Cognitive perspectives on counting

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Introduction

Counting

- common $\Rightarrow$ everyday experience
- cognitive $\sim$ linguistic perspectives
- three different though related concepts
  - count list $\Rightarrow$ recitation
  - arithmetic $\Rightarrow$ abstract operations
  - quantification $\Rightarrow$ cardinality of a set

(1) a. one, two, three, four, five, six,...
b. Three times two equals six.
c. three cats

Number sense

Two cognitive systems

Hyde (2011)

- OTS $\Rightarrow$ object tracking system
- ANS $\Rightarrow$ approximate number system

Figure 1: Object tracking

Figure 2: Approximate number

Number sense

Object tracking system

Carey (1998, 2009), Piazza (2010)

- mental ability to immediately enumarate small sets
- no counting via individuation
- manifests in infants

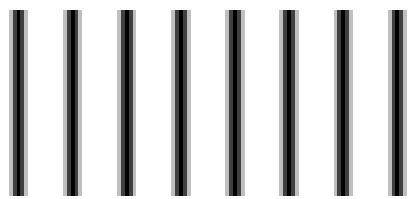

Figure 3: How many marks?

Number sense

Object tracking system

Carey (1998, 2009), Piazza (2010)

- mental ability to immediately enumarate small sets
- no counting via individuation
- manifests in infants

Figure 4: How many marks?

Number sense

Approximate number system

Feigenson et al. (2004), Nieder & Dehaene (2009), Cantlon et al. (2006)

- estimation of the magnitude of a collection
- no reliance on symbolic representation
- manifests in infants $\Rightarrow$ develops with age

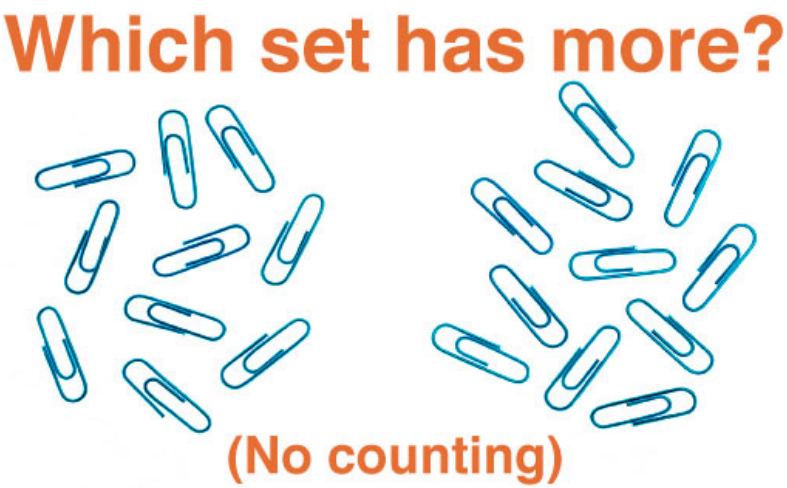

Figure 5: Compare

Number sense

Number sense in non-human animals

Davis & Pérusse (1998), Gallistel (1989), Dehaene (1997)

- primates $\Rightarrow$ operations on quantities
  - apprehension
  - comparison
  - approximate addition
- other mammals: dolphins, cats, rats
- also: birds, fish
- botanics $\Rightarrow$ plant arithmetic
- however, no evidence for symbolic addition except for the chimpanzee after long training

`https://www.youtube.com/watch?v=t-SQisIYPh4`

Psychology of counting

Implicit knowledge of counting in children

Gelman & Gallistel (1978)

- intuitive understanding of the cardinality of a set
- and its conservation under changes not affecting quantity
- each entity must be count once and once only
- 1 number cannot be associated with more than 1 entity
- no explicit formulation $\Rightarrow$ children are never taught that

Figure 6: Enumerating sets

Psychology of counting

Innate principles of counting

Gelman & Gallistel (1978)

- stable order $\Rightarrow$ ordered list of symbols
- 1-1 correspondence $\Rightarrow$ symbols related to objects
- cardinality $\Rightarrow$ determined by the last symbol

