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Indeterminist time and truth-value gaps[1]

by

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1.

Owing primarily to the efforts of A. N. Prior, the theory of tenses is now a well-established branch of symbolic logic. In this theory the future and past tenses are treated like modal operators. Where A is a formula of a formalized language L with future-tense operator F and past-tense operator P, the formulas FA and PA will translate the future and past tenses, respectively, of a sentence of natural language translated by A.[2] Many of the techniques developed in the study of modal logic have been used with good effect in tense logic, especially semantic or model-theoretic techniques involving assignments of truth-values to formulas in a variety of "possible worlds" or "points of reference."[3]

In tense logic, of course, these points of reference are times. For our semantic purposes in this paper, all we need to know about these times is the relation $<$ which orders them. The fundamental semantic idea is that the formulas of a given formal language L will be assigned truth-values which may differ from time to time, and that the truth-values of tensed formulas at a given time

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[2] I will adhere as closely as possible to the terminology and notation of Thomason [5]. 'A,' 'B,' etc. are metavariables ranging over formulas, 'P,' 'Q,' etc. over sentence variables. This means that we will use italic 'P' for sentence variables and Roman 'P' for the past tense operator, and will thus have formulas such as PP. But this should cause no confusion.

[3] See Hintikka [1] and Montague [2] for accounts of this idea and references to further accounts.

will depend on the truth-values taken by that formula at other times. The components of a tense-theoretic model needed to achieve a full assignment of truth-values to formulas are thus: (1) a set $\mathcal{K}$ of *times*, ordered by a relation $<$, and (2) a valuation V which assigns truth-values $V_\alpha(A)$ to formulas A of L, for each $\alpha \in \mathcal{K}$. We will call such a set $\mathcal{K}$, together with an ordering relation $<$, a *model structure*. Such a structure will serve to represent the underlying form of time.

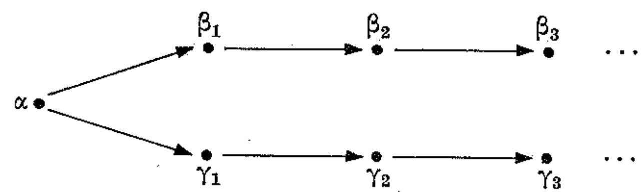
Two factors influence validity on this account: the properties possessed by model structures and the way in which valuations V are supposed to act on complex formulas. It has generally been assumed in tense logic that the temporal ordering $<$ of a model structure must be linear; this means that given any times α and β, if $\alpha \neq \beta$ then $\alpha < \beta$ or $\beta < \alpha$. However, for many philosophical (and perhaps even some scientific) purposes it is more interesting to consider the case in which time may be nonlinear. Such an account of time will permit instances in which a time α has alternative possible futures. The linear conception of time can countenance "alternative futures" as epistemic possibilities; although according to this conception each time α can have only one possible future, we may not have full knowledge at α of what the future will be like so that for all we know there are many alternative futures for α. These alternatives, however, being generated by incomplete knowledge rather than by the nature of time itself, do not enter into the truth-conditions for tenses. But nonlinear time puts these alternatives into the ontological structure of time, so that they must be taken into account in reckoning the truth-values of tensed formulas. We will call temporal model structures of this kind *indeterministic* model structures.

This paper will consider the problem of working out a semantic theory of tenses for indeterministic model structures. Its principal contribution is in developing a rigorous form of the traditionally popular view that "future contingent" statements can be neither true nor false.

2.

In dealing with nonlinear model structures it is natural to suppose that $<$ is a "treelike" ordering: i.e. for all α, β, $\gamma \in \mathcal{K}$ if $\beta < \alpha$ and $\gamma < \alpha$, then $\beta < \gamma$ or $\gamma < \beta$ if $\beta \neq \gamma$. The reason for this condition is that it would be highly counterintuitive to suppose that times α and β could be identical unless they shared a unique past. Their having different pasts would make them different times, for different "alternative pasts" can arise only through gaps in our knowledge about the past. We will also posit, as usual in tense logic, that $<$ must be transitive: if $\alpha < \beta$ and $\beta < \gamma$ then $\alpha < \gamma$.

