Chapter Four
Mathematics Challenges Philosophy: Galileo, Kepler, and the Surveyors

I Natural philosophy – the only game in town?

Bacon's notion of an operational natural philosophy took its lead from the kinds of natural philosophy taught in the schools. Bacon attempted a radical reformation of natural philosophy, but it was still a reformation rather than a completely different enterprise. This fact might suggest that the available scope for rethinking the study of nature was severely restricted – as indeed it was. But natural philosophy was not the only model provided by learned culture for the study of nature. There were other relevant areas of inquiry too, areas that could be turned to account by people dissatisfied by (or uninterested in) the enterprise of the physicists.

Recall that Aristotelian physics aimed at understanding qualitative processes. Quantities were at best peripheral to it, because they failed to speak of the essences of things – of what *kinds* of things they were. Measurements, whether of dimensions or of numbers, were purely descriptive, while the natural philosopher's job was defined by its attempt to *explain*, not merely describe.

During the sixteenth century, certain Aristotelian philosophers had denigrated the mathematical enterprise on precisely these grounds. Scholars like Alessandro Piccolomini, and prominent natural philosophers like Benito Pereira, published critiques of mathematics that contrasted it unfavourably with physics. Mathematics, they said, did not demonstrate its conclusions through *causes*. This disqualified mathematical proofs from being scientific in Aristotle's sense, because Aristotle had specified that true scientific demonstration always proceeded through the identification of a relevant explanatory cause for its conclusion. Such causes, falling under one of Aristotle's four categories of formal, final, efficient, and material, were what made a proof into a piece of science.[1] None of these kinds of cause was utilized in mathematics, its critics claimed, and hence mathematics was not a scientific discipline. Indeed, the most damning shortcoming of all was mathematics' failure to speak of *formal* causes, that is, explanatory causes that relied on specifying the *kind* of thing that was involved. In other words, mathematics did not get at the true *natures* of its objects, and was restricted to discussing only superficial quantitative properties (in Aristotelian terminology, quantitative *accidents* unrevealing of a thing's nature, or *essence*).

Needless to say, there were contemporary mathematicians who resented such assertions. They wished to portray their own discipline as a "science" because that was the highest grade of knowledge; they did not want second-class status behind the physicists. Accordingly, several mathematical writers in the later sixteenth century and the early seventeenth century produced counter-arguments to establish, against the natural philosophers, that mathematical proofs were indeed causal and properly scientific. Foremost among them were mathematicians belonging to the Catholic religious order called the Society of Jesus – the Jesuits.

During the second half of the sixteenth century the Jesuits (founded by Ignatius Loyola in 1540) became the foremost teaching order in the Catholic world. Their colleges quickly sprang up all over Europe, with a reputation for excellence that was second to none. The education that the Jesuit colleges offered was comparable to the arts education available at universities. Apart from the explicitly religious aspects, which underlay the whole, Jesuit education thus consisted of a great deal of humanist training in ancient languages and literature, as well as education in the traditional scholastic subjects based on the texts of Aristotle – physics, metaphysics, and ethics, together with the subjects of the quadrivium, that is, mathematics.[2] The Jesuit mathematicians were frequently different people from those who taught natural philosophy, and some of them objected to the belittling characterizations of their specialty found even in the writings of their own philosophical brothers, such as Pereira. The earliest concerted defence came from the leading Jesuit mathematician of the late sixteenth century, Christoph Clavius, professor of mathematics at the Jesuits' flagship college in Rome, the Collegio Romano. Clavius explicitly rejected the claims of the philosophers concerning mathematics, and pointed out the pedagogical harm that could be caused by their teachings on the subject. There were those, he complained in the 1580s, who told their pupils that "mathematical sciences are not sciences, do not have demonstrations, abstract from being and the good, etc.".[3] Clavius wanted the teachers of mathematics to be accorded as much respect as the teachers of natural philosophy and metaphysics, and the scurrilous charges against mathematical knowledge hindered this goal. As regards substantive responses to the hated arguments, Clavius himself was less effective, although he established a position in support of mathematics that was subsequently widely imitated by other Jesuit mathematicians. He relied especially on Aristotle's own discussions, pointing out that Aristotle had included mathematics as an integral part of philosophy alongside natural philosophy, thereby implying that it had an equivalent cognitive status, and that Aristotle had described the mixed mathematical disciplines (astronomy, music, and so on) as being "subordinate sciences"; that is, sciences that relied on results borrowed from other higher sciences – meaning arithmetic and geometry. There could thus be no doubt that Aristotle regarded mathematics as truly scientific.

