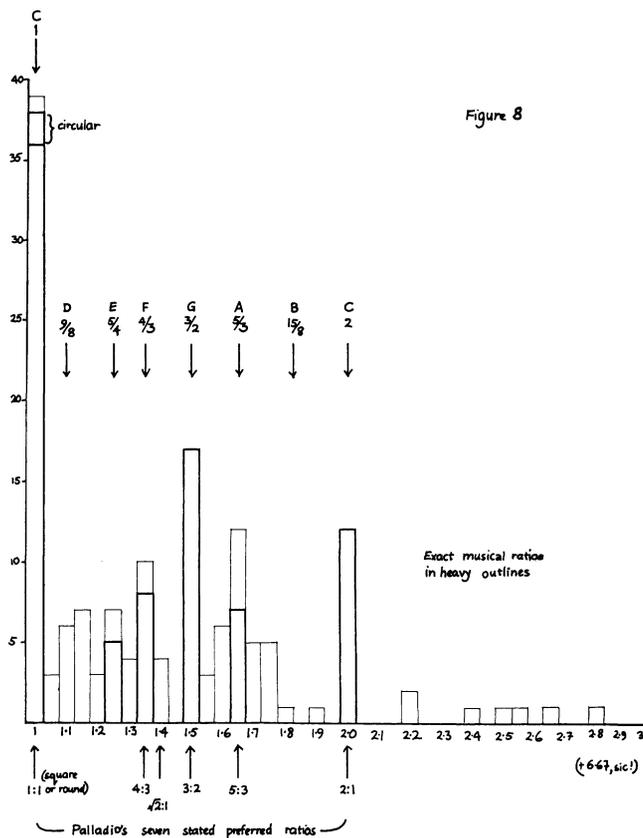


Fig. 8. Histogram showing the frequency of different room shapes in the Book II plans. Each ratio is counted once only in each plan.

cannot have any harmonic connection with 16 or 24 although $41\frac{2}{3}$ would have done so.

If we count only those rooms which have precisely Palladio's preferred ratios, we find that 82 out of 153 are accounted for. We may add to this number those which approximate very closely to his preferred ratios. In that case a total of 93 out of 153, or 61% of the rooms, have proportions very close to his preferred ratios. It is interesting to note how few of the rooms with exact preferred ratios are not measured in harmonic numbers. There are only 9 of these, 6 being in the ratio 1:1, 2 in the ratio 2:1, and one in the ratio 3:2.

This is not at all surprising, of course. All Palladio's preferred ratios contain the numbers, 2, 3, and 5, out of which all the harmonic numbers can be generated. It seems entirely feasible that Palladio could have developed a system of proportion based upon harmonic numbers simply because more often than not he would be able to use room dimensions corresponding to his preferred ratios.

There is, in principle, a way of distinguishing whether Palladio's own preferred proportions were more important to him than harmonic ratios in the choice of room shapes. This is because the harmonic scheme includes other ratios of small numbers which were not mentioned by Palladio in his series of recommended room shapes. For example, if Palladio had been interested

in ensuring that the dimensions of individual rooms were equivalent to musical ratios, then 6:5 and 5:4 should appear as frequently as 5:3 and 4:3. In fact there are only two examples of 6:5 and five of 5:4, compared with eight of 4:3, seventeen of 3:2, and seven of 5:3. Another test is to look at the number of times the major intervals of the just scale are found. If Palladio had harmonic ratios in mind with the degree of subtlety implied by Wittkower, we might expect to see some ratios corresponding to C-D and C-B as well as those of C-E, C-F, C-G, C-A, and C-C. In fact, there are no rooms with dimensions precisely 9:8 or 15:8.

Not a great deal of weight can be given to this evidence, but it does suggest that, overall, Palladio's preferred room shapes were indeed more important to him than simple harmonic ratios.

V. CONCLUSION

The many anomalies and contradictions arising from the foregoing analysis are not entirely surprising. As in so many other respects, the ideas behind the *Quattro libri* remain tantalizingly elusive. Despite the apparently consistent tone of the treatise, its pages betray no coherent underlying system which could have governed Palladio's principles of design. The subtlety and elusiveness of the *Quattro libri* are just two reasons for the enduring fascination of this very remarkable work.

Nevertheless, certain definite conclusions can be drawn from the results of our investigations:

(1) Overall Palladio shows a clear preference for the "harmonic" numbers which we listed in Table 1, that is, those numbers which can be incorporated into the system of musical harmonies expounded by Wittkower. About two-thirds of all the dimensions greater than five in Book II are "harmonic" numbers, whereas in a random sample less than half would be harmonic. This predominance of harmonic numbers cannot be attributed to the presence of a few totally harmonic buildings, but is a general tendency in all his buildings.