The chief problem arising in indeterministic tense logic is that of finding truth-conditions for the future tense: how are we to define $V_\alpha(FA)$?[4] In linear model strucutures, the obvious defining condition is that $V_\alpha(FA) = T$ (i.e., V gives the value *true* to FA at α) if $V_\beta(A) = T$ for some β such that $\alpha < \beta$, but in indeterministic structures this yields unacceptable results. For instance, it renders $\{\sim FP, \sim PP, \sim P, PFP\}$ simultaneously satisfiable. It can be true to say that a thing is never so, while at the same time it was true to say that it will be true. To render every formula in this set true, take a model structure arranged as follows,



and let P be false in α and all the β_i, but true in all the γ_i. Then in β_1 all the members of the set are true.

This absurdity could be put more strikingly in a language equipped with metric tense operators: e.g. an operator Fh for 'will be the case one hour hence.' Then if we say that $V_\alpha(FhA) = T$ if $V_\beta(A) = T$ for some β one hour after α, FhP and $Fh \sim P$ can be made true at once in an indeterministic model structure.

Responding to this, one might think that FA should be true at α if A is true at some time in every possible future for α. This can be made rigorous by letting a (possible) *history* of a model structure with a set $\mathcal{K}$ of times and ordering $<$ be a linear pathway through the structure. A history for the model structure is a subset h of $\mathcal{K}$ such that (1) for all $\alpha, \beta \in h$, if $\alpha \neq \beta$ then $\alpha < \beta$ or $\beta < \alpha$ and (2) if g is any subset of $\mathcal{K}$ such that for all $\alpha, \beta \in g$, if $\alpha \neq \beta$ then $\alpha < \beta$ or $\beta < \alpha$, then $g = h$ if $h \subseteq g$. (Mathematicians would call such histories *maximal chains* on the model structure.) Where α is a member of $\mathcal{K}$, let $\mathcal{H}_\alpha$ be the set of histories containing α. Now, where h is a member of $\mathcal{H}_\alpha$, the segment of h beyond α corresponds to a *possible future* for α. Therefore, according to the truth-condition now under consideration,

(2.1) $V_\alpha(FA) = T$ if for all $h \in \mathcal{H}_\alpha$, $V_\beta(A) = T$ for some $\beta \in h$ such that $\alpha < \beta$.

This proposal, discussed in Prior [3] as the "Peircian" theory,[5] again does not represent very accurately the English future tense. Here the trouble is that $FP \vee F{\sim}P$ is invalid. (It will fail to be true at any time which is located on histories g and h such that P is true in all $\beta \in h$ such that $\alpha < \beta$, and false in all $\beta \in g$ such that $\alpha < \beta$.) But 'It will or it won't' has the force of tautology. It is invariably true to say things such as 'Either it will rain tomorrow or it won't,' even in cases where there is no more justification for saying it will than for saying it won't rain.

A further anomaly of this proposal is that PFP would not be a semantic consequence of P. (Even though P is true at a time α, it

[4] This problem can also spill over in a derivative way to the past tense. But if we can find a solution to the problem of the future tense, the past will take care of itself.

[5] The indeterministic tense logics discussed by Prior in [3] differ from ours in that their tense operators are metric. I have used the unadorned past and future tenses in my formal languages because I do not believe metric tense logics to correspond very well to natural language. Although there are many temporal locations in natural language which involve metric considerations, I feel these most often are best represented by pragmatic restrictions which confine the domain of the simple tense operators to parts of a larger model structure.

may still be the case that every time prior to α has a possible future in which P is never true.) Again, this clashes with common sense; arguments such as 'There is space travel; therefore it was the case that space travel would come about.' strike us as valid on logical grounds.

3.

These two semantic characterizations of the future tense in indeterministic model structures are perhaps the first that come to mind, yet both are defective. Our last suggestion, that P*FA* should be a consequence of *A*, is a good place to begin seeking a remedy. Why does this entailment hold?

Suppose that $\alpha<\beta$. From the perspective of α, β may well be located in only some of many possible alternative futures, all of which are equally likely. But from the perspective of β, it is apparent that many of these alternative futures for α have not been realized: in particular, all those that do not include β. From this later perspective all these other alternatives have been nullified by the course of events. Therefore (still from the perspective of β) if *A* is true then *A must* be true at some time subsequent to α (by the time we've reached β we have verified this), and any reasonable semantic theory must make F*A* true at α.

This suggests that, rather than making formulas true or false with respect only to the times at which they are true or false, we make their being true or false relative to subsequent times as well. Where $\alpha<\beta$, $V_\alpha^\beta(A)$ will be the value T if *A* is true in α from the future perspective β. We define this notion by the following conditions.