Later Jesuit mathematical writers supplemented Clavius's appeals for fair play with philosophically-based refutations of the anti-mathematical arguments. A former student of Clavius, Giuseppe Biancani, in a work of 1615, wrote at some length on the question, denying the view that mathematical demonstrations did not employ causal proofs and that mathematical objects (geometrical figures or numbers) lacked true essences – in effect, that they were not real things. Biancani says that, on the contrary, geometry defines its objects in such a way as to express their essences. He means that a triangle, for example, is a figure composed of three right lines in the same plane that intersect one another to yield three internal angles – that is what a triangle *is*. Similarly, geometrical figures have their own matter (the subject of material-cause explanations), in this case *quantity*. Using such arguments, Biancani attempted to refute the philosophical critics of mathematics, while also following Clavius in claiming a certain *superiority* for mathematical demonstrations over those of natural philosophy. This superiority flowed from the generally accepted certainty of mathematical proofs, which by common consent exceeded that of other kinds of philosophical argument.

Thanks initially to Clavius, these sorts of arguments were well known, especially among Jesuit mathematicians, in the early seventeenth century. They served as a means of increasing the confidence of mathematicians that their sciences were not only on a par with natural philosophy but were perhaps in some ways even better at making reliable knowledge of nature. One such mathematician was an Italian friend of Clavius, Galileo Galilei.

II Galileo the mathematical philosopher

Galileo was born at Pisa, the second city of the Grand Duchy of Tuscany in northern Italy, in 1564. He was the son of a musician, Vincenzo Galilei, who was from Tuscany's capital city, Florence, and Giulia Ammannati, and the family held minor noble status derived from its Florentine forebears. Galileo attended the University of Pisa to study medicine, but his lack of vocation conspired with his aptitude for mathematics to cause him to leave in 1585; he subsequently returned to the university in 1589 to take up a chair in mathematics. The chair had been secured on the strength of personal recommendations from established mathematicians, especially Guidobaldo dal Monte (Galileo had also met Clavius by this time, on a visit to Rome in 1587).[4]

Much of Galileo's subsequent career must be explained by reference to his aggressive and ambitious personality. His approach, however, and the values that he expressed, were not idiosyncratic, but can be understood as part of the outlook of a university mathematician of his time and place. Although other people in similar positions failed to acquire Galileo's fame, Galileo did what many of them would no doubt have liked to achieve – he stood up to the higher-paid, more prestigious natural philosophers and refused to concede to their expertise.

The earliest example of this dates from around 1590, during Galileo's professorship at Pisa. An early manuscript treatise surviving from that period, usually known as *De motu* ("On Motion," composed in Latin), signals by its very title that Galileo intends to take on the despised Aristotelian physicists. Motion, as an example of change, was a central topic of Aristotelian physics. The natural philosopher spoke of motion so as to explain why things moved, and one of the typical kinds of such explanations invoked an appropriate final cause. In particular, to explain the free fall of a heavy body, Aristotle had described it as a *natural* motion, since it is in the nature of heavy bodies to fall when unimpeded. But *why* do they fall? Aristotle decided that they fell because they were seeking their proper place at the centre of the universe. Fall thus appeared as a *process* of travel, wherein the moving body set off from its starting place in an endeavour to reach its goal. That goal, the centre of the universe, coincided in Aristotle's cosmos with the centre of the earth – because the earth is simply the accretion of all heavy bodies bunched together around their natural place, towards which they strive.

One of the Aristotelian rules governing fall that emerged from this conceptualization was that the heavier a body, the faster it falls. Weight expressed the motive tendency of the body, so if weight increased, so too should the speed of descent. A body that weighs twice as much as another ought therefore to descend twice as quickly as the lighter body. Galileo, in *De motu*, argues that this familiar Aristotelian claim is false, and he provides a number of arguments intended to show it. One, for example, imagines two independently falling bodies becoming linked together by a piece of cord as they fall. Becoming connected, they should now constitute a single aggregate body. Such a body, being heavier than either of its original components, would, according to Aristotelian doctrine, fall more rapidly than either one. And yet, Galileo urges, it is not conceivable that the two pieces would suddenly speed up as soon as the cord linked them.