However, it should be remembered that all the harmonic numbers are multiples of 2, 3, and 5. The practical advantages to an architect of using easily divisible whole numbers are obvious, especially in the design of symmetrical buildings such as Palladio's villas and palaces.

(2) As well as a preference for harmonic numbers in general, we find a conspicuous group of plans in Book II which are entirely or almost entirely composed in harmonic numbers. That is to say, in these designs all or almost all the dimensions can be related in a sequence of ratios to be found in musical harmonies. We could detect no particular characteristic which distinguished these plans from others, except for the fact that all the completely harmonic plans were those of buildings designed after about 1556 (the year in which Daniele Barbaro's edition of Vitruvius with Palladio's illustrations was published). It is also likely that their patrons shared a definite interest in musical or architectural theory.

(3) The variations in the thicknesses of dividing walls in the Book II plans cannot be attributed to any conscious attempt on Palladio's part to achieve a greater degree of musical harmony in the *Quattro libri*.

(4) Comparison between the measurements of the actual buildings and those recorded in the *Quattro libri* shows that the overall degree of harmony which exists in the buildings themselves is slightly less than that found in the published plans, but nonetheless significantly greater than would be expected by chance. A few plans, in particular the Villa Barbaro at Maser, are noticeably more harmonic in the treatise than in reality. However, in most cases there is little evidence of conscious idealization of the proportions of the actual buildings for the purpose of publication. Moreover, the unexecuted schemes in the treatise are not significantly more harmonic than those which were built.

(5) A clear majority of the main living rooms are given dimensions corresponding to Palladio's seven preferred room shapes. Yet as many as 39% of the principal rooms have different proportions from those he recommends. The fact that certain simple musical ratios are absent or poorly represented on the plans suggests that Palladio tended to use his own recommended ratios in preference to musical harmonic ratios.

In conclusion we should consider the wider implications of these results. In its simplest terms our analysis shows that Palladio had a definite preference for multiples of 2, 3, and 5 in the dimensions which appear in the plans in Book II of the *Quattro libri*. In interpreting these results, various hypotheses could be made, including: (a) that Palladio used a system of musical harmonies, as Wittkower suggests; or (b) that he adhered to his own simpler recommendations concerning room shapes; or (c) that he recognized the practical advantages of using simple, easily divisible numbers. Obviously in the ideas of an architect as sophisticated and flexible as Palladio one should not search for a simple answer.

Nevertheless one can suggest tentative answers to the question. All three hypotheses seem to have governed Palladio's choice of proportions at different times and in different circumstances. Vitruvius alone would have given enough advice on the proportions of domestic buildings to provide Palladio with most of his own preferred room shapes. For the first half of his career these simple ratios could have served as an adequate guide, although he never subordinated the beauty or efficiency of the overall design to rigid proportional rules. The combined effect of using his preferred room ratios and the fact that most of the numbers on the plans are whole numbers would be sufficient to ensure that the plans contain a much higher proportion of harmonic numbers than would be expected by chance. Later, however, the picture changes. It is unlikely that those plans, dating from the late 1550s onwards, in which every dimension can be incorporated into a single system of musical harmonies, could have been designed in this form if Palladio had had no knowledge of the

theory of musical ratios. He could of course have learned of the theory either through his collaboration with Daniele Barbaro or from his close friend Silvio Belli.

Almost certainly Palladio already knew of the theory of harmonic proportion when he began work on the *Quattro libri* in the early 1550s.³⁸ Despite this, our analysis shows that he did not attempt to idealize his designs from all periods to fit the precise demands of the theory. The fact that *even in the published versions* later projects are generally more harmonic than those designed before the treatise was begun suggests that in this respect the process of revision for publication was only carried out to a rather limited extent.

Finally we should stress once again that Book II of Palladio's *Quattro libri* is not a straightforward catalogue of his own buildings, nor is it a work of pure theory. It is something far more subtle and effective. In the book Palladio's own projects provide models more convincing and more adaptable than any theoretical discourse could offer. In the process they are modified, standardized, and removed from the individual surroundings which stamp each real building with its own unmistakable character. Yet they are not transformed into perfect expositions of a single proportional theory. As we have seen, the brief, commonsense

texts which accompany the striking visual images rarely allude to questions of proportion. More often, we are told of the practical conditions which the architect had to satisfy—site, function, patron, disposition of rooms, and so on. At the same time every villa or palace is a work of art in its own right, and the *Quattro libri* another work of art of a very different kind.³⁹ Palladio's intelligence and experience would not have allowed him to suggest that a single proportional theory alone would enable one to design a beautiful building, any more than a musician could compose a great symphony merely with a knowledge of harmony and counterpoint.