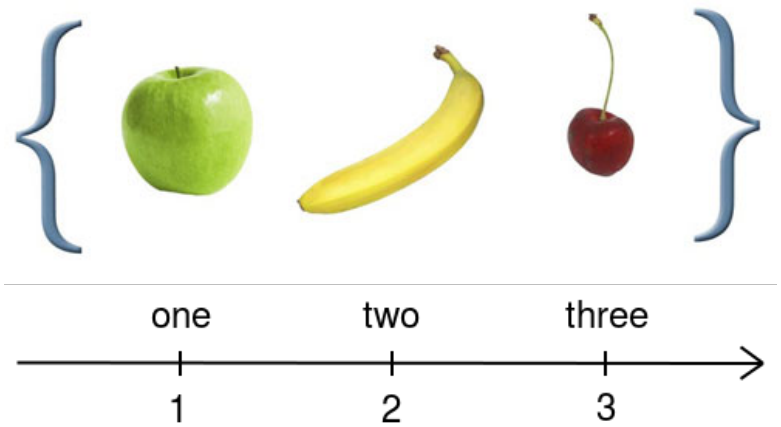

Figure 7: Counting and order

Psychology of counting

Acquisition of counting

Wynn (1990)

- children 6–18 months
  - stable order and 1-1 correspondence observed
  - fail when asked to give ‘two’ or ‘three’ objects
- 2,5 years
  - understanding that counting is an abstract procedure
  - applicable to different kinds of objects
- 3,5 years
  - order of recitation $\Rightarrow$ crucial
  - order of pointing at objects $\Rightarrow$ irrelevant
  - children indicate and correct subtle errors
- 4 years
  - counting can be generalized to novel situations

Psychology of counting

Quinean bootstrapping $\Rightarrow$ crucial linguistic component

Carey (2009)

- learning the ordered list $\Rightarrow$ relative order
- learning the meaning of symbols
- learning how the list represents number

(2) a. eeny, meeny, miny, mo,...
b. one, two, three, four, five, six,...

(3) $[\![\text{three}]\!] = 3$

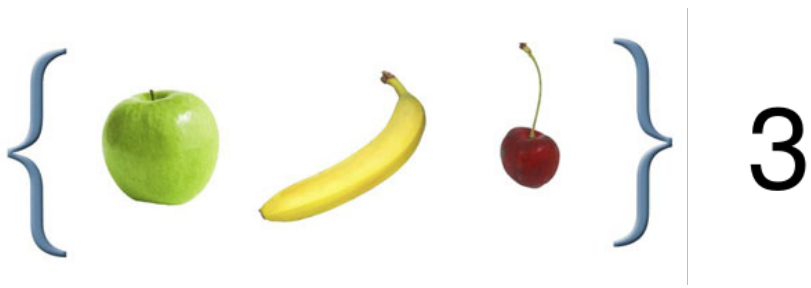

Figure 8: Cardinality

Spatial integrity in counting

Object/substance distinction

Soja et al. (1991), Hauser & Carey (2003), Hauser & Spaulding (2006)

- innate ontological commitments
- manifested in infants
- assumptions ⇒ nature of objects
  - boundedness ⇒ natural boundaries
  - cohesion ⇒ parts stick together
  - movement across space along continuous paths
- substances ⇒ not expected to have those properties
- also in non-human animals

`https://www.youtube.com/watch?v=hwgo2O5Vk_g&t=2s`

Spatial integrity in counting

Broken object experiments

Shipley & Shepperson (1990), Dehaene (1997), Melgoza et al. (2008)

