

$-7x_1 - x_2 - 4x_3 \rightarrow \min$
 $x_1 + 4x_2 + 2x_3 = 6$
 $x_1 + x_2 + 2x_3 = 5/2$
 $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

	Σw	z	x_1	x_2	x_3	w_1	w_2	
w_1	0	0	1	4	2	1	0	6
w_2	0	0	1	1	2	0	1	2.5
z	0	1	7	1	4	0	0	0
Σw	1	0	2	5	4	0	0	8.5

	Σw	z	x_1	x_2	x_3	w_1	w_2	
x_2	0	0	0.25	1	0.5	0.25	0	1.5
w_2	0	0	0.75	0	1.5	-0.25	1	1
z	0	1	6.75	0	3.5	-0.25	0	-1.5
Σw	1	0	0.75	0	1.5	-1.25	0	1

	Σw	z	x_1	x_2	x_3	w_1	w_2	
x_2	0	0	0	1	0	0.333333	-0.333333	1.166667
x_3	0	0	0.5	0	1	-0.166667	0.666667	0.666667
z	0	1	5	0	0	0.333333	-2.333333	-3.833333
Σw	1	0	0	0	0	-1	-1	0

	z	x_1	x_2	x_3	
x_2	0	0	1	0	1.166667
x_3	0	0.5	0	1	0.666667
z	1	5	0	0	-3.833333

	z	x_1	x_2	x_3	
x_2	0	0	1	0	1.166667
x_1	0	1	0	2	1.333333
z	1	0	0	-10	-10.5

1.5

2.5

3

0.666667

$$\frac{2}{3}x_1 - x_2 \rightarrow \min$$

$$-x_1 + x_2 \leq 2$$

$$-\frac{1}{2}x_1 + x_2 \leq \frac{5}{2}$$

$$\frac{1}{3}x_1 - x_2 \leq \frac{1}{3}$$

$$x_1 \geq 0, x_2 \geq 0$$

	Z	x_1	x_2	w_1	w_2	w_3		
w_1	0	-1	1	1	0	0	2	2
w_2	0	-0.5	1	0	1	0	2.5	2.5
w_3	0	0.333333	-1	0	0	1	0.333333	
Z	1	-0.66667	1	0	0	0	0	
	Z	x_1	x_2	w_1	w_2	w_3		
x_2	0	-1	1	1	0	0	2	
w_2	0	0.5	0	-1	1	0	0.5	
w_3	0	-0.66667	0	1	0	1	2.333333	
Z	1	0.333333	0	-1	0	0	-2	
	Z	x_1	x_2	w_1	w_2	w_3		
x_2	0	0	1	-1	2	0	3	
x_1	0	1	0	-2	2	0	1	
w_3	0	0	0	-0.333333	1.333333	1	3	
Z	1	0	0	-0.333333	-0.66667	0	-2.333333	

$$\frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 + x_3^2 \rightarrow \min$$

$$x_1 + x_2 + x_3 = 1$$

$$2x_2 - x_3 = 0$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

	ΣW	x_1	x_2	x_3	y_1	y_2	u_1	u_2	u_3	Z_{11}	Z_{12}	Z_{13}	Z_{21}	Z_{22}
Z_{11}	0	1	0	0	1	0	-1	0	0	1	0	0	-1	0
Z_{12}	0	0	1	0	1	2	0	-1	0	0	1	0	0	-1
Z_{13}	0	0	0	2	1	-1	0	0	-1	0	0	1	0	0
W_1	0	1	1	1	0	0	0	0	0	0	0	0	0	0
W_2	0	0	2	-1	0	0	0	0	0	0	0	0	0	0
ΣW	1	1	3	0	0	0	0	0	0	0	0	0	0	0
Z_{11}	0	1	0	0	1	0	-1	0	0	1	0	0	-1	0
Z_{12}	0	0	0	0.5	1	2	0	-1	0	0	1	0	0	-1
Z_{13}	0	0	0	2	1	-1	0	0	-1	0	0	1	0	0
W_1	0	1	0	1.5	0	0	0	0	0	0	0	0	0	0
x_2	0	0	1	-0.5	0	0	0	0	0	0	0	0	0	0
ΣW	1	1	0	1.5	0	0	0	0	0	0	0	0	0	0
Z_{11}	0	1	0	0	1	0	-1	0	0	1	0	0	-1	0
Z_{12}	0	-0.33	0	0	1	2	0	-1	0	0	1	0	0	-1
Z_{13}	0	-1.33	0	0	1	-1	0	0	-1	0	0	1	0	0
x_3	0	0.667	0	1	0	0	0	0	0	0	0	0	0	0
x_2	0	0.333	1	0	0	0	0	0	0	0	0	0	0	0
ΣW	1	0	0	0	0	0	0	0	0	0	0	0	0	0