Galileo's strategy becomes clearer when he calls on the precedent of the ancient mathematician Archimedes to aid him.[5] In *On Floating Bodies*, Archimedes considered the relationship between the specific gravity (or density) of a body and that of the medium in which it was immersed. He used this relationship to determine whether the body should float or sink: if the body was denser than the medium, it sank; if less dense, it floated. Galileo takes the same approach in his own discussion of falling bodies –

in effect, he treats falling bodies as if they were all *sinking* in a common medium, the air, and compares their rates of fall by comparing their specific gravities in relation to air's.

Galileo, notably, does *not* ask the question "*why* do heavy bodies fall?" That would have been a natural philosopher's question. Galileo, the mathematician, asks only how fast they fall, and what the relationship is between their densities and that of the medium; like Archimedes, Galileo does not ask what weight *is*. Against Aristotle, he concludes, first of all, that two bodies of differing weights – say, differently sized iron balls – will nonetheless fall at identical speeds. The speeds are a function of the balls' specific gravities in a common medium, air; since both balls are made of the same material, solid iron, their speeds too are the same.

In 1591 Galileo left the university at Pisa to take up a similar, although rather more illustrious, professorship at the great thirteenth-century university of Padua. The city of Padua, in north-eastern Italy, was at this time a part of the independent republic of Venice, and Galileo's academic position fell under the control of the Venetian senate. For nearly two decades Galileo remained at Padua, lecturing on mathematical subjects and engaging in occasional controversies with Aristotelian philosophers there. He supplemented his income by making and selling mathematical instruments designed for surveying work, an activity that was a common feature of practical mathematical pursuits at the time.[6] By 1609 he had developed to a high degree his work on the motion of heavy bodies, including the famous doctrines of the uniform acceleration of freely falling bodies and the parabolic paths of projectiles. This work, however, was not to be published until 1638, in his *Discorsi* ("Discourses and Demonstrations Concerning Two New Sciences," often referred to in English as the *Two New Sciences*).[7] His aversion, as a mathematician, to the natural philosophy of his Aristotelian colleagues continued to motivate him, and probably contributed to his readiness, from the 1590s onwards, to entertain the unorthodox doctrines of another mathematician, Nicolaus Copernicus.

Galileo's interest in Copernicanism existed from at least 1597, when he mentions Copernicus in two letters. One of these letters was sent to the great astronomer Johannes Kepler, acknowledging receipt of the latter's Copernican book *Mysterium cosmographicum* ("Cosmographical Mystery") of 1596; Galileo, famously, claims to Kepler that he too was a Copernican, and had been "for many years."[8] It was not until the first decade of the seventeenth century, however, that Galileo took up astronomical and cosmological issues in a serious way, especially from 1609 onwards when he began to use a telescope to make astronomical observations.[9] Copernicanism seems to have appealed to Galileo above all because it was a useful tool for attacking the Aristotelian physicists. First, it advocated the acceptance of a sun-centred universe, which would tear to shreds the physical world-picture on which the entire Aristotelian system was based. If the earth were no longer at the centre of the universe, for example, the fall of heavy bodies (and the rise of light bodies) could no longer be explained by their desire to reach a destination defined in terms of the centre of the universe, because the latter would no longer coincide with the earth's centre.[10] Secondly, the chief arguments in favour of Copernicanism were astronomical rather than cosmological: that is, they were the arguments of a mathematician, concerned with reducing the apparent motions of the heavens to order, rather than those of the physicist, concerned with the nature of the heavens and the explanation of their movements. At the same time, Copernicus and a few followers of his doctrine, such as Kepler, had embraced the cosmological inferences that they nonetheless dared to draw from the new astronomical system.[11]