Palladio was a gifted communicator, as the enduring popularity of his treatise demonstrated. If he had really intended the *Quattro libri* to serve as a guide to the application of musical theory to architecture, then it is remarkable that he did not make the point more explicitly. Nevertheless, as we have shown in this article, his choice of proportions was far more systematic and deliberate than a quick glance at the *Quattro libri* plans would suggest.

Acknowledgment

We should like to thank Martin Kubelik for his perceptive and constructive criticisms of an earlier draft of this article.

38. On the genesis of the *Quattro libri* see G. G. Zorzi, "La preparazione de 'I Quattro libri' . . .," in *I disegni delle antichità di Andrea Palladio*, Venice, 1959, 145ff., and Burns et al., *Andrea Palladio*, 101–110.

39. It will by now be apparent that we do not fully share the views expressed by Douglas Lewis in his recent criticism of the *Quattro libri* and its influence, published in his exhibition catalogue *The Drawings of Andrea Palladio*, 5–6.

APPENDIX TABLE A I

Dates adopted for *Quattro libri* buildings

Notes

- (i) The dates aim to give the date as close as possible to the start of the building work, if the building was executed, or to the final design.
- (ii) It should be remembered that this is still a subject of much controversy, since few of the dates are securely documented. As a guide to the reliability of the dates we have assigned the following categories:
- A = accurate to within 3 years
B = accurate to within 3–10 years
C = may not be accurate to within 10 years.
- (iii) In the space available only the briefest possible information can be given here.

- (iv) References are given to Puppi's catalogue unless his dates are convincingly altered by later research. For ease of reference, only standard works are cited.

Puppi = L. Puppi, *Andrea Palladio*, 2 vols., Milan, 1973, vol. II (catalogue).

Burns = H. Burns, L. Fairburn, B. Boucher, *Andrea Palladio 1508–1580: The portico and the farmyard*, Arts Council exh. cat., London, 1975.

Marini = P. Marini et al., *Palladio e Verona*, exh. cat., Verona, 1980.

Lewis = D. Lewis, *The Drawings of Andrea Palladio*, exh. cat., Washington, D.C., 1981.

C.P. = *Corpus Palladianum*, English editions, University Park, Pa. & London.

Page in QL	Name of building	Location	Suggested date of final design	Category	Evidence for date given	Source
5	Pal. Antonini	Udine	1556	B	Palladio's visit to Udine in 1556.	Puppi, 306.
6	Pal. Chiericati	Vicenza	1550	A	Documented date of final design.	Puppi, 281–282.
8	Pal. Iseppo Porto	Vicenza	c. 1543?	C	Iseppo Porto's marriage, c. 1542. Façade inscription 1552.	Forssman, C.P., VIII, 13 & 23.
11	Pal. della Torre	Verona	c. 1555	C	B. Ridolfi, stuccoist, left for Poland 1563.	Marini, 232–233.
13	Pal. Thiene	Vicenza	1542–6	B	First contract with builders 1542. Palladio a witness only.	Puppi, 251–254.
16	Pal. Valmarana	Vicenza	1565	A	Building contract 1565. Foundation stone laid 1566.	Puppi, 369–371.