	ΣZ	x_1	x_2	x_3	y_1	y_2	u_1	u_2	u_3	Z_1	Z_2	Z_3
Z_1	0	1	0	0	1	0	-1	0	0	1	0	0
Z_2	0	0.333	0	0	-1	-2	0	1	0	0	1	0.333
Z_3	0	1.333	0	0	-1	1	0	0	1	0	0	1.333
x_2	0	0.333	1	0	0	0	0	0	0	0	0	0.333
x_3	0	0.667	0	1	0	0	0	0	0	0	0	0.667
ΣZ	1	2.667	0	0	-1	-1	-1	1	1	0	0	1.667
x_1	0	1	0	0	1	0	-1	0	0	1	0	0
Z_2	0	0	0	0	-1.33	-2	0.333	1	0	-0.33	1	0.333
Z_3	0	0	0	0	-2.33	1	1.333	0	1	-1.33	0	1.333
x_2	0	0	1	0	-0.33	0	0.333	0	0	-0.33	0	0.333
x_3	0	0	0	1	-0.67	0	0.667	0	0	-0.67	0	0.667
ΣZ	1	0	0	0	-3.67	-1	1.667	1	1	-2.67	0	1.667
x_1	0	1	0	0	0	-1.5	-0.75	0.75	0	0.75	0.75	0.25
y_1	0	0	0	0	1	1.5	-0.25	-0.75	0	0.25	-0.75	-0.25
Z_3	0	0	0	0	0	4.5	0.75	-1.75	1	-0.75	-1.75	0.75
x_2	0	0	1	0	0	0.5	0.25	-0.25	0	-0.25	-0.25	0.25
x_3	0	0	0	1	0	1	0.5	-0.5	0	-0.5	-0.5	0.5

Σz	1	0	0	0	0	4.5	0.75	-1.75	1	-1.75	-2.75	0	0.75
x_1	0	1	0	0	0	0	-0.5	0.167	0.333	0.5	0.167	0.333	0.5
y_1	0	0	0	0	1	0	-0.5	-0.17	-0.33	0.5	-0.17	-0.33	-0.5
y_2	0	0	0	0	0	1	0.167	-0.39	0.222	-0.17	-0.39	0.222	0.167
x_2	0	0	1	0	0	0	0.167	-0.06	-0.11	-0.17	-0.06	-0.11	0.167
x_3	0	0	0	1	0	0	0.333	-0.11	-0.22	-0.33	-0.11	-0.22	0.333
Σz	1	0	0	0	0	0	0	0	0	-1	-1	-1	0

Z ₂₃	W ₁	W ₂	
0	0	0	0
0	0	0	0
-1	0	0	0
0	1	0	1
0	0	1	0
0	0	0	1
0	0	0	0
0	0	-0.5	0
-1	0	0	0
0	1	-0.5	1
0	0	0.5	0
0	0	-1.5	1
0	0	0	0
0	-0.33	-0.33	-0.33
-1	-1.33	0.667	-1.33
0	0.667	-0.33	0.667
0	0.333	0.333	0.333
0	-1	-1	0