Galileo therefore attempted to use Copernican astronomy as a mathematician's means of subverting Aristotelian cosmology. He trampled on the usual demarcation between physics and mathematics by stressing that the natural philosopher had to take into account the discoveries of the mathematical astronomer, since the latter concretely affected the content of the natural philosopher's theorizing – the astronomer told the physicist what the phenomena were that required explanation. In his *Letters on Sunspots* (1613), Galileo made this point strongly in arguing for the presence of variable blemishes on the sun's surface. The Aristotelian heavens were held to be perfect and substantively unchanging; all they did was to wheel around eternally, exhibiting no generation of new things or passing away of old. The marks first seen on the face of the sun by Galileo and others in 1611 did not appear to show the permanence and cyclicity characteristic of celestial bodies, and Galileo took the opportunity to argue that they were, in fact, dark blemishes that appeared, changed, and disappeared irregularly on the surface of the sun. It was important to the argument that the spots be located precisely on the sun's surface itself. The Jesuit Christoph Scheiner, Galileo's main rival for the glory of their discovery, at first thought that the spots were actually composed of small bodies akin to moons, which orbited around the sun in swarms so numerous as to elude, thus far, reduction to proper order. Accordingly, Galileo presented careful, geometrically couched observational reasoning to show, first of all, that there was an apparent shrinkage of the spots' width as they moved across the face of the sun from its centre towards the limb (and corresponding widening as they appeared from the other limb and approached the centre); and secondly, that this effect, interpreted as foreshortening when the spots were seen near the edges of the sun's disc, was consistent with their having a location on the very surface of the sun itself. The precise appearances, he argued, would be noticeably different if these necessarily flat patches were any distance above the sun.[12]

Galileo's argument leads to the following point: if it is established that the sun's surface is blemished by dark patches that manifestly appear from nothing and ultimately vanish, then it becomes undeniable that there is, contrary to Aristotelian doctrine, generation and corruption in the heavens. Galileo has moved from a "mathematical" explication of the

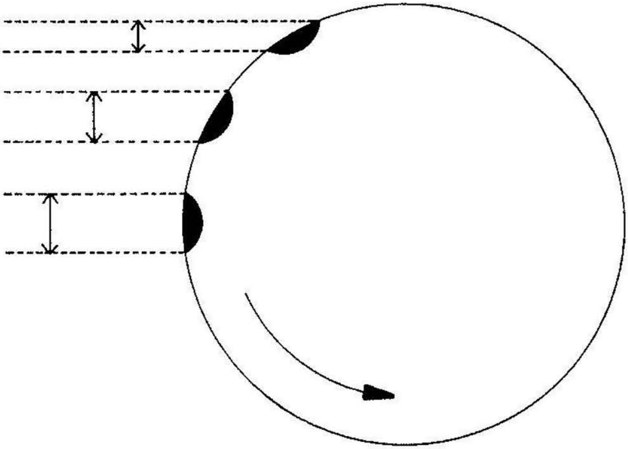

Figure 4.1 *Galileo's reasoning concerning the foreshortening of sunspots as they approach the sun's limb, to show that they are on the sun's surface.*

external properties of things (here, the apparent size, shape, and motion of the sunspots) to a properly *physical* conclusion about the matter of the heavens.

As he explained elsewhere in his published contributions to the debate with Scheiner, the true essences of things as distant as the celestial bodies cannot be determined by the senses, and indeed the same should be understood also of bodies near at hand: "I know no more about the true essences of earth or fire than about those of the moon or sun, for that knowledge is withheld from us, and is not to be understood until we reach the state of blessedness."[13] Hence all that remains to us is knowledge of those manifest properties which *are* accessible to the senses.

> Hence I should infer that although it may be in vain to seek to determine the true substance of the sunspots, still it does not follow that we cannot know some properties of them, such as their location, motion, shape, size, opacity, mutability, generation, and dissolution. These in turn may become the means by which we shall be able to philosophize better about other and more controversial qualities of natural substances.[14]

Not only could the manifest (and measurable) properties of bodies be known, but such knowledge would enable better philosophizing. The work of the mathematician, that is, could guide that of the physicist.

III The rising status and cognitive ambitions of the mathematical sciences: Galileo and Kepler

Galileo sometimes used the self-descriptive label "philosophical astronomer"[15] to represent the kind of work that he purported to be achieving in his work on sunspots and on the Copernican world-system. There is a hint of continuing deference to the category of natural philosopher, if not to natural philosophers themselves, in the way he liked to characterize himself. While negotiating with the Tuscan court in 1610 over the terms of his new service to the Medici (see Chapter 6, section II, below), Galileo insisted that his official title be that of court "philosopher and mathematician." It was common for a princely court to retain a mathematician (Tycho Brahe and Kepler both played that role), but this was clearly insufficient for Galileo. He wanted to be recognized also, and perhaps first, as a philosopher, someone who had things to say about the nature, not just the disposition, of the universe.

