

Hygrometry

The objective of atmospheric humidity measurements is to determine the amount of water vapor present in the atmosphere by weight, by volume, by partial pressure, or by a fraction (percentage) of the saturation (equilibrium) vapor pressure with respect to a plane surface of pure water. The measurement of atmospheric humidity in the field has been and continues to be troublesome. It is especially difficult for automatic weather stations where low cost, low power consumption, and reliability are common constraints.

5.1 Water Vapor Pressure

Pure water vapor in equilibrium with a plane surface of pure water exerts a pressure designated e'_s . This pressure is a function of the temperature of the vapor and liquid phases and can be obtained by integration of the Clausius–Clapeyron equation, assuming linear dependence of the latent heat of vaporization on temperature, $L = L_0[1 + \alpha(T - T_0)]$,

$$e'_s = e'_{s0} \exp \left[\frac{L_0}{R_v} \left(\frac{T - T_0}{T T_0} + \alpha \ln \left(\frac{T}{T_0} \right) - \frac{\alpha(T - T_0)}{T} \right) \right] \quad (5.1)$$

where $T_0 = 273.15$ K, $L_0 = 2.5008 \times 10^6$ J kg⁻¹, the latent heat of water vapor at T_0 , $R_v = 461.51$ J kg⁻¹ K⁻¹, the gas constant for water vapor, $e'_{s0} = 611.21$ Pa, the equilibrium water vapor pressure at $T = T_0$, and $\alpha = -9.477 \times 10^{-4}$ K⁻¹ = average rate of change coefficient for the latent heat of water vapor with respect to temperature.

Table 5-1 Coefficients for the empirical equation 5.2 for equilibrium vapor pressure over a plane surface of pure water and over ice.

Coefficient	Water	Ice
c_1	-2991.272	0.0
c_2	-6017.0128	-5865.3696
c_3	18.876 438 54	22.241 033
c_4	-0.028 354 721	0.013 749 042
c_5	$0.178 383 01 \times 10^{-4}$	$-0.340 317 75 \times 10^{-4}$
c_6	$-0.841 504 17 \times 10^{-9}$	$0.269 676 87 \times 10^{-7}$
c_7	$0.444 125 43 \times 10^{-12}$	0.0
c_8	2.858 487	0.691 865 1

Since water vapor is not a perfect gas, the above equation is not an exact fit. The vapor pressure as a function of temperature has been determined by numerous experiments. Wexler (1976, 1977) fitted an empirical equation to the experimental vapor pressure data,

$$e'_s = \exp\left(\frac{c_1}{T^2} + \frac{c_2}{T} + c_3 + c_4 T + c_5 T^2 + c_6 T^3 + c_7 T^4 + c_8 \ln T\right) \quad (5.2)$$

(T in K, e'_s in Pa) and the coefficients for vapor pressure over water and over ice are given in table 5-1.

Both eqns. 5.1 and 5.2 are cumbersome; an equation that is readily inverted but with sufficient accuracy would be preferable. Buck (1981) developed an equation that is easy to use and sufficiently precise over the temperature range -30 to 50°C ,

$$e'_s = 6.1121 \exp\left(\frac{17.502T}{240.97 + T}\right) \quad (5.3)$$

where T is in degrees Celsius and e'_s is in units of hPa (or mb). Equations 5.1, 5.2, and 5.3 are contrasted in table 5-2 and fig. 5-1. The error in eqn. 5.1 is tolerable but eqn. 5.3 is preferred because it is easier to invert to obtain the dew-point temperature given the ambient vapor pressure.

As noted above, the term equilibrium vapor pressure is more accurate but the term saturation vapor pressure is commonly used. We will use the term saturation vapor pressure.

Contrary to Dalton's¹ law of partial pressures, the total air pressure does have a small affect on the saturation vapor pressure; this is called the enhancement effect (Buck, 1981); see table 5-3. Saturation vapor pressure in a mixture of dry air and water vapor (moist air) is the saturation vapor pressure of pure water vapor times the enhancement factor: $e_s = e'_s(T)f(T, p)$.