18	Villa Almerico (Rotonda)	Vicenza	c. 1566	A	Villa inhabited by 1569. Built rapidly.	Puppi, 381-382; Burns, 198.
21	Pal. Capra	Vicenza	1563-4?	C	Stylistic evidence only. Not executed.	Puppi, 349-350.
22	Pal. Barbarano	Vicenza	1570	A	Foundation medal 1570. Design revised just before publication of <i>Quattro libri</i>	Puppi, 393-395.
30	Convento della Carità	Venice	1560	A	Work had been in progress for one year in 1561.	Bassi, <i>C.P.</i> , VI, 39.
47	Villa Pisani	Bagnolo	1541-2	A	Patron came of age 1541, married 1542. Villa complete 1544.	Puppi, 254-257.
48	Villa Badoer	Fratta Polesine	c. 1557	B	Site prepared 1556. Villa complete 1566.	Puppi, <i>C.P.</i> , VII, 27 & 35.
49	Villa Zeno	Cesalto	c. 1559	C	Patron Podestà of Vicenza in 1559. Villa existed by 1566.	Puppi, 374; Burns, 190-191.
50	Villa Foscari (Malcontenta)	Mira	early 1550s?	B	Must predate death of N. Foscari in 1560 and of fresco painter Battista Franco in 1561.	Lewis, 149-150; Puppi, 328-330. ⁺
51	Villa Barbaro	Maser	c. 1557-8	B	Probably follows Vitruvius ed. 1556. Part of villa existed by 1558.	Puppi, 314-318.
52	Villa Pisani	Montagnana	1552	A	Last part of site acquired 1552. Close to completion 1553.	Puppi, 288-289; Lewis, 166.
53	Villa Cornaro	Piombino Dese	1551	A	Patron inherited site 1551. Bdg. work documented 1552-3.	Burns, 194.
54	Villa Mocenigo	Marocco	1562	A	Design not settled 1561-2. Inscribed tablet 1562.	Lewis, 100; Burns, 223.
55	Villa Emo	Fanzolo	c. 1564	C	Early maturity of patron between 1554 and 1567.	Puppi, 352-353.
56	Villa Saraceno	Finale	before 1545	B	Villa already existed in 1546.	Puppi, 258-259.
57	Villa Ragona	Ghizzole	1553-5?	C	Stylistic evidence only.	Puppi, 295.
58	Villa Poiana	Poiana Maggiore	1548-9	B	Patron had land at Poiana from 1547. Villa documented as not yet finished in 1555.	Puppi, 274.
59	Villa Valmarana	Lisiera	1563-4?	C	Mentioned by Vasari, who met Palladio in 1566.	Puppi, 350-351.
60	Villa Trissino	Meledo	c. 1566	B	Vasari refers to villa as begun.	Puppi, 385-388.
61	Villa Repeta	Campiglia	c. 1557	B	Patron built villa in memory of father, who died 1556. Vasari refers to it as under construction.	Puppi, 318.
62	Villa Thiene	Cicogna	1556-63	C	Francesco Thiene, named as one of patrons in <i>Quattro libri</i> , died 1556. Construction in progress by 1563.	Puppi, 311-312.
63	Villa Angarano	Angarano	1548	B	Palladio visited Angarano in 1548.	Puppi, 272-273.
64	Villa Thiene	Quinto	1545-6	B	One of brothers left Vicenza 1547.	Puppi, 262; Lewis, 93.
65	Villa Godi	Lonedo	1537	B	Patron inherited property 1536. Villa complete by 1542 (inscription on portico).	Puppi, 238-240.
67	Villa Sarego	Sta. Sofia	1565	B	Marc'Antonio Sarego, named as patron, inherited villa 1552. Payments for substantial building work 1565-9.	Marini, 249*

⁺ Possible confirmation of the suggested early date of the Villa Malcontenta is afforded by the result of dating brick samples by thermoluminescence (see C. Goedicke, K. Slusallek and M. Kubelik "Thermoluminescence dating in architectural history: Venetian villas," *JSAH*, XI, 1981, 207-208).

* We do not view this date as incompatible with the results of dating of the building by thermoluminescence. One of the brick samples taken from the villa gave a date of about 1384, confirming the existence of an older building on the site (Goedicke et al., 210-212). The three other samples yielded a date of about 1534, which would seem to indicate that renovation or extension of the villa occurred at around this time. In his will of 1536 the then owner, Brunoro Sarego, expressed his wish that the building be finished (Marini, 249), suggesting that work was either in progress or planned in the 1530s. There seems to be no need to involve Palladio in this phase of the work since considerable building activity is documented at the villa in the late 1560s (Marini, 249), and the style of the existing façade and of the *Quattro libri* project are characteristic of Palladio's late work. The interior disposition of the rooms in the present building is quite different from that illustrated in the *Quattro libri* plan, suggesting that Palladio's contribution was mainly confined to the portico area.

68	Villa Sarego	La Miega	c. 1562	B	Annibale Sarego inherited property 1552. Payment to Palladio 1562, probably for design.	Marini, 244.
74	Pal. Trissino	Vicenza	c. 1558	C	One of patrons in contact with Palladio 1558–9 over work on Duomo, Vicenza. Patrons already owned site 1559.	Puppi, 323–324.
75	Pal Angarano	Vicenza	1564	B	1565 house was “ruinata per fabricarla.” (Not executed)	Puppi, 354.
76	Pal. della Torre	Verona	c. 1565	B	Site mapped by Sorte 1563. Patron died 1568.	Marini, 235–236.
77	Pal. Garzadore	Vicenza	1555–6?	C	Stylistic evidence only.	Puppi, 304.
“66”	Villa Mocenigo	Brenta	1561	A	Start of building work documented 1561. Materials assembled 1554 onwards.	Lewis, 100.

APPENDIX TABLE A2

Dimensions appearing on the plans in Book II of Palladio's *Quattro libri dell'architettura*. The dimensions of main rooms are set in boldface type. An asterisk beside a number indicates that it is a harmonic number.