The Jesuit Biancani's arguments for the full causal character of mathematical demonstration expressed very much the same sentiment. In Biancani's case, however, there was no real attempt (Clavius's paean to the peculiar certainty of mathematics notwithstanding) to set up the techniques of mathematicians as potentally superior alternatives to those of the physicists. The Jesuit mathematicians' goal seems to have been one of achieving parity with their natural-philosophical colleagues; Galileo's goal was to reform natural philosophy itself, so that it would be recognized as a discipline for mathematicians. Either way, such promotion of mathematical sciences as exemplary ways of learning about the natural world typifies a widespread movement in the first half of the seventeenth century. It was a movement that began to be recognizable through its gradual adoption of an identifying label: "physico-mathematics."

The value of this label sprang from its imprecision. It served to unite the notion of the physical with that of the mathematical, but the nature of the juxtaposition was ambiguous. It apparently designated a kind of mathematics (in the broad contemporary understanding of that word) that was in some way of physical relevance. There were older, pre-existent terms for what looks like the same thing, as we have seen in Chapter 1, section II: "mixed mathematics" was perhaps the most common. And yet there seems to have been a felt need for the new term. Why?

This is where Galileo is such a useful figure. His endeavours help us to understand what the spread of "physico-mathematics" meant to those who eagerly adopted the term. Galileo's polemics and propaganda set into relief, perhaps in exaggerated form, those issues the debating of which form the core of what we can call the Scientific Revolution. These issues concerned the question of the proper character of natural philosophy: what should it be about, how should it be pursued, and why? Chapter 3 considered the attempts of people like Francis Bacon to reform notions of what the purpose of natural philosophy should be. In arguing that it ought to be directed towards practical utility, Bacon at the same time effectively altered the ways in which it should be conducted, as well as how its knowledge-claims should be constituted and presented (his new definition of "forms"). The endeavours of the mathematicians, while different in focus and scope, acted in concert with this new stress on knowledge for practical use to promote a view of natural philosophy that emphasized the operational. In doing so, they came close to rejecting natural philosophy in its old sense in favour of an entirely different enterprise, simply applying to it an old name borrowed from the rejected discipline.

The case of Galileo illustrates how this complete break in fact failed to take place. He, in common with users of the contemporary term "physico-mathematics," retained a claim to the label of natural philosopher. The properties that he and other mathematicians wished to attribute to mathematical knowledge, properties that they resented the physicists for denying to it, were lifted from natural philosophy itself. Mathematicians did not simply declare the virtues of the mathematical sciences in isolation from those of physics; the relative status of the two disciplinary areas meant that mathematicians would still have been left – however certain their demonstrations – in command of what most others saw as an inferior kind of knowledge. In this regard the mathematicians resembled the craftsmen. The change in values expressed by Bacon involved the investing of practical, artisanal knowledge with a higher social status. It had been (and to a considerable extent continued to be) associated with low-status work – manual labour. Bacon in particular argued for a higher evaluation of utility by claiming its importance for the state, as well as through moral and religious arguments that associated it with Christian charity. And yet he wanted this newly-upgraded practical knowledge to receive the prestige already possessed by natural philosophy. His solution was to argue as if "natural philosophy" were a category much broader in scope than usually admitted by academics, one that *included* practical knowledge; he then chased out purely contemplative knowledge by criticizing the goals of the latter, thus leaving the field to his own proposed endeavour.

Similarly, Galileo and other mathematicians rejected the disciplinary boundary between natural philosophy and mathematics by arguing that mathematics was crucially important in drawing legitimate *physical* conclusions. In effect, the label "physico-mathematics" served to signal that the mathematicians' own expertise would not thereby be subsumed to that of the natural philosophers. Instead, the cuckoo's egg of physico-mathematics would (if Galileo had his way) serve to expel most of the original occupants of the natural-philosophical nest, so as to leave the mathematicians in the position formerly occupied by the physicists. In both this and the previous case, the established category "natural philosophy" was a valuable resource for those who wanted to raise the status of their own favoured kind of knowledge.

