

Zadania:

1. Riešte DR $y^{(5)} - 4y^{(4)} + 5y^{(3)} - 4y'' + 4y' = 4e^x + 8x + 4$.

2. Určte $\rho_C(f, g)$ a $\rho_I(f, g)$, ak je $f(x) = x^2$ a $g(x) = 1 - x^2$ a

$$\rho_C(f, g) = \max_{x \in [0, a]} |f(x) - g(x)| \quad \text{a} \quad \rho_I(f, g) = \int_0^a |f(x) - g(x)| dx, \text{ kde } a \in \mathbb{R}^+.$$

3. Určte vnútro, uzáver a hranicu množiny $A = \{(x, y) : (x - 1)^2 + (y - 1)^2 \leq 1\} \cup \{(x, y) : x = y\}$ v \mathbb{E}^2 .

Riešenia:

1.

Korene rovnice $r^5 - 4r^4 + 5r^3 - 4r^2 + 4r = 0$ sú $0, 2, 2, i, -i$. Riešenie homog. DR je $y_0 = c_1 + c_2 e^{2x} + c_3 x e^{2x} + c_4 \cos x + c_5 \sin x$. Partikulárne riešenia hľadáme v tvare $y_1 = d e^x$ a $y_2 = x(ax + b)$, vyjde $d = 2, a = 1, b = 3$.

Riešenie je $y = y_0 + y_1 + y_2 = c_1 + c_2 e^{2x} + c_3 x e^{2x} + c_4 \cos x + c_5 \sin x + 2e^x + x^2 + 3x$.

2.

$$|f(x) - g(x)| = 1 - 2x^2 \text{ ak } x < \frac{1}{\sqrt{2}} \quad \text{a} \quad |f(x) - g(x)| = 2x^2 - 1 \text{ ak } x \geq \frac{1}{\sqrt{2}}.$$

$$\rho_C(f, g) = \max_{x \in [0, a]} |f(x) - g(x)| = \max_{x \in \{0, a\}} |f(x) - g(x)| = \max\{1, |1 - 2a^2|\} = \begin{cases} 1, & a \leq 1 \\ 2a^2 - 1, & a > 1 \end{cases}$$

$$\rho_I(f, g) = \int_0^a |f(x) - g(x)| dx = \int_0^{\frac{1}{\sqrt{2}}} 1 - 2x^2 dx + \int_{\frac{1}{\sqrt{2}}}^a 2x^2 - 1 dx = \frac{2}{3}a^3 - a + 2\sqrt{2}. \text{ Alebo, pre}$$

$$a < \frac{1}{\sqrt{2}}, \text{ je } \rho_I(f, g) = \int_0^a 1 - 2x^2 dx = a - \frac{2}{3}a^3.$$

3.

$$A^\circ = \{(x, y) : (x - 1)^2 + (y - 1)^2 < 1\},$$

$$\bar{A} = A,$$

$$h(A) = \{(x, y) : (x - 1)^2 + (y - 1)^2 = 1\} \cup \{(x, y) : x = y\}.$$