There is a pressure effect and a weak temperature dependence. For $p > 800$ hPa, we can use $f = 1.004$. The enhancement factor has been incorporated into the following equations for the vapor pressure, and so eqn. 5.3 becomes

$$e_s = 6.1365 \exp\left(\frac{17.502T}{240.97 + T}\right) \quad (5.4)$$

and the equilibrium vapor pressure over an ice surface is

Table 5-2 Comparison of the theoretical equation (5.1) for water vapor pressure with the expression obtained from experimental results (5.2) and the more convenient approximation (5.3).

Temperature (°C)	Experimental results, eqn. 5.2 (hPa)	Error in Buck approx., eqn. 5.3 (hPa)	Error in eqn. 5.1 (hPa)
0	6.1121	0.0	0.0
30	42.4520	-0.0169	-0.0789
50	123.4476	0.2447	-0.5831

Table 5-3 Enhancement factor for various temperatures and pressures.

T(°C)	Enhancement factor f (dimensionless)		
	$p = 1000$ hPa	$p = 500$ hPa	$p = 250$ hPa
-50	1.0058	1.0029	1.0014
-40	1.0052	1.0026	1.0013
-30	1.0047	1.0024	1.0012
-20	1.0044	1.0022	1.0012
-10	1.0041	1.0022	1.0012
0	1.00395	1.00219	1.00132
10	1.00388	1.00229	
20	1.004	1.00251	
30	1.00426	1.00284	
40	1.00467	1.00323	
50	1.00519		

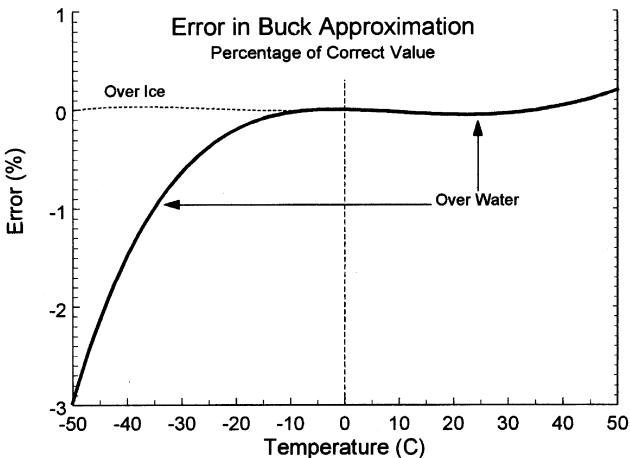


Fig. 5-1 Error in the Buck approximation.

$$e_i = 6.1359 \exp\left(\frac{22.452T}{272.55 + T}\right) \quad (5.5)$$

where, as before, T is in units of $^{\circ}\text{C}$ for both of the above equations.

Water vapor saturation pressure varies over two orders of magnitude in the normal temperature range; see fig. 5-2. On the basis of this figure, one would expect the accuracy of almost any humidity instrument to decrease with decreasing temperature.

Figure 5.2 can be used to illustrate several humidity relationships. Let point A represent ambient temperature and vapor pressure. Then the saturation vapor pressure is e_s (point B). If the air parcel were cooled, at constant pressure, until condensation just starts to occur, the new air temperature would be the dew-point temperature T_d and the ambient vapor pressure would be unchanged and would now be equal to the saturation vapor pressure at T_d (point D). Starting at point A again, a thermal bulb covered with water would be cooled by evaporation and the vapor pressure in the immediate vicinity would increase, due to the increased evaporation rate of water molecules, until the temperature of the wet bulb becomes the wet-bulb temperature T_w and the new vapor pressure would be the saturation vapor pressure at T_w , e_{sw} (point C).

5.2 Definitions

There are many variables commonly encountered in the study of humidity.

Absolute humidity, d_v , is the ratio of the mass of water vapor m_v to the total volume of moist air V in units of kg m^{-3} .