Another important advocate of the central place of mathematics in natural philosophy was the Copernican astronomer Johannes Kepler. Kepler's approach to astronomy was, like any astronomer of the time, fundamentally mathematical. But he went much further in his promotion of mathematics than most of his colleagues: for Kepler, the mathematics that structured astronomical theory was the very mathematics that underlay the structure of the universe itself. Thus, in his work as a mathematical astronomer, Kepler at the same time endeavoured to create a mathematical *physics*. For Kepler, the universe is properly intelligible in mathematical terms; it is mathematics, especially geometry, which allows insight into the mind of God, the Creator, and hence into the deepest realms of natural philosophy. In one of his last publications, a work of 1618 called *Epitome astronomiae Copernicanae* ("Epitome of Copernican Astronomy"), Kepler describes his own special field as a *part* of physics:

> What is the relation between this science [astronomy] and others? 1. It is a part of physics, because it seeks the causes of things and natural occurrences, because the motion of the heavenly bodies is amongst its subjects, and because one of its purposes is to inquire into the form of the structure of the universe and its parts. . . . To this end, [the astronomer] directs all his opinions, both by geometrical and by physical arguments, so that truly he places before the eyes an authentic form and disposition or furnishing of the whole universe.[16]

Kepler put these principles into effect in his restructuring of Copernican astronomy. As a student at the Lutheran university in the German town of Tübingen, he had become convinced of the truth of the new Copernican cosmology from his teacher in astronomy, Michael Mästlin. Belief in the literal truth of the Copernican system, as opposed to a recognition of the value of Copernicus's *De revolutionibus* in the practical computational work of mathematical astronomy, was not widespread among astronomers at this time, and Kepler's early guidance by one of the exceptions to this rule is therefore noteworthy. Kepler's metaphysical and theological predilections expressed themselves in relation to Copernican astronomy in his first publication, the *Mysterium cosmographicum* ("Cosmographical Mystery") of 1596, when Kepler was working as a school teacher in Austria. The most noteworthy feature of the work is its presentation of Kepler's proud discovery of a relationship between the dimensions of the planetary orbits (calculated according to the Copernican system) and certain interrelationships among the so-called "perfect" or "Platonic" or "regular" solids.

The latter were solid figures that had been demonstrated by Euclid to be restricted to precisely five in number. They were solids that are contained by identical facets which are themselves regular polygons, such as equilateral triangles, squares, or pentagons. The five solids, as Euclid had shown, were the tetrahedron, the cube, the octahedron, the dodecahedron,

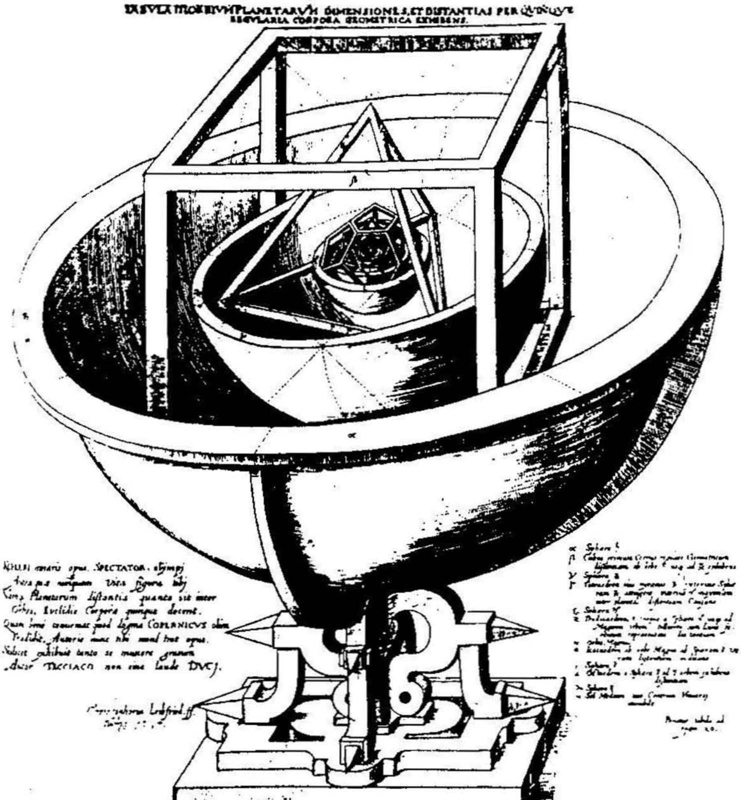

Figure 4.2 *The nested perfect solids structuring the universe, from Kepler's* Mysterium cosmographicum.

and the icosahedron, of four, six, eight, twelve, and twenty faces respectively. The fact that these five solids were unique of their kind implied to Kepler that they represented something profound about the nature of space and of the geometrical principles on the basis of which God had created the universe. In the *Mysterium cosmographicum,* he shows that (imaginary) spheres used to represent the relative sizes of the various Copernican planetary orbits around the sun are separated by various distances that closely accommodate the perfect solids as spacers between the spheres. Using available data, Kepler was able to show that the sizes of the planetary orbits closely fit the sizes allowed by the intercalated solids, to within an error of around five per cent. In 1600 he joined Tycho Brahe in Prague so as to gain access to Tycho's famed data on planetary motions, which Kepler hoped would enable him to reduce the error still further. Furthermore, Kepler's model accounted for there being, in a Copernican universe, precisely six planets – the number that could be adequately spaced by five intervening solids.