(3.1) $V_\alpha^\beta(A)=V_\alpha(A)$ if A contains no tense operators.
$V_\alpha^\beta(\mathrm{F}A)=\mathrm{T}$ if $V_\alpha(A)=\mathrm{T}$ for some γ such that $\alpha<\gamma\leq\beta$.
$V_\alpha^\beta(\mathrm{F}A)=\mathrm{F}$ otherwise.

I do not propose to take this seriously as a theory of the future tense, because it doesn't really solve any of the problems with which we have been concerned; it merely postpones them. The trouble here is with iterated future tenses. In evaluating a formula such as FF*P*, we know that $V_\alpha^\beta(\mathrm{FF}P)=\mathrm{T}$ if $V_\gamma(\mathrm{F}P)=\mathrm{T}$ for some γ such that $\alpha<\gamma\leq\beta$. But it was the calculation of truth-values such as $V_\gamma(\mathrm{F}P)$ which led us in the first place to make the future tense true relative to a future perspective.

Although this is a crippling objection it should not make us lose sight of the good points of our latest idea. Its most outstanding virtue is that it successfully represents the way we do evaluate future-tense statements, by waiting to see whether or not they are fulfilled.[6] Rather than dismissing the idea, then, we'll seek to repair this difficulty. The source of the trouble seems to be that in evaluating the truth at α of formulas F*A* from a perspective β in the future of α, we have placed an upper limit on the times at which *A* can be fulfilled: it is illegitimate to consider times after β. But iterated future tenses suggest that we should take these times into consideration too.

This problem can be remedied by adopting a whole possible future for α as our perspective, rather than a single time in the future of α. To say that F*A* is true in α relative to a particular possible future of α is to say that *A* is realized at some time in this possible future. This idea can be carried out by speaking of the truth-value $V_\alpha^h(A)$ of a formula *A* at the time α, relative to a history h containing α.[7] The truth-condition for F*A* is then as follows.

(3.2) $V_\alpha^h(\mathrm{F}A)=\mathrm{T}$ if $V_\beta^h(A)=\mathrm{T}$ for some $\beta\in h$ such that $\alpha<\beta$.
$V_\alpha^h(\mathrm{F}A)=\mathrm{F}$ otherwise.

[6] An objection to this is that the method does not serve to falsify future tense statements, since time has no end. Most predictions we make, however, have a time-limit built into them, implicitly or explicitly. Those that do not have an oracular flavor. At any rate, I do not mean to say the method is always satisfactory in practice, only that it is the definitive method; as far as the evaluation of predictions goes, there is no appeal from its results.

[7] We use histories here rather than alternative futures because this leaves more room for generalizations not discussed in the present paper. Histories and alternative futures are interchangeable in this context because, in view of the condition of "treelikeness" on model structures, each alternative future for α corresponds to a unique history containing α, and vice versa.

Our train of thought has brought us at length to still another idea of Prior's: 3.2 corresponds to what he calls the "Ockhamist" theory of tense.[8] This latest definition yields a highly satisfactory notion of validity: it renders, e.g., both $A \supset \mathrm{FP}A$ and $\mathrm{F}A \vee \mathrm{F}{\sim}A$ valid. Indeed, if we deal with a formal language that has F and P as its only operators whose truth-conditions depend on temporal ordering, the notion of validity generated by this theory is coextensive with that given by linear time.

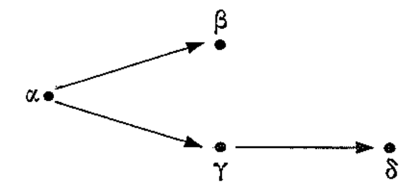
Nevertheless, this account is not above criticism.[9] It says that more is needed to assign a truth-value to a formula at time α than a model structure and assignment of truth-values to formulas not involving tense operators. Besides these a possible future for α must be specified, for on this view statements in the future tense do not in general take a truth-value at α unless a possible future for α is given.