Dew-point temperature, T_d , is the temperature at which ambient water vapor condenses. The *frost-point temperature*, T_f , is the temperature at which ambient water

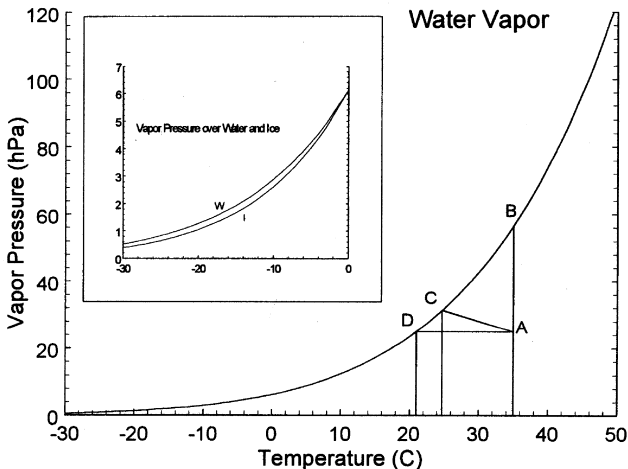


Fig. 5-2 Saturation vapor pressure as a function of temperature. Inset shows saturation vapor pressure with respect to water (top curve) and with respect to ice (bottom curve) for $T < 0^{\circ}\text{C}$.

vapor freezes. The dew- and frost-point temperatures can be obtained from the ambient vapor pressure by inverting eqns. 5.4 and 5.5:

$$\left. \begin{aligned} T_d &= 240.97 \ln(e/6.1365)/(17.502 - \ln(e/6.1365)) \\ T_f &= 272.55 \ln(e/6.1359)/(22.452 - \ln(e/6.1359)) \end{aligned} \right\} \quad (5.6)$$

Mixing ratio, w , is the ratio of the mass of water vapor m_v to the mass of dry air m_d .

Relative humidity, U , is defined as the ratio, expressed as a percentage, of the actual vapor pressure e to the saturation vapor pressure e_s at the air temperature T :

$$U = 100e/e_s \quad (5.7)$$

This definition always uses saturation vapor pressure with respect to a plane surface of pure water, even for temperatures below freezing. Some of the earliest humidity sensors, and still the most common, are the class of sorption sensors which, as will be shown later, generate an output proportional to relative humidity.

Specific humidity, q , also known as the mass concentration, is the ratio of the mass of water vapor m_v to the mass of moist air, $m_v + m_d$.

Temperature or dry-bulb temperature is the ambient air temperature T as measured, for example, by the dry-bulb thermometer of a psychrometer.

Vapor pressure, e , is the partial pressure of water vapor expressed in hPa.

Virtual temperature, T_v , is the temperature that dry air would have if the dry air had the same density as moist air at the same pressure. $T_v \geq T$:

$$T_v = \frac{T}{1 - \frac{e}{p}(1 - 0.622)}$$

Wet-bulb temperature, T_w , is the temperature indicated by the wet bulb of a psychrometer, that is, the temperature of a sensor covered with pure water that is evaporating freely into an ambient air stream.

The following relations are useful approximations that are sufficiently accurate for most meteorological applications. The temperature is in degrees Celsius.

$$w = 0.622e/(p - e)$$

$$e = wp/(0.622 + w)$$

$$q = w/(1 + w); \text{ when } e \ll p, w \approx q \approx 0.622 e/p$$

$$d_v = 0.2167e/(t + 273.15) \text{ kg m}^{-3}.$$

The formulae for mixing ratio w and specific humidity q are dimensionless; from the definition of these variables, the units are kg/kg. Frequently, w and q are multiplied by 1000 because it is easier to write 15.2 than 0.0152, and then the assigned units are g/kg. The constant 0.622 in the expression for w is the ratio of the gas constant for dry air to the gas constant for water vapor.

EXAMPLE

Given $p = 1000$ hPa, $T = 35.00^\circ\text{C}$ and $e = 24.85$ hPa, find the relative humidity and the dew-point temperature. Compute the saturation vapor pressure using eqn. 5.4: