Zadania:

1. Riešte DR $y^{(5)} - 4y^{(4)} + 5y^{(3)} - 4y'' + 4y' = 4e^x + 8x + 4$.

2. Určte $\rho_C(f,g)$ a $\rho_I(f,g)$, ak je $f(x) = x^2$ a $g(x) = 1 - x^2$ a

$\rho_C(f,g) = \max\limits_{x\in[0,a]} |f(x) - g(x)|$ a $\rho_I(f,g) = \int\limits_0^a |f(x) - g(x)| dx$, kde $a \in \mathbb{R}^+$.

3. Určte vnútro, uzáver a hranicu množiny $A = \{(x,y) : (x-1)^2 + (y-1)^2 \leq 1\} \cup \{(x,y) : x = y\}$ v $\mathbb{E}^2$.

Riešenia:

1.

Korene rovnice $r^5 - 4r^4 + 5r^3 - 4r^2 + 4r = 0$ sú $0, 2, 2, i, -i$. Riešenie homog. DR je $y_0 = c_1 + c_2 e^{2x} + c_3 x e^{2x} + c_4 \cos x + c_5 \sin x$. Partikulárne riešenia hľadáme v tvare $y_1 = de^x$ a $y_2 = x(ax+b)$, vyjde $d = 2$, $a = 1, b = 3$.

Riešenie je $y = y_0 + y_1 + y_2 = c_1 + c_2 e^{2x} + c_3 x e^{2x} + c_4 \cos x + c_5 \sin x + 2e^x + x^2 + 3x$.

2.

$|f(x) - g(x)| = 1 - 2x^2$ ak $x < \frac{1}{\sqrt{2}}$ a $|f(x) - g(x)| = 2x^2 - 1$ ak $x \geq \frac{1}{\sqrt{2}}$.

$$\rho_C(f,g) = \max_{x\in[0,a]} |f(x) - g(x)| = \max_{x\in\{0,a\}} |f(x) - g(x)| = \max\{1, |1 - 2a^2|\} = \begin{cases} 1, & a \leq 1 \\ 2a^2 - 1, & a > 1 \end{cases}$$

$\rho_I(f,g) = \int\limits_0^a |f(x) - g(x)| dx = \int\limits_0^{\frac{1}{\sqrt{2}}} 1 - 2x^2 dx + \int\limits_{\frac{1}{\sqrt{2}}}^a 2x^2 - 1 dx = \frac{2}{3}a^3 - a + 2\sqrt{2}$. Alebo, pre $a < \frac{1}{\sqrt{2}}$, je $\rho_I(f,g) = \int\limits_0^a 1 - 2x^2 dx = a - \frac{2}{3}a^3$.

3.

$A^\circ = \{(x,y) : (x-1)^2 + (y-1)^2 < 1\}$,

$\bar{A} = A$,

$h(A) = \{(x,y) : (x-1)^2 + (y-1)^2 = 1\} \cup \{(x,y) : x = y\}$.