

Řešte lineární diferenciální rovnici prvního řádu $y' + a(x)y = b(x)$:

$$1) (x+1)y' - 2y = (x+1)^4 \quad x > -1$$

$$[y = (\frac{x^2}{2} + x + c)(x+1)^2]$$

$$2) (2x+1)y' + y = \sqrt{2x+1} + 3$$

$$[y = \frac{1}{\sqrt{2x+1}}(c + x + 3 \cdot \sqrt{2x+1})]$$

$$3) y' \cos x + y \sin x = 1 \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$[y = (c + \operatorname{tg} x) \cdot \cos x]$$

$$4) y' + \frac{1}{x+1} \cdot y = \ln(x+1) \quad x > -1$$

$$[y = \frac{x+1}{2} \cdot \ln(x+1) - \frac{x+1}{4} + \frac{c}{x+1}]$$

$$5) y' + y \cos x = 2 \sin x \cos x$$

$$[y = 2(-1 + \sin x) + ce^{-\sin x}]$$

$$6) y' \cos x = (y + 2 \cos x) \sin x \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$[y = \frac{1}{2} \left(\frac{\sin^2 x}{\cos x} - \cos x + \frac{c}{\cos x} \right)]$$

$$7) y' = 1 + \frac{2x-1}{x^2} \cdot y$$

$$[y = x^2 + cx^2 e^{\frac{1}{x}}]$$

$$8) y' + y = x$$

$$[y = x - 1 + ce^{-x}]$$

$$9) y' + y = \cos x$$

$$[y = \frac{1}{2}(\sin x + \cos x) + c \cdot e^{-x}]$$

$$10) xy' + y = x^2 \quad x > 0$$

$$[y = \frac{x^2}{3} + \frac{c}{x}]$$

$$11) (1+x^2) \cdot y' - xy = 1$$

$$[y = x + c \cdot \sqrt{x^2 + 1}]$$

$$12) xy' + 2y = e^{-x^2}$$

$$[y = \frac{1}{2x^2} \left(c - e^{-x^2} \right)]$$

$$13) y' \cos x - y \sin x = \sin 2x \quad x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

$$[y = \frac{1}{\cos x} \left(c - \frac{\cos 2x}{2} \right)]$$

$$14) xy' - \frac{y}{x+1} = x \quad x > 0$$

$$[y = \frac{x}{x+1} (c + x + \ln x)]$$

$$15) y' + \frac{x}{x^2-1}y = \frac{\arcsin x}{1-x^2}, x \in (-1, 1)$$

$$[y = \frac{1}{2\sqrt{1-x^2}} [c + \arcsin^2 x]]$$

$$16) y' - y = \frac{1+x^2}{x} e^x$$

$$[y = e^x (c + \frac{x^2}{2} + \ln |x|)]$$

$$17) y' - y \operatorname{tg} x = \sin x$$

$$[y = -\frac{\cos x}{2} + \frac{c}{\cos x}]$$