This requirement may be construed in two ways. The difference between these two is explained most clearly by supposing that we occupy the time α and are seeking to evaluate certain predictions made at α. According to the first interpretation we do this by *provisionally positing* a possible future for α, because a prediction made at α can only be true or false relative to such a future. According to the second interpretation, just one of the possible futures for α is the right one—the one that will be actualized. This second view does not square very well with the whole project of indeterministic tense logic. For if a time α can have only one "real" future, times located in other alternative futures cannot really bear any temporal relation to α. They can bear an *epistemic* relation, being futures for a situation which for all we know is the actual one α, but strictly speaking this is not a temporal relation. Thus, indeterministic tense logic collapses on this interpretation to deterministic tense logic.

[8] See [3], pp. 122—127.

[9] The following arguments are perhaps best viewed as dialectical, in that they provide a transition to a competing theory of the future tense. I am not convinced that they constitute a final refutation of the "Ockhamist" theory; Stalnaker has a philosophical account of this theory with which I am in agreement in many respects, though I still prefer the account I present below. When all is said and done, the basic issue here seems to be whether or not one is prepared to accept as meaningful the assertion that there is always, whether we know it or not, a single possible future which, from the perspective of a given time will be its actual future.

The same point can be made in a different way by considering a part of an indeterministic model structure at which there is a fork.



Suppose that β is in the real future of α; then what of the point γ? It isn't in real time, and yet in order to evaluate tensed formulas at γ we must provide it a real future. At this point we begin to lose track of what a "real future" is, and plainly it would be better to just return to a linear conception of time.

No doubt it is this consequence that leads Prior to speak of *"prima facie"* assignments of values to variables, a form of words suggesting our first interpretation of the alternative futures, that they are posited provisionally. But this too is an unstable view, for its import is that statements in the future tense may be neither true nor false. In particular, they will be neither true nor false unless a unique possible future is posited. Since we may often be in situations in which we have made no suppositions concerning which of a variety of possible futures will come about, it should also often be the case that certain statements in the future tense are neither true nor false. But the present theory provides us with no way to accomplish this.

4.

I now want to prepare a response to this last objection. None of the materials used in my proposal are new, though as far as I know the combination is one that has not been suggested. It is plain, first of all, that we need a strategy for introducing truth-value gaps, since we are now determined that formulas of the kind FA should be neither true nor false under certain conditions. Recently a very general way of providing for such truth-value gaps has been developed by B. van Fraassen, and applied with striking success to subjects such as existence presuppositions and the paradoxes of self-reference.[10]

In order to apply this method to a particular problem, say to a formal language L, we must have at hand a clear notion of a *bivalent* (or *classical*) *valuation* of the language. Such a valuation V assigns a truth-value $V(A)=T$ or $V(A)=F$ to each formula A of L, and so fills in all truth-value gaps. However, e.g. in ascribing properties to nonexistents, V may fill these gaps *arbitrarily*. There will thus arise equivalence classes S of valuations which are alike in all nonarbitrary respects, i.e. which differ only in arbitrary assignments of truth-values to formulas.

Van Fraassen's idea is to define truth (i.e. nonarbitrary truth) relative to such equivalence classes. Such a class is called a *supervaluation*, and $S(A)$ is said to be T if $V(A)=T$ for all $V \in S$ and to be F if $V(A)=F$ for all $V \in S$. $V(A)$ is undefined otherwise. A formula A is *satisfied* by a supervaluation S if $S(A)=T$.

This definition has a number of important consequences. First, it preserves excluded middle while rejecting bivalence. Any formula $A \vee \sim A$ will be valid (i.e. made true by all supervaluations), whereas in many cases it will be possible for neither A nor $\sim A$ to be true. Second, it yields a notion of validity that coincides with that given by the bivalent valuations; A is satisfied by all supervaluations if and only if A is satisfied by all bivalent valuations. Third, it provides a distinction between two ways in which B may be said to follow logically from A: the material implication $A \supset B$ may be valid, or it may hold for all supervaluations that if A is true then B is true. Adopting one of the usual notations, let's say that $A_1, \ldots, A_n \Vdash B$ if $S(B)=T$ for all supervaluations S such that $S(A_i)=T$ for all i, $1 \leq i \leq n$. Then, in particular, $\Vdash B$ if and only if B is valid. The two ways in which B may follow from A are then

[10] See van Fraassen [6] and [7].

(4.1) $\Vdash A \supset B$

and

(4.2) $A \Vdash B$.

Although I'm not entirely happy with this terminology, I'll use the term 'implication' to speak of 4.1, so that A implies B if and only if $\Vdash A \supset B$, and 'semantic consequence' to speak of 4.2, so that B is a *semantic consequence* of A (or just a *consequence* of A) if and only if $A \Vdash B$.