Palaces

p. 5	Pal. Antonini, Udine	6* 8* 11 12^{1/4} 12^{1/2}* 17 24* 28 32*
p. 6	Pal. Chiericati, Vicenza	11 ^{1/4} * 12* 13 15 ^{1/2} 16* 18* 19 30* 54*
p. 8	Pal. Porto, Vicenza	7 9* 10* 20* 30*
p. 11	Pal. della Torre, Verona	10* 11 15* 17 18* 19 22^{1/2}* 30* 32* 34 50*
p. 13	Pal. Thiene, Vicenza	12* 20* 30* 34^{1/2} 55 56 74 102
p. 16	Pal. Valmarana, Vicenza	8^{1/2} 10* 14 15* 17^{1/2} 18* 19 20* 20^{1/2} 35 36* 38 44 60* 120*
p. 19	Villa Rotonda, Vicenza	6* 11 12* 15* 26 30*
p. 21	Pal. Capra, Vicenza	13 17 24* 27* 32*
p. 22	Pal. Barbarano, Vicenza	7 8^{1/2} 9* 11 12* 16* 23 24* 25* 33 41^{1/2}

Antique reconstructions

p. 25	Atrio Toscano	7 8* 9* 11 18* 20* 25* 28 30* 38 40* 60* 66 85
p. 26	Atrio Toscano (detail)	6 ^{5/6} 18* 19 45* 67^{1/2}
p. 28	Atrio di 4 colonne	5 ^{1/2} 16* 17^{1/2} 35 52^{1/2}
p. 30	Convento della Carità, Venice	8* 9 ^{1/2} 13 14^{1/2} 26 30* 40* 54* 63 78
p. 34	Atrio Testugginato	6* 12* 18* 20* 21 27* 36* 48* 72* 99
p. 35	Atrio Testugginato (detail)	20* 21 22^{1/2}* 50* 83^{1/3}*

Villas for Venetian patrons

p. 47	Villa Pisani, Bagnolo	8* 12 ^{1/2} * 16* 18* 19 22 24* 32* 42
p. 48	Villa Badoer, Fratta Polesine	8* 10* 12* 14 16* 20* 26^{1/2} 32* 34
p. 49	Villa Zenò, Cesalto	12* 14 20* 21^{1/2} 29
p. 50	Villa Foscari (Malcontenta), Mira	6* 7 12* 16* 24* 32* 46^{1/2}
p. 51	Villa Barbaro, Maser	6* 8* 9* 10* 12* 14 16* 18* 20* 32* 46 60*
p. 52	Villa Pisani, Montagnana	8^{1/2} 10* 11 ^{1/4} * 16* 26 28
p. 53	Villa Cornaro, Piombino	10* 11 16* 24* 26^{1/2} 27^{1/4} 32*

p. 54	Villa Mocenigo, Marocco	10* 16* 20* 26 32*
p. 55	Villa Emo, Fanzolo	9* 12* 15* 16* 20* 24* 27* 48*

Villas for Terraferma patrons

p. 56	Villa Saraceno, Finale	12* 15* 16* 26^{1/2} 28 30*
p. 57	Villa Ragona, Ghizzole	12* 15* 17 18* 21^{1/4} 50*
p. 58	Villa Poiana, Poiana Maggiore	8* 10* 15* 17 18* 20* 30* 36*
p. 59	Villa Valmarana, Lisiera	6 ^{1/2} 7 12* 15* 17 25* 27* 32^{1/2} 45*
p. 60	Villa Trissino, Meledo	8* 12* 14 15* 18* 20* 30* 36*
p. 61	Villa Repeta, Campiglia	8* 10* 18* 20* 115 214
p. 62	Villa Thiene, Cicogna	12* 16* 18* 36*
p. 63	Villa Angarano, Angarano	10* 13 15* 18* 20* 22 36* 44
p. 64	Villa Thiene, Quinto	6 ^{1/2} 11 12 ^{1/2} * 14 ^{1/2} 17 ^{1/2} 18* 20* 28 30* 41^{1/2} 44 56 68 102^{1/3} 130
p. 65	Villa Godi, Lonedo	10* 12* 13 ^{1/2} 16* 20* 21 24* 32* 36* 56 60* 100*
p. 67	Villa Sarego, Santa Sofia	10* 12* 18* 20* 24* 36*
p. 68	Villa Sarego, Miega	6 ^{1/2} 8* 9* 10* 12* 15* 16* 20* 24* 27* 40*

