

ence for  $C = 1$ . In other words, the sign of difference  $I_{ij}^2 - I_{ij}^2$  coincides with the sign of expression

$$\sqrt{\frac{(r^1)^2}{N^{i_1} \cdot C}} - \sqrt{\frac{(r^2)^2}{N^{i_2} \cdot C}}$$

for  $C = 1$ , that is, with the sign of difference

$$\sqrt{\frac{(r^1)^2}{N^{i_1}}} - \sqrt{\frac{(r^2)^2}{N^{i_2}}}.$$

Taking into consideration the identity

$$\frac{(r^1)^2}{N^{i_1}} - \frac{(r^2)^2}{N^{i_2}} = \left( \sqrt{\frac{(r^1)^2}{N^{i_1}}} - \sqrt{\frac{(r^2)^2}{N^{i_2}}} \right) \times \left( \sqrt{\frac{(r^1)^2}{N^{i_1}}} + \sqrt{\frac{(r^2)^2}{N^{i_2}}} \right)$$

we easily arrive at the following conclusion: the signs of differences

$$I_{ij}^2 - I_{ij}^2$$

and

$$\frac{(r^1)^2}{N^{i_1}} - \frac{(r^2)^2}{N^{i_2}}$$

in fact coincide. We note further that the ordering of the values of a quantity (in decreasing order) in fact reduces to the determination and analysis of signs of differences in pairs of obtained values of the mentioned quantity. For example, if the sign of the difference of a pair of values is positive, then the first value of the given pair is greater than the second, and so on. Therefore, taking into account that for any two search methods ( $i_1$  and  $i_2$ ), the signs of differences

$$I_{ij}^2 - I_{ij}^2$$

and

$$\frac{(r^1)^2}{N^{i_1}} - \frac{(r^2)^2}{N^{i_2}}$$

coincide, it is possible to assert that the ordering of available search methods in decreasing order of obtained values of characteristic  $I_2$  and in decreasing order of obtained values of criterion  $r^2/N$  proves to be the same; that is, the necessary ordering in fact can be accomplished using the criterion  $r^2/N$ . In other words, the order established by means of criterion  $r^2/N$  is preserved also in the case of ordering by means of characteristic  $I_2$ . This was the reason why a property leading to this result was called the order preservation property.

Now we will show that some of the complex search characteristics  $I_1$

through  $I_{12}$  discussed earlier, namely  $I_2, I_3, I_4$ , and  $I_5$ , possess the order preservation property. To show this, assuming that the condition of this property is fulfilled for each analyzed CSC, we will consider for any two search methods ( $i_1$  and  $i_2$ ) the following differences:  $I_{ij}^2 - I_{ij}^2, I_{ij}^2 - I_{ij}^2, I_{ij}^2 - I_{ij}^2$ , and  $I_{ij}^2 - I_{ij}^2$ . Thus,

$$1. I_{ij}^2 - I_{ij}^2 = \sqrt{R^{i_1} \cdot P^{i_1}} - \sqrt{R^{i_2} \cdot P^{i_2}} = \sqrt{\frac{(r^1)^2}{N^{i_1} \cdot C}} - \sqrt{\frac{(r^2)^2}{N^{i_2} \cdot C}}$$

$$= \frac{1}{\sqrt{C}} \left( \sqrt{\frac{(r^1)^2}{N^{i_1}}} - \sqrt{\frac{(r^2)^2}{N^{i_2}}} \right);$$

$$2. I_{ij}^2 - I_{ij}^2 = C + N^{i_1} - 2r^{i_1} - C - N^{i_2} + 2r^{i_2}$$

$$= N^{i_1} - 2r^{i_1} - N^{i_2} + 2r^{i_2};$$

$$3. I_{ij}^2 - I_{ij}^2 = \sqrt{I_{ij}^2} - \sqrt{I_{ij}^2} = \frac{(\sqrt{I_{ij}^2} - \sqrt{I_{ij}^2})(\sqrt{I_{ij}^2} + \sqrt{I_{ij}^2})}{\sqrt{I_{ij}^2} + \sqrt{I_{ij}^2}}$$

$$= \frac{I_{ij}^2 - I_{ij}^2}{\sqrt{I_{ij}^2} + \sqrt{I_{ij}^2}};$$

$$4. I_{ij}^2 - I_{ij}^2 = 1 - \frac{I_{ij}^2}{N_0} + \frac{I_{ij}^2}{N_0} - \frac{I_{ij}^2}{N_0}.$$

In Case 1, the sign of difference  $I_{ij}^2 - I_{ij}^2$  coincides with the sign of expression

$$\sqrt{\frac{(r^1)^2}{N^{i_1}}} - \sqrt{\frac{(r^2)^2}{N^{i_2}}}$$

for any permissible value of  $C$ , because for any of these values  $\sqrt{C} > 0$ . In other words, the sign of difference  $I_{ij}^2 - I_{ij}^2$  is determined by the sign of expression

$$\sqrt{\frac{(r^1)^2}{N^{i_1}}} - \sqrt{\frac{(r^2)^2}{N^{i_2}}}.$$

This expression does not contain quantity  $C$ ; that is, the sign of this expression and, hence, the sign of the difference  $I_{ij}^2 - I_{ij}^2$  does not depend on value of  $C$ . It follows from this statement that complex search characteristic  $I_2$  possesses order preservation property.

In Case 2, the expression for difference  $I_{ij}^2 - I_{ij}^2$  does not contain quantity  $C$ , from which it is obvious that complex search characteristic  $I_3$  has the order preservation property.

In Case 3, quantity  $\sqrt{I_{ij}^2} + \sqrt{I_{ij}^2}$  is nonnegative. Therefore, the sign of difference  $I_{ij}^2 - I_{ij}^2$  is determined by the sign of  $I_{ij}^2 - I_{ij}^2$ , and it was established that the sign of the latter does not depend on the value of  $C$ . Hence, the sign of