It always is the case that if 4.1 holds so does 4.2, but, depending on the nature of the language L and the bivalent valuations of L, the converse may fail. A simple example is the case in which L has quantifiers and one wants to maintain that if a term t lacks a referent then any assignment of a truth-value to Pt is arbitrary. Then $(\exists x)Px$ will be a consequence of Pt, but Pt will not imply $(\exists x)Px$.

Van Fraassen relies continually on this distinction in applying and defending his theory. For instance, one of the objections to his view of truth-value gaps is that excluded middle together with Tarski's principle entails bivalence. To formalize this argument, however, we must have an operator T in our formal language expressing truth. Recognizing this, van Fraassen seeks to disarm the argument by claiming that Tarski's principle holds only as a consequence, $A \Vdash TA$, and not as an implication, so that for some A, $\nVdash A \supset TA$.[11]

5.

Our development of indeterministic tense logic up to this point has provided us with all the materials needed to apply van Fraassen's method: the V^h_α are the bivalent valuations, and these are arbitrary insofar as they depend on a particular history h. Thus V^h_α and V^g_α are to be assigned to the same equivalence class if and only if $\alpha = \beta$. As a special case of van Fraassen's theory we then have the following characterization of truth and falsity at a time α.

[11] '$A \nVdash B$' means that B is not consequence of A, so that '$\nVdash B$' means that B is invalid.

(5.1) $V_\alpha(A)=T$ if and only if $V_\alpha^h(A)=T$ for all $h\in\mathcal{H}_\alpha$.
$V_\alpha(A)=F$ if and only if $V_\alpha^h(A)=F$ for all $h\in\mathcal{H}_\alpha$.
$V_\alpha(A)$ is undefined otherwise.

The special case in which A is FB then works out according to 5.1 and 3.6 in the following way.

(5.2) $V_\alpha(FB)=T$ if and only if for all $h\in\mathcal{H}_\alpha$ there is a $\beta\in h$ such that $\alpha<\beta$ and $V_\beta^h(B)=T$.
$V_\alpha(FB)=F$ if and only if for all $h\in\mathcal{H}_\alpha$ there is no $\beta\in h$ such that $\alpha<\beta$ and $V_\beta^h(B)=T$.
$V_\alpha(FB)$ is undefined otherwise.

We will call the assignment V of 5.1 simply a valuation rather than a supervaluation. (We'll call the V^h *bivalent valuations*.) A formula A is a (semantic) consequence of a set Γ of formulas, in symbols $\Gamma \Vdash A$, if for all model structures $\mathfrak{M}$, for all points of reference α of $\mathfrak{M}$, $V_\alpha(A)=T$ if $V_\alpha(B)=T$ for all $B\in\Gamma$, for all valuations V on $\mathfrak{M}$. A formula A is *valid* if $\Vdash A$.

6.

As we have said, the notion of validity given by these definitions coincides with that given by the bivalent valuations V^h. We are therefore not recommending any departure from Prior's "Ockhamist" theory as far as validity goes. For certain restricted formal languages we can say much more than this. If L contains P and F as its only operators whose truth-conditions depend on temporal ordering, then *consequence* in our theory coincides with consequence on the "Ockhamist" theory, and this in turn coincides with consequence for *linear model structures*. Let's pause to express this result clearly. For the moment, let '$\Vdash_1$' represent the relation of consequence defined in Section 5, above; let '$\Vdash_2$' stand for "Ockhamist" consequence (i.e. consequence for bivalent valuations on indeterministic model structures), and '$\Vdash_3$' for consequence in linear model structures. Then, for languages L with P and F their only temporally determined operators, $\Gamma \Vdash_1 A$ if and only if $\Gamma \Vdash_2 A$ if and only if $\Gamma \Vdash_3 A$.

Thus, for those interested primarily in the consequence relation yielded by a semantic theory there is little difference between these three accounts of tense. But as soon as we consider richer formal languages in which temporally determined operators other than P and F are taken into account, significant differences do appear among these theories.

The first such operator to come to mind is one corresponding to unavoidability or *inevitability*; a thing is inevitable if it is the case with respect to all alternative futures. Such an operator clearly is needed if we are to be capable of formalizing philosophical arguments concerning determinism, and again Prior has preceded us in including an inevitability operator in his formal language. We will use 'L' for inevitability; the truth-condition for this operator is as follows.