Kepler was enormously proud of this result, which he believed brought him nearer to an intimate understanding of the structure of God's Creation. The rôle of geometry in his argumentation was fundamental: geometry was not simply a tool for calculating dimensions and motions in astronomy; it was capable of providing *explanations* of why things in the world are as they are. The geometry of the five perfect solids serves not only to *describe* the number of the planets and their distances from the sun, but to make *sense* of those facts. Kepler believed in a fundamentally mathematical constitution to the universe, in the sense that mathematical intelligibility of the kind provided by the perfect solids accounted for *why* certain things are as they are. The nature of such an explanation is not, in the present case, one that provides mathematical, demonstrative *necessity* to the things that it explains (as with showing, as Euclid does, *why* the base angles of an isosceles triangle are equal to one another); but it does show, Kepler believed, what was in God's mind when He chose to create things in the way that He did. In many respects, in fact, Kepler's entire astronomical career was one directed towards gaining an understanding of God's mind, of coming closer to God through the medium of astronomical study. This was natural philosophy in its starkest, most theocentric form.

Kepler's major work was the *Astronomia nova* of 1609. It was the published result of a project that he had originally undertaken at the behest of Tycho, to determine a satisfactory astronomical model for the motion of Mars. Mars had always been a planet whose motion was particularly troublesome to model with exactness, and since Tycho's great observational project had been designed as the foundation for much more accurate planetary models, the continuing recalcitrance of Mars was a source of especial concern to him. Tycho was particularly interested in having Kepler solve the difficulties in terms of Tycho's own favoured cosmological system, a kind of compromise between Ptolemy and Copernicus that he had first published in a book of 1588. This scheme had the moon and sun in orbit around a central, stationary earth, but with the planets orbiting that moving sun. The resultant relative motions thus remained the same as in Copernicus's system (disregarding the issue of the fixed stars), with Copernicus's annual orbit of the earth around the sun being exactly mirrored in the annual orbit of the sun around the earth. Kepler responded to the challenge by producing models that could be expressed in Ptolemaïc, Copernican, or Tychonic terms (simply by shifting reference-frames). But, for Kepler, the Copernican remained the true account.

Several years of intensive work by Kepler resulted in an achievement that was remarkable in several ways. First, Kepler produced a model for the motion of Mars of unparalleled accuracy, both as determined by comparison with Tycho's observations and as confirmed over time by its predictions. Second, in doing so, he had come to abandon the classical Greek astronomical requirement, followed proudly by Copernicus as well as by Tycho himself, that the component motions used in creating astronomical models each be a uniform motion around a circle. Third, Kepler developed his new laws governing planetary motion on a basis that involved speculation about the *physical* causes that brought about that motion.

His new planetary orbits around the sun took the form of ellipses, with one focus of each ellipse located on the sun itself. He knew the geometry of the ellipse, one of the conic sections, from the treatise on conic sections written by the Greek astronomer and mathematician Apollonius of Perga, and Kepler's desire to find mathematics written in the fabric of the universe was thoroughly satisfied by this result, even though it meant abandoning circles. Furthermore, his elliptical orbits were traversed by the planets (including the earth) in such a way that the space swept out by the line joining the planet to the sun was uniform – equal areas swept out in equal times.

Equally importantly for Kepler, however, he had achieved these results in continual dialogue with ideas on the causes of planetary motions. These included the idea of a motive force emanating from the sun that drove the planets around in their orbits, together with an idea about a kind of magnetic attraction and repulsion between the sun and the two poles of each planet that served to explain why planetary orbits were not perfectly circular. Making explicit reference to William Gilbert, Kepler used his notion of the earth as a giant magnet to explain why planets successively approach and depart from the sun in the course of their elliptical orbits. The celestial spheres were gone (Tycho had already rejected them); Kepler's planets moved independently through space.