- children between 3 and 4 years
- count only discrete integrated objects

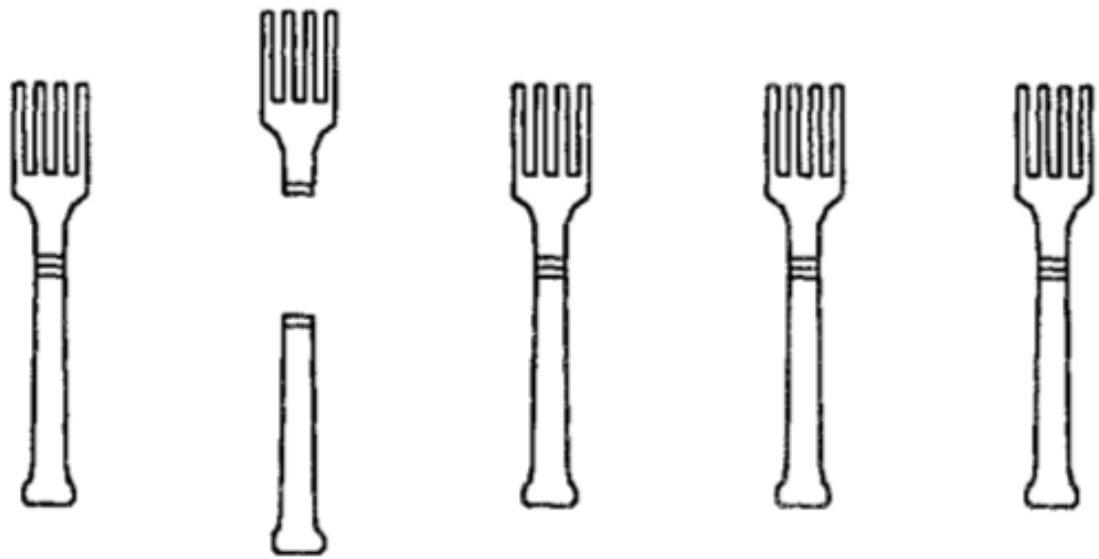

Figure 9: Relevance of integrity in counting

Spatial integrity in counting

Broken object experiments

Shipley & Shepperson (1990), Dehaene (1997), Melgoza et al. (2008)

- other forms of linguistic quantification
- comparative constructions and pluralization

Figure 10: Integrity in quantity comparison and pluralization

Part-whole structures

Ontological intuition

Varzi (2016), Priest (2014)

- Pre-Socratics ⇒ roots of mereology
  - entities ⇒ made up of smaller entities (parts)
- Plato ⇒ *Parmenides* and *Theaetetus*
  - unity ∼ arbitrary sum of parts
  - structure ⇒ arrangement of parts

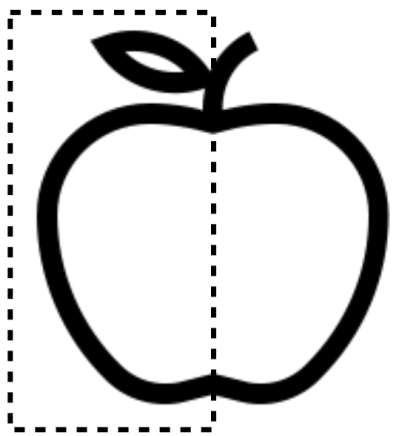

Figure 11: Material parthood

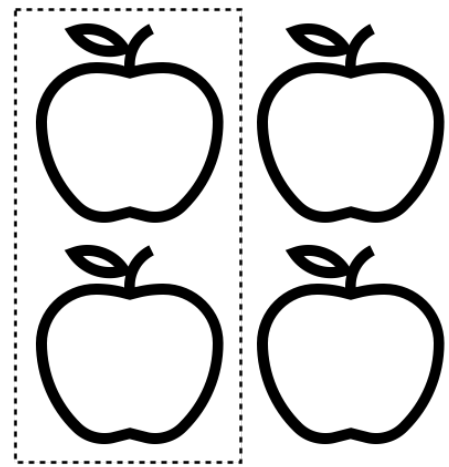

Figure 12: Individual parthood

Part-whole structures

Part-whole perception

Elkind et al. (1964), Kimchi (1993), Boisvert et al. (1999)

- simultaneous perception $\Rightarrow$ wholes $\sim$ collections of parts
- manifests in young children

Figure 13: Part-whole perception

Mass/count distinction

Countability $\Rightarrow$ mass nouns $\sim$ count nouns

Jespersen (1913) among many others

- uncountable $\sim$ countable nouns
- grammatical category
- pluralization, compatibility with numerals
- intuition $\Rightarrow$ object/substance distinction

(4)
a. cat
b. cats
c. two cats

(5)
a. mud
b. *muds
c. *two mud/muds

Mass/count distinction

Object mass nouns

Barner & Snedeker (2005), Chierchia (2010), Landman (2011)

- grammatical category $\Rightarrow$ mass nouns
- denote discrete objects
- clash $\Rightarrow$ grammar $\sim$ perception

(6)
a. furniture
b. silverware
c. footwear

(7)
a. nábytek
b. bižuterie
c. obuv

Czech

Mass/count distinction

Object mass nouns

Barner & Snedeker (2005), Chierchia (2010), Landman (2011)

- quantity comparison task
- object mass nouns pattern with count nouns
- attested in several typologically distinct languages