(6.1) $V_\alpha^h(LA)=T$ if and only if $V_\alpha^g(A)=T$ for all $g\in\mathcal{H}_\alpha$.
$V_\alpha^h(LA)=F$ otherwise.

From 6.1 it follows at once that inevitability will be a modal operator satisfying the laws of the Lewis system S5. We also have the following properties for L.

(6.2) $A \Vdash LA$
(6.3) $FP \Vdash LFP$
(6.4) $\Vdash P\supset LP$
(6.5) $\nVdash FP\supset LFP$
(6.6) $\nVdash PFP\supset PLFP$

These call for some explanation. It is an immediate consequence of 6.1 that $V_\alpha(LA)=T$ if and only if $V_\alpha(A)=T$; in other words, L expresses the property of being true.[12] Thus, we have 6.2; whenever A is true, so is LA. On the other hand, there are formulas of the kind $A\supset LA$ which are invalid; e.g. $FP\supset LFP$ will be false in a situation α that is placed on two alternative histories g and h such that $V_\alpha^h(FP)=T$ and $V_\alpha^g(FP)=F$. Then $V_\alpha^h(FP\supset LFP)=F$ and hence $V_\alpha(FP\supset LFP)\neq T$.

[12] We will qualify this later, in Section 8.

Here we have a case in which implication differs from consequence; L*FP* is a consequence of *FP* but does not imply *FP*. Intuitively this means that the argument from *FP* to L*FP* is a valid one, for if it is already true that a thing will come to be, it is inevitable that it will come to be. But at the same time, it does not follow from supposing that a thing will come to be that it will inevitably come to be. To suppose that *P* will be is to posit that we will be in a situation in which *P* is true, that we will follow a history h in which *P* is sooner or later satisfied. But this is quite different from positing that such histories are the only alternatives now open; this would amount to positing that *P* is inevitable. In our semantic theory this difference between supposing that *P* will be and supposing that it is now true that *P* will be is represented by the difference between making *FP* an antecedent of an implication as in 6.5 and making it a premiss of the consequence relation as in 6.3.[13]

This feature of our tense logic reflects well the problems that have perennially arisen in the history of debate over determinism. It is intuitively very plausible and, as Prior points out, has been held by many indeterminists that whatever is presently true is presently unavoidable; this is our principle 6.2. The problem for such an indeterminist is then to explain how this does not entail that whatever *will* be will *inevitably* be. We have done this by denying bivalence, thus making possible the distinction between implication and semantic consequence.

7.

To make the above presentation more readable—and more general—I did not formulate the proposed semantic theory in full detail for a specific formal language. In order to make the theory quite explicit I will now do this. Readers not interested in technical details may go on to the next section without loss of continuity.

[13] It is worth mentioning that 6.5 is a formula of the sort $A \supset TA$ when inevitability is equated with truth. It is thus an expression at the implicative level of Tarski's principle that T*A* follows from *A*. Van Fraassen is forced to deny the validity of such a formula in connection with the Liar paradox.

Let L consist of sentence variables *P*, *Q*, *R*, etc. and have four singulary sentence connectives $\sim$, P, F, and L and one binary sentence connective $\supset$. (Predicates and quantifiers afford no special difficulties for this theory, but are omitted in order to focus our attention on the tense operators.) A *model structure* $\mathcal{M}$ consists of a nonempty set $\mathcal{K}$ and a binary relation $<$ on $\mathcal{K}$, subject to the two conditions mentioned in Section 2, above. It is also reasonable to require that for all $\alpha \in \mathcal{K}$ there is a $\beta \in \mathcal{K}$ such that $\alpha < \beta$. A *valuation* V of L on $\mathcal{M}$ is a function which for each $\alpha \in \mathcal{K}$ and sentence variable *P* of L assigns *P* a unique value T or F in α. We say $V_\alpha(P) = T$ or $V_\alpha(P) = F$.