1

0

$$2x_1^2 + x_2^2 - 16x_1 - 12x_2 \rightarrow \min$$

$$3x_1 + 3x_2 + x_3 = 18$$

$$x_1 - x_2 + x_4 = 2$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$$

	ΣW	x_1	x_2	x_3	x_4	y_1	y_2	u_1	u_2	u_3	u_4	Z_{11}	Z_{12}	Z_{13}
Z_{11}	0	4	0	0	0	3	1	-1	0	0	0	1	0	0
Z_{12}	0	0	2	0	0	3	-1	0	-1	0	0	0	1	0
Z_{13}	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
Z_{14}	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
W_1	0	3	3	1	0	0	0	0	0	0	0	0	0	0
W_2	0	1	-1	0	1	0	0	0	0	0	0	0	0	0
ΣW	1	4	2	1	1	0	0	0	0	0	0	0	0	0
Z_{11}	0	0	4	0	-4	3	1	-1	0	0	0	1	0	0
Z_{12}	0	0	2	0	0	3	-1	0	-1	0	0	0	1	0
Z_{13}	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
Z_{14}	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
W_1	0	0	6	1	-3	0	0	0	0	0	0	0	0	0
x_1	0	1	-1	0	1	0	0	0	0	0	0	0	0	0
ΣW	1	0	6	1	-3	0	0	0	0	0	0	0	0	0
Z_{11}	0	0	0	-0.67	-2	3	1	-1	0	0	0	1	0	0
Z_{12}	0	0	0	-0.33	1	3	-1	0	-1	0	0	0	1	0
Z_{13}	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
Z_{14}	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
x_2	0	0	1	0.167	-0.5	0	0	0	0	0	0	0	0	0
x_1	0	1	0	0.167	0.5	0	0	0	0	0	0	0	0	0
ΣW	1	0	0	0	0	0	0	0	0	0	0	0	0	0

	ΣZ	x_1	x_2	x_3	x_4	y_1	y_2	u_1	u_2	u_3	u_4	Z_1	Z_2	Z_3
Z_1	0	0	0	0.667	2	-3	-1	1	0	0	0	1	0	0
Z_2	0	0	0	0.333	-1	-3	1	0	1	0	0	0	1	0
Z_3	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
Z_4	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
x_1	0	1	0	0.167	0.5	0	0	0	0	0	0	0	0	0
x_2	0	0	1	0.167	-0.5	0	0	0	0	0	0	0	0	0
ΣZ	1	0	0	1	1	-5	1	1	1	-1	-1	0	0	0
Z_1	0	0	0	0	4	3	-3	1	-2	0	0	1	-2	0
x_3	0	0	0	1	-3	-9	3	0	3	0	0	0	3	0
Z_3	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
Z_4	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
x_1	0	1	0	0	1	1.5	-0.5	0	-0.5	0	0	0	-0.5	0
x_2	0	0	1	0	0	1.5	-0.5	0	-0.5	0	0	0	-0.5	0

Σz	1	0	0	0	4	4	-2	1	-2	-1	-1	0	-3	0
x_4	0	0	0	0	1	0.75	-0.75	0.25	-0.5	0	0	0.25	-0.5	0
x_3	0	0	0	1	0	-6.75	0.75	0.75	1.5	0	0	0.75	1.5	0
z_3	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
z_4	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
x_1	0	1	0	0	0	0.75	0.25	-0.25	0	0	0	-0.25	0	0
x_2	0	0	1	0	0	1.5	-0.5	0	-0.5	0	0	0	-0.5	0
Σz	1	0	0	0	0	1	1	0	0	-1	-1	-1	-1	0
x_4	0	0	0	0	1	0	-0.75	0.25	-0.5	0.75	0	0.25	-0.5	-0.75
x_3	0	0	0	1	0	0	0.75	0.75	1.5	-6.75	0	0.75	1.5	6.75
y_1	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
z_4	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
x_1	0	1	0	0	0	0	0.25	-0.25	0	0.75	0	-0.25	0	-0.75
x_2	0	0	1	0	0	0	-0.5	0	-0.5	1.5	0	0	-0.5	-1.5
Σz	1	0	0	0	0	0	1	0	0	0	-1	-1	-1	-1
x_4	0	0	0	0	1	0	0	0.25	-0.5	0.75	-0.75	0.25	-0.5	-0.75
x_3	0	0	0	1	0	0	0	0.75	1.5	-6.75	0.75	0.75	1.5	6.75
y_1	0	0	0	0	0	1	0	0	0	-1	0	0	0	1
y_2	0	0	0	0	0	0	1	0	0	0	-1	0	0	0
x_1	0	1	0	0	0	0	0	-0.25	0	0.75	0.25	-0.25	0	-0.75
x_2	0	0	1	0	0	0	0	0	-0.5	1.5	-0.5	0	-0.5	-1.5
Σz	1	0	0	0	0	0	0	0	0	0	0	-1	-1	-1