Another antique reconstruction

p. 70	Villa of the ancients	6* 7 8* 10* 16* 19 25* 40* 44^{1/2} 60* 73 111 ^{1/4}
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Inventions for various sites

p. 71	Venetian palace	7 12* 14 14^{3/4} 16* 18* 27* 30* 30 ^{1/2}
p. 72	Venetian palace	5 ^{1/2} 8* 10* 11 16* 20* 24* 30* 34^{1/2} 37^{1/2}* 40* 45*
p. 74	Pal. Trissino, Vicenza	7 8* 10* 12 ^{1/2} * 14 15* 18* 20* 23 40* 46 62
p. 75	Pal. Angarano, Vicenza	12* 18* 30* 50*
p. 76	Pal. della Torre, Verona	15* 20* 36* 40*
p. 77	Pal. Garzadore, Vicenza	8* 12* 16* 18^{1/2} 20* 28^{1/2}
p. “66”	Villa Mocenigo, Brenta	6* 8* 15* 16* 20* 24* 30* 40* 59 75* 76

APPENDIX TABLE A3
Fractions and percentages of harmonic numbers in the plans in Book II
of Palladio's *Quattro libri*

	<i>Palaces</i>	<i>All dimensions</i>		<i>Main rooms only</i>	
p. 5	Palazzo Antonini, Udine	$\frac{5}{9}$	56%	$\frac{4}{7}$	57%
p. 6	Palazzo Chiericati, Vicenza	$\frac{6}{9}$	67%	$\frac{4}{4}$	100%
p. 8	Palazzo Porto, Vicenza	$\frac{4}{5}$	80%	$\frac{3}{4}$	75%
p. 11	Palazzo della Torre, Verona	$\frac{7}{11}$	64%	$\frac{4}{7}$	57%
p. 13	Palazzo Thiene, Vicenza	$\frac{3}{8}$	38%	$\frac{2}{5}$	40%
p. 16	Palazzo Valmarana, Vicenza	$\frac{7}{15}$	47%	$\frac{3}{8}$	38%
p. 19	Villa Rotonda, Vicenza	$\frac{4}{6}$	67%	$\frac{2}{4}$	50%
p. 21	Palazzo Capra, Vicenza	$\frac{3}{5}$	60%	$\frac{2}{4}$	50%
p. 22	Palazzo Barbarano, Vicenza	$\frac{5}{11}$	45%	$\frac{4}{6}$	67%
<i>Antique reconstructions</i>					
p. 25	Atrio Toscano	$\frac{8}{14}$	57%	$\frac{6}{9}$	67%
p. 26	Atrio Toscano (detail)	$\frac{2}{5}$	40%	$\frac{2}{3}$	67%
p. 28	Atrio di 4 colonne	$\frac{1}{5}$	20%	$\frac{1}{4}$	25%
p. 30	Convento della Carità, Venice	$\frac{4}{10}$	40%	$\frac{3}{5}$	60%
p. 34	Atrio Testugginato	$\frac{8}{10}$	80%	$\frac{6}{7}$	86%
p. 35	Atrio Testugginato (detail)	$\frac{4}{5}$	80%	$\frac{3}{4}$	75%
<i>Villas for Venetian Patrons</i>					
p. 47	Villa Pisani, Bagnolo	$\frac{6}{9}$	67%	$\frac{5}{6}$	83%
p. 48	Villa Badoer, Fratta Polesine	$\frac{6}{9}$	67%	$\frac{3}{4}$	75%
p. 49	Villa Zeno, Cesalto	$\frac{2}{5}$	40%	$\frac{1}{3}$	33%
p. 50	Villa Foscari (Malcontenta)	$\frac{5}{7}$	71%	$\frac{4}{6}$	67%
p. 51	Villa Barbaro, Maser	$\frac{10}{12}$	83%	$\frac{5}{6}$	83%
p. 52	Villa Pisani, Montagnana	$\frac{3}{6}$	50%	$\frac{1}{3}$	33%
p. 53	Villa Cornaro, Piombino	$\frac{4}{7}$	57%	$\frac{3}{5}$	60%
p. 54	Villa Mocenigo, Marocco	$\frac{4}{5}$	80%	$\frac{3}{4}$	75%
p. 55	Villa Emo, Fanzolo	$\frac{8}{8}$	100%	$\frac{2}{2}$	100%
<i>Villas for Terraferma patrons</i>					
p. 56	Villa Saraceno, Finale	$\frac{4}{6}$	67%	$\frac{2}{4}$	50%
p. 57	Villa Ragona, Ghizzole	$\frac{4}{6}$	67%	$\frac{3}{5}$	60%
p. 58	Villa Poiana, Poiana Maggiore	$\frac{7}{8}$	88%	$\frac{3}{4}$	75%
p. 59	Villa Valmarana, Lisiera	$\frac{5}{9}$	56%	$\frac{4}{6}$	67%
p. 60	Villa Trissino, Meledo	$\frac{7}{8}$	88%	$\frac{4}{5}$	80%
p. 61	Villa Repeta, Campiglia	$\frac{4}{6}$	67%	$\frac{1}{1}$	100%
p. 62	Villa Thiene, Cicogna	$\frac{4}{4}$	100%	$\frac{3}{3}$	100%
p. 63	Villa Angarano, Angarano	$\frac{5}{8}$	63%	$\frac{3}{5}$	60%
p. 64	Villa Thiene, Quinto	$\frac{4}{15}$	27%	$\frac{1}{3}$	33%
p. 65	Villa Godi, Lonedo	$\frac{9}{12}$	75%	$\frac{3}{3}$	100%
p. 67	Villa Sarego, Santa Sofia	$\frac{6}{6}$	100%	$\frac{3}{3}$	100%
p. 68	Villa Sarego, Miega	$\frac{10}{11}$	91%	$\frac{7}{7}$	100%
<i>Another antique project</i>					
p. 70	Villa of the ancients	$\frac{7}{12}$	58%	$\frac{5}{7}$	71%
<i>Inventions for various sites</i>					
p. 71	Venetian palace	$\frac{5}{9}$	56%	$\frac{5}{7}$	71%
p. 72	Venetian palace	$\frac{9}{12}$	75%	$\frac{8}{9}$	89%
p. 74	Palazzo Trissino, Vicenza	$\frac{7}{12}$	58%	$\frac{6}{8}$	75%
p. 75	Palazzo Angarano, Vicenza	$\frac{4}{4}$	100%	$\frac{4}{4}$	100%
p. 76	Palazzo della Torre, Verona	$\frac{4}{4}$	100%	$\frac{3}{3}$	100%
p. 77	Palazzo Garzadore, Vicenza	$\frac{4}{6}$	67%	$\frac{4}{6}$	67%
p. "66"	Villa Mocenigo, Brenta	$\frac{9}{11}$	82%	$\frac{4}{5}$	80%