A valuation V is extended so as to give values to complex formulas of L by first defining the values V gives to these formulas relative to histories in $\mathcal{M}$. A history h in $\mathcal{M}$ is a maximal chain in $\mathcal{M}$ (see Section 2). $\mathcal{H}_\alpha$ is the set of all histories in $\mathcal{M}$ that contain α. The values $V_\alpha(A)$ given by V to a formula *A* at α relative to h, where $h \in \mathcal{H}_\alpha$, are defined inductively by the following conditions.

$$V^h_\alpha(P) = V_\alpha(P).$$
$$V^h_\alpha(A \supset B) = T \text{ if } V^h_\alpha(A) = F \text{ or } V^h_\alpha(B) = T.$$
$$V^h_\alpha(A \supset B) = F \text{ otherwise.}$$
$$V^h_\alpha(\sim A) = T \text{ if } V^h_\alpha(A) = F.$$
$$V^h_\alpha(\sim A) = F \text{ otherwise.}$$
$$V^h_\alpha(FA) = T \text{ if } V^h_\beta(A) = T \text{ for some } \beta \in h \text{ such that } \alpha < \beta.$$
$$V^h_\alpha(FA) = F \text{ otherwise.}$$
$$V^h_\alpha(PA) = T \text{ if } V^h_\beta(A) = T \text{ for some } \beta \in h \text{ such that } \beta < \alpha.$$
$$V^h_\alpha(PA) = F \text{ otherwise.}$$
$$V^h_\alpha(LA) = T \text{ if } V^g_\alpha(A) = T \text{ for all } g \in \mathcal{H}_\alpha.$$
$$V^h_\alpha(LA) = F \text{ otherwise.}$$

8.

The above theory, like all semantic theories, has the virtue of being readily extendible. By adding domains to model structures and devising semantic rules for quantifiers, one gets a theory of time and existence. By adding a relation $\mathcal{R}$ among situations such that $\alpha\mathcal{R}\beta$ if β is possible for all that is known in α one can develop an account of time and knowledge. I will mention several other extensions of the basic theory in Section 9, below. However, in order to forestall an important objection, I wish to give particular attention to an extension of our formal language to include an operator corresponding to truth.

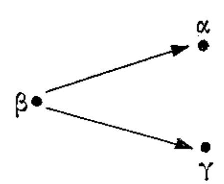
The objection is this. On the account of inevitability given in Section 6 above, it turned out that inevitability was identical to truth. However, suppose I make a prediction that comes out true: on Friday I say it will snow the next day, and on Saturday it snows. We then say (on Saturday) that what I said was true. But we don't say it was inevitable. This shows that truth cannot be the same as inevitability, contrary to what seems to be entailed by the proposed theory.

In considering this point we must first realize that it requires us to formalize a locution of the form '... was true,' so that we must bring an operator T for truth into our formal language. The truth-condition for this operator is evidently the following.[14]

$$(8.1)\quad V^h_\alpha(\mathrm{T}A) = \mathrm{T} \text{ if and only if } V^h_\alpha(A) = \mathrm{T}.$$
$$V^h_\alpha(\mathrm{T}A) = \mathrm{F} \text{ otherwise.}$$

We then have $\mathrm{T}A \Vdash \mathrm{L}A$ and $\mathrm{L}A \Vdash \mathrm{T}A$. In this sense, truth and inevitability are coincident.

However, PLF*P* is not a consequence of PTF*P*, as is shown by taking a portion of a model structure



and letting $V_\alpha(P) = V_\beta(P) = \mathrm{T}$ but $V_\gamma(P) = \mathrm{F}$. Where $h = \{\alpha, \beta\}$ and $g = \{\alpha, \gamma\}$, $V^h_\alpha(\mathrm{PTF}P) = \mathrm{T}$, but since $V^g_\beta(\mathrm{F}P) = \mathrm{F}$, $V^h_\beta(\mathrm{LF}P) = \mathrm{F}$ so that

[14] As Tarski has shown, for languages with sufficient self-referential power we must modify this. If we followed van Fraassen's strategy here, we could do this by making the V^h_α themselves not bivalent, so that truth-value gaps would arise at two levels.

$V^h_\alpha(\mathrm{PLF}P) = \mathrm{F}$. Since here h is the only history containing α, $V_\alpha(\mathrm{PTF}P) = \mathrm{T}$ while $V_\alpha(\mathrm{PLF}P) = \mathrm{F}$.

Our theory thus allows (indeed, forces) us to say that *having been true* is different from *having been inevitable*, at least as far as future-tense statements go. The latter is not a consequence of the former, $\mathrm{PTF}P \nVdash \mathrm{PLF}P$, because in an assertion that it was *true* that a thing would come about, truth is relative to events up to the present, whereas in an assertion that it was *inevitable* that a thing would come about, inevitability is judged relative to some time in the past.