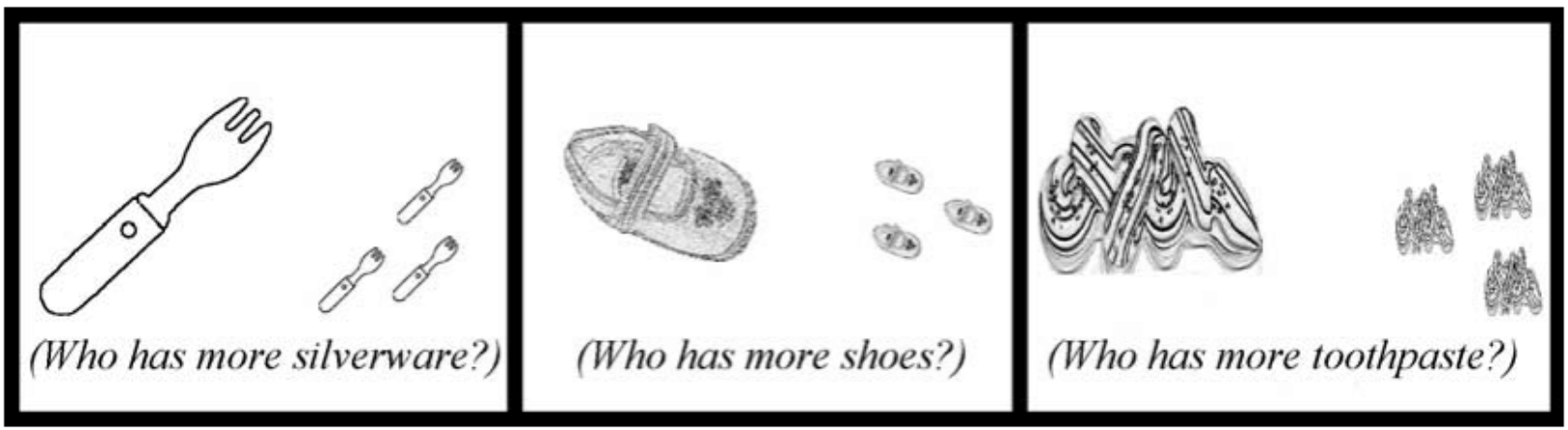

Figure 14: Object mass – count – mass

Counting and measuring

Counting and measuring are independent operations
Rothstein (2017), Wągiel (2018)

- distinct syntax and semantics
- counting indicates integrity $\Rightarrow$ measuring does not

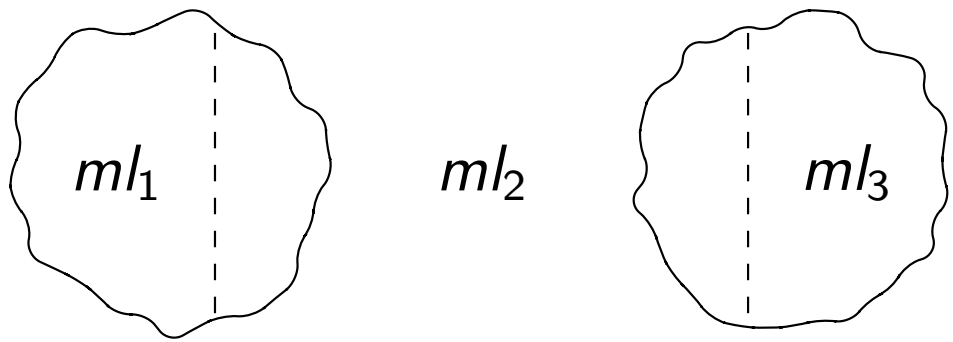

Figure 15: Inegrity in measuring and counting

(8) a. There are three mililiters of liquid on the table.
b. #There are three objects on the table.

Counting and measuring

Measuring is not sensitive to integrity

Wągiel (2018)

- numeral phrases ⇒ counting/measuring ambiguity
- counting ⇒ measuring shift (possible but restricted)

(9) CONTEXT: John is cooking with his child. They put three whole apples on a table. John says:

a. There are three apples on the table...
b. Let's count them together: one, two, three.

(10) CONTEXT: John is cooking with his child. They sliced three apples and put the slices into a bowl. John says:

a. There are three apples in the bowl...
b. #Let's count them together: one, two, three.

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