Řešte metodou separace proměnných.

1) $(x-1)y^3 - e^x y' = 0$

$$y = \begin{cases} \frac{\pm 1}{\sqrt{2xe^{-x}-2c}} & c \in \mathbb{R} \\ 0 \end{cases}$$

2) $y' = \frac{y-1}{x^2y^2}$

$$\frac{1}{2}y^2 + y + \ln|y-1| = -\frac{1}{x} + c$$

3) $y' - xy^2 = 2xy$

$$y = \begin{cases} -2 \\ \frac{2K}{e^{-x^2}-K} & K \in \mathbb{R} \end{cases}$$

4) $2(1+e^x)yy' = e^x$

$$y = \begin{cases} \sqrt{\ln(1+e^x)+c} \\ -\sqrt{\ln(1+e^x)+c} \end{cases}$$

5) $(1+e^x)y' + e^x y = 0$

$$y = \frac{K}{1+e^x}, K \in \mathbb{R}$$

6) $xy(1+y^2) - (1+x^2)y' = 0$

$$y = \begin{cases} (1+x^2)(1+y^2) = Ky^2 & K \in \mathbb{R}^+ \\ 0 \end{cases}$$

7) $y' = \frac{2x+1}{2(y-1)}$

$$y = \begin{cases} 1 + \sqrt{x^2+x+c} \\ 1 - \sqrt{x^2+x+c} \end{cases}$$

8) $y' = \frac{2x-1}{x^2}y$

$$y = Kx^2 e^{\frac{1}{x}}, K \in \mathbb{R}$$

9) $\frac{1}{x+1} - \frac{1}{y-1}y' = 0$

$$y = K(x+1)+1, K \in \mathbb{R} \setminus \{0\}$$

10) $\sin x \sin y y' = \cos x \cos y;\ y(\frac{\pi}{4}) = 0$

$$y = \arccos \frac{\sqrt{2}}{2\sin x}$$

11) $\frac{x^2+1}{x} + \frac{yy'}{y^2-1} = 0$

$$\ln|y^2-1| = -x^2 - \ln x^2 + c$$

12) $(y^2-1) + yy'(x^2-1) = 0$

$$y = \begin{cases} \sqrt{1 + K \cdot \frac{x-1}{x+1}} \\ -\sqrt{1 + K \cdot \frac{x-1}{x+1}} \end{cases}$$

13) $y' = 2\sqrt{y}\ln x;\ y(e) = 1$

$$y = (x\ln x - x + 1)^2$$

14) $x^2 + y'\cos y + 1 = 0$

$$\sin y = -(\frac{x^3}{3} + x) + c$$

15) $y'\cos^2 x = (1+\cos^2 x)\sqrt{1-y^2}$

$$y = \begin{cases} \sin(\operatorname{tg} x + x + c) \\ 1 \\ -1 \end{cases}$$

16) $2y - x^3 \cdot y' = 0$

$$y = K \cdot e^{-\frac{1}{x^2}}, K \in \mathbb{R}$$

17) $y' = e^x y$

$$y = Ke^{e^x}, K \in \mathbb{R}$$

18) $y'e^{x^2+y} = -\frac{x}{y}$

$$2ye^y - 2e^y = e^{-x^2} + C, C \in \mathbb{R}$$

19) $\theta' = \frac{\sin t}{\cos^2 \theta}, \qquad \theta = \theta(t)$

$$2\theta + \sin(2\theta) = -4\cos t + C$$

20) $y' + xy = y$

$$y = Ke^{x-\frac{x^2}{2}}, K \in \mathbb{R}$$

21) $y' = x^2(1+y^2)$

$$y = \operatorname{tg}\left(\frac{x^3}{3} + C\right)$$

22) $y - y^2 + xy' = 0$

$$y = \begin{cases} \frac{1}{1-Kx} & K \in \mathbb{R} \\ 0 \end{cases}$$

23) $e^{-y}(1+y') = 1$

$$y = -\ln(1-Ke^x), K \in \mathbb{R}$$

24) $y\ln y + xy' = 0$

$$y = e^{K/x}, K \in \mathbb{R}$$