Far from being an objection to our proposal, this result is *support* for it, inasmuch as without any changes the theory accounts for our intuitive judgments regarding truth and inevitability in this case.

9.

The main purpose of this paper, the application of van Fraassen's theory of truth to Prior's "Ockhamist" tense logic, has now been accomplished. I didn't think it appropriate to get so deep into technical details that it became necessary to prove metatheorems, or to venture into applications of the theory that would take us afield from the central point. In conclusion, though, I would like to describe the form that I expect further developments of the theory to take.

Proof theory has not been mentioned in this paper, because I think it contributes little to the understanding of the questions here under consideration. However, it is relatively easy to axiomatize the notion of semantic consequence described in Section 7, i.e. to define by means of axioms and rules of inference a relation $\vdash$ such that $\vdash A$ if and only if $\Vdash A$.[15]

[15] In view of the equivalence discussed at the beginning of Section 6, above, nothing new would be gained by axiomatizing the consequence relation of our theory for a language with only *P*, *F*, and truth-functional operators. I mean to present an axiomatization of the theory discussed in Section 7 in a forthcoming paper. Several of the suggestions made below will also, I hope, be the topics of forthcoming papers.

As Prior points out, such an axiomatization cannot satisfy an unrestricted rule of substitution for sentence variables, in view of examples such as our 6.4 and 6.5. In his presentation of the "Ockhamist" theory he also introduces more general sentence variables which do satisfy substitution. Such sentence variables can be handled semantically by assigning them truth-values relative to both a time and a history containing that time. Thus, where G is a general sentence variable we can have $V^h_\alpha(G) \neq V^g_\alpha(G)$, whereas for a variable P, $V^h_\alpha(P)$ must equal $V^g_\alpha(P)$. I am not certain that this extension of the formal language provides any advantages in translating natural language, because it is difficult for me to imagine how truth-values of a statement of natural language can vary relative to possible futures unless this statement, like 'He is a future millionaire,' contains a tense operator implicitly. But it is an interesting idea from a technical point of view, and worth developing. (If it were to happen that in dealing with sentences from natural language which vary in the way described, it were not a superficial and easy task to reconstruct them so that this variation were seen to depend on tense operators, I would gladly amend the proposed theory, treating its variables as general variables. Formulas such as $P \supset LP$ would then no longer be valid.)[16]

I mentioned earlier several ways in which the present theory can yield accounts of the relation of time to other things, by enriching the formal language so as to include a vocabulary for expressing these things. Notions which are particularly worth studying in this way are knowledge, mentioned above, and obligation. A mixed temporal-epistemic language, for instance, is particularly useful for explaining the behavior of the English word 'might,' which displays temporal as well as epistemic traits. A semantic interpretation of *conditionals* can be added to tense logics such as the one described here by adding selection functions of the kind described in [4] to models. This renders our logic capable of handling mixed temporal and conditional statements

[16] When the theory is revised in this way, we can use the distinction between implication and consequence to good effect in explaining how the inevitability of a thing follows from the thing itself. We have $A \Vdash LA$, but $\nVdash P \supset LP$. Thus, a thing has its inevitability as a consequence, but does not imply its inevitability.

such as 'If you hadn't turned left there we wouldn't be lost now,' which would be translated by a formula $P \sim P > \sim Q$, where $>$ is the conditional connective of [4].

It has been insufficiently realized, I think, that many of the puzzles that deontic logicians have encountered in formalizing the notion of obligation arise in part from the interaction of obligation with temporal factors. I suspect that these puzzles will only be solved with reference to a combined tense and deontic language. Indeterministic tense logic is particularly attractive for this purpose in view of the traditional position that obligations presuppose freedom.

Finally, I feel that the present theory may be useful in clarifying issues in the history of philosophy. A comparison of the theory, and of indeterministic tense logics generally, with deterministic tense logics in which "alternative" futures are regarded as mere epistemic possibilities should provide us with a much sharper conception of the difference between determinism and indeterminism. Such clarification could hardly help but be useful to the historian of philosophy. It may also be that the theory presented here in fact coincides with the views of previous philosophers on truth and the future tense. Here, Aristotle is the man who comes first to mind; his "sea-battle" passage is, at first glance anyway, in very good accord with the modeling of the future tense propounded here.

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