APPENDIX TABLE A4

List of the sizes and proportions of the main rooms in the plans in Book II of the *Quattro libri*.
Each room size is counted only once.

Page	Name of Building	Measurable room sizes	Both dimensions of room harmonic	Equivalent to one of Palladio's preferred ratios	Very close
p. 5	Palazzo Antonini, Udine	17×17		1:1	
		24×17			$\sqrt{2}:1$
		28×17			5:3
		32×28			
		12 $\frac{1}{2}$ ×8	Yes		
		12 $\frac{1}{2}$ ×12 $\frac{1}{4}$			1:1
p. 6.	Palazzo Chiericati, Vicenza	18×12	Yes	3:2	
		18×18	Yes	1:1	
		30×18	Yes	5:3	
p. 8	Palazzo Porto, Vicenza	30×20	Yes	3:2	
		20×20	Yes	1:1	
		20×9	Yes		
		9×7			
		30×30	Yes	1:1	
p. 11	Palazzo della Torre, Verona	19×19		1:1	
		19×11			
		19×15			
		30×19			
		22 $\frac{1}{2}$ ×18	Yes		
		19×17			
p. 13	Palazzo Thiene, Vicenza	20×20	Yes	1:1	
		56×20			
		34 $\frac{1}{2}$ ×20			
		30×20	Yes	3:2	
p. 16	Palazzo Valmarana, Vicenza	20 $\frac{1}{2}$ ×17 $\frac{1}{2}$			
		18×15	Yes		
		38×19		2:1	
p. 19	Villa Rotonda, Vicenza	26×15			
		15×11 circle diameter 30	Yes	circular	
p. 21	Palazzo Capra, Vicenza	27×17			
		13×13		1:1	
		32×32	Yes	1:1	
p. 22	Palazzo Barbarano, Vicenza	12×7			
		41 $\frac{1}{2}$ ×25			5:3
		24×16	Yes	3:2	
		16×16	Yes	1:1	
		16×12	Yes	4:3	
p. 25	Atrio Toscano	20×20	Yes	1:1	
		60×40	Yes	3:2	
		25×20	Yes		
		38×30			
		28×20			$\sqrt{2}:1$
p. 26	Atrio Toscano (detail)	67 $\frac{1}{2}$ ×45	Yes	3:2	
p. 28	Atrio di 4 colonne	17 $\frac{1}{2}$ ×16			
		52 $\frac{1}{2}$ ×35		3:2	
p. 30	Convento della Carità, Venice	26×26		1:1	
		54×40			4:3
p. 34	Atrio Testugginato	27×18	Yes	3:2	
		36×18	Yes	2:1	
		21×18			
		20×18	Yes		
		36×36	Yes	1:1	
		72×36	Yes	2:1	