Kepler's views on the place of mathematics in understanding the physical world were thus more directly related to a purely philosophical, as opposed to practical, conception of natural knowledge than were Galileo's. The very nature of the mixed mathematical sciences, however, was such as to encourage, even in Kepler, a concern with some operational criteria of knowledge. The instrumental function of optics in assisting astronomical investigations was a major part of his justification for publishing *Ad Vitellionem paralipomena quibus astronomiae pars optica traditur* ("Additions to Witelo, in which the Optical Part of Astronomy is Treated"), in 1604.[17] Kepler considers the imperfection of sciences such as astronomy and optics, as compared to the demonstrative ideal of geometry, but argues

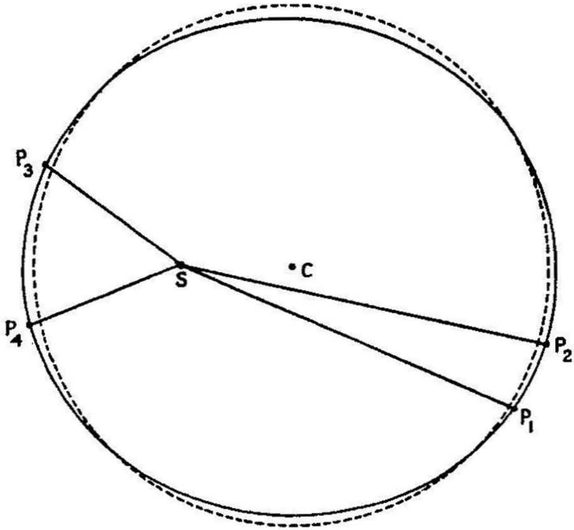

Figure 4.3 *Kepler's elliptical orbit for the planets, and his area-law. The planet, P, pursues its elliptical path with the sun, S, at one focus. The line joining the planet to the sun sweeps out equal areas in equal times, so that the distance traversed by the planet when nearer to the sun (P_3–P_4) is greater than that traversed when farther from the sun (P_1–P_2). From Marie Boas,* The Scientific Renaissance 1450–1630 *(New York: Harper and Brothers, 1962), © 1962 by Marie Boas.*

that optical theorems should be sufficient to satisfy an astronomer's needs.[18]

IV Knowing, doing, and mathematics

Mathematics was itself traditionally related to practical endeavours such as land-surveying or the building of fortifications. Both fell under the heading of "mixed mathematics," along with such others as astronomy and mechanics. The latter too were of great practical importance. Astronomy had been valued in Latin Europe since the Middle Ages for its use in marine navigation and in astrology, a practical art much used in learned medieval medicine. Mechanics concerned machines themselves (such as wind or water mills), but more especially discussed the classical domain of the so-called simple machines, which considered certain devices and techniques (such as levers or pulleys) that made work easier. The practical and artisanal associations of many of the mathematical sciences were thus very hard to miss.

During the second half of the sixteenth century, mathematicians, especially in England, had begun to make strong claims for their discipline that revolved around its practical dimensions rather than focusing on the more philosophical justifications preferred by increasing numbers of bookish mathematicians. In 1570 there appeared a new translation into English of Euclid's *Elements*, bearing a preface written by John Dee of Mortlake. He used this opportunity to praise the branches of mathematics for their usefulness "in the Common lyfe and trade of men," as witnessed by the practices of many and diverse occupations.[19] Dee had himself already had dealings with one such endeavour, navigation; the interrelated concerns of navigation (including in this period increasing interest in the magnetic compass and terrestrial magnetism) and of cartography were important, and unassailably mathematical, subjects of books by a number of English authors in the decades around 1600, such as Robert Recorde, Thomas Digges, and Edward Wright. Most such authors wrote in English rather than Latin, and presented themselves as men of practical rather than contemplative bent. Typical examples of the genre include works on surveying techniques, the demand for which seems to have grown during the second half of the sixteenth century, in concert with the increasing enclosure of formerly common land and the surveying of church lands now seized by the Crown following the English Reformation.

Mathematics thus had, besides its association with learned classical treatises and the niceties of formal demonstration, a practical, computational image somewhat at odds with the academic, philosophical discipline promoted by scholars such as Clavius. At the same time, its leaning towards practicality enabled it to appeal to the same sensibilities that Bacon's propaganda exploited. The kind of knowledge that mathematical practices tended to promote was not simply utilitarian, however: its elevation to philosophical importance by such as Galileo implied a revaluing of mathematical characteristics as being peculiarly important to true understanding of nature.