<i>Page</i>	<i>Name of Building</i>	<i>Measurable room sizes</i>	<i>Both dimensions of room harmonic</i>	<i>Equivalent to one of Palladio's preferred ratios</i>	<i>Very close</i>
p. 35	Atrio Testugginato (detail)	21×20 83 ¹ / ₃ ×50	Yes	5:3	
p. 47	Villa Pisani, Bagnolo	16×16 24×16	Yes Yes	1:1 3:2	
p. 48	Villa Badoer, Fratta Polesine	26 ¹ / ₂ ×16 16×16 32×16 16×8	Yes Yes Yes Yes	1:1 2:1 2:1	5:3
p. 49	Villa Zeno, Cesalto	21 ¹ / ₂ ×14 14×12 14×14		1:1	
p. 50	Villa Malcontenta (Foscari)	16×12 16×16	Yes Yes	4:3 1:1	
p. 51	Villa Barbaro, Maser	20×9 20×18 20×12 12×6	Yes Yes Yes Yes	5:3 2:1	
p. 52	Villa Pisani, Montagnana	28×16 16×16 16×8 ¹ / ₂ 28×28	Yes	1:1 1:1	
p. 53	Villa Cornaro, Piombino	32×27 ¹ / ₄ 26 ¹ / ₂ ×16 16×10	Yes		5:3
p. 54	Villa Mocenigo, Marocco	16×10 16×16 26×16 32×32	Yes Yes Yes	1:1 1:1	
p. 55	Villa Emo, Fanzolo	16×16 27×16 27×27	Yes Yes Yes	1:1 1:1	
p. 56	Villa Finale, Saraceno	26 ¹ / ₂ ×16 16×12	Yes	4:3	5:3
p. 57	Villa Ragona, Ghizzole	18×17 15×12 21 ¹ / ₄ ×15	Yes		√2:1
p. 58	Villa Poiana, Poiana Maggiore	30×17 18×17 30×18 30×30	Yes Yes	5:3 1:1	
p. 59	Villa Valmarana, Lisiera	12×12 15×15 15×12 27×17 32 ¹ / ₂ ×25	Yes Yes Yes	1:1 1:1	
p. 60	Villa Trissino, Meledo	circle diameter 36 18×14 18×12	Yes Yes	circular 3:2	
p. 61	Villa Repeta, Campiglia	no rooms with two dimensions marked			
p. 62	Villa Thiene, Cicogna	18×18 36×36	Yes Yes	1:1 1:1	
p. 63	Villa Angarano, Angarano	13×10 22×18 36×18	Yes	2:1	
p. 64	Villa Thiene, Quinto	no rooms with two dimensions marked			
p. 65	Villa Godi, Lonedo	24×16 36×24	Yes Yes	3:2 3:2	

<i>Page</i>	<i>Name of Building</i>	<i>Measurable room sizes</i>	<i>Both dimensions of room harmonic</i>	<i>Equivalent to one of Palladio's preferred ratios</i>	<i>Very close</i>
p. 67	Villa Sarego, Sta. Sofia	24×24 24×10 24×18	Yes Yes Yes	1:1 4:3	
p. 68	Villa Sarego, Miega	40×20 24×9 16×12 16×16 27×16	Yes Yes Yes Yes Yes	2:1 4:3 1:1	
p. 70	Villa of the Ancients	44 ^{1/2} ×44 ^{1/2} 6 (sic!)×40 40×40	 Yes Yes	 1:1 1:1	
p. 71	Venetian palace	14×7 30×18 30×30 27×18 18×12	 Yes Yes Yes Yes	 2:1 5:3 1:1 3:2 3:2	
p. 72	Venetian palace	34 ^{1/2} ×20 20×20 30×20 20×16 20×8 45×37 ^{1/2}	 Yes Yes Yes Yes Yes	 1:1 3:2	
p. 74	Palazzo Trissino, Vicenza	23×20 20×18 20×15 14×8 40×20	 Yes Yes Yes	 4:3 2:1	
p. 75	Palazzo Angarano, Vicenza	30×18 50×30 50×50 18×12	Yes Yes Yes Yes	5:3 5:3 1:1 3:2	
p. 76	Palazzo della Torre, Verona	20×20 20×15 36×20	Yes Yes Yes	1:1 4:3	
p. 77	Palazzo Garzadore, Vicenza	20×18 ^{1/2} 18 ^{1/2} ×16 18 ^{1/2} ×12 16×8 28 ^{1/2} ×20	 Yes	 2:1	
p. "66"	Villa Mocenigo, Brenta	40×20 20×15 30×20 76×30 20×20	Yes Yes Yes Yes	2:1 4:3 3:2 1:1	$\sqrt{2}:1$