	x_1	x_2	x_3	x_4	y_1	y_2	u_1	u_2	u_3	u_4	μ			
x_4	0	0	0	0	1	0	0	0.25	-0.5	0.75	-0.75	-2	2	
x_3	0	0	0	1	0	0	0	0.75	1.5	-6.75	0.75	30	18	
y_1	0	0	0	0	0	1	0	0	0	-1	0	0	0	
y_2	0	0	0	0	0	0	1	0	0	0	-1	0	0	
x_1	0	1	0	0	0	0	0	-0.25	0	0.75	0.25	-4	0	
x_2	0	0	1	0	0	0	0	0	-0.5	1.5	-0.5	-6	0	
	1	0	0	0	0	0	0	0	0	0	0	1	0	
x_4	0	0	0	0.067	1	0	0	0.3	-0.4	0.3	-0.7	0	3.2	10.67
μ	0	0	0	0.033	0	0	0	0.025	0.05	-0.23	0.025	1	0.6	
y_1	0	0	0	0	0	1	0	0	0	-1	0	0	0	
y_2	0	0	0	0	0	0	1	0	0	0	-1	0	0	
x_1	0	1	0	0.133	0	0	0	-0.15	0.2	-0.15	0.35	0	2.4	
x_2	0	0	1	0.2	0	0	0	0.15	-0.2	0.15	-0.35	0	3.6	24
	1	0	0	-0.03	0	0	0	-0.03	-0.05	0.225	-0.03	0	-0.6	
u_3	0	0	0	0.222	3.333	0	0	1	-1.33	1	-2.33	0	10.67	
μ	0	0	0	0.083	0.75	0	0	0.25	-0.25	0	-0.5	1	3	
y_1	0	0	0	0.222	3.333	1	0	1	-1.33	0	-2.33	0	10.67	
y_2	0	0	0	0	0	0	1	0	0	0	-1	0	0	

x_1	0	1	0	0.167	0.5	0	0	0	0	0	0	0	0	4
x_2	0	0	1	0.167	-0.5	0	0	0	0	0	0	0	0	2
	1	0	0	-0.08	-0.75	0	0	-0.25	0.25	0	0.5	0	-3	

x_1	x_2	x_3	x_4	μ
2.4	3.6	0	3.2	0.6
4	2	0	0	3
2.667	3.333	0	2.667	1

$\alpha = 0.833$

Z_{14}	Z_{21}	Z_{22}	Z_{23}	Z_{24}	W_1	W_2	μ	
0	-1	0	0	0	0	0	-16	0
0	0	-1	0	0	0	0	-12	0
0	0	0	-1	0	0	0	0	0
1	0	0	0	-1	0	0	0	0
0	0	0	0	0	1	0	0	18
0	0	0	0	0	0	1	0	2
0	0	0	0	0	0	0	0	20
0	-1	0	0	0	0	-4	-16	-8
0	0	-1	0	0	0	0	-12	0
0	0	0	-1	0	0	0	0	0
1	0	0	0	-1	0	0	0	0
0	0	0	0	0	1	-3	0	12
0	0	0	0	0	0	1	0	2
0	0	0	0	0	0	-4	0	12
0	-1	0	0	0	-0.67	-2	-16	-16
0	0	-1	0	0	-0.33	1	-12	-4
0	0	0	-1	0	0	0	0	0
1	0	0	0	-1	0	0	0	0
0	0	0	0	0	0.167	-0.5	0	2
0	0	0	0	0	0.167	0.5	0	4
0	0	0	0	0	-1	-1	0	0

6

2

Z_4	μ	
0	16	16
0	12	4
0	0	0
1	0	0
0	0	4
0	0	2
0	28	20
0	-8	8
0	36	12
0	0	0
1	0	0
0	-6	2
0	-6	0

24

12

0	-8	8
0	-2	2
0	30	18
0	0	0
1	0	0
0	-4	0
0	-6	0
0	0	0
0	-2	2
0	30	18
0	0	0
1	0	0
0	-4	0
0	-6	0
0	0	0
0.75	-2	2
-0.75	30	18
0	0	0
1	0	0
-0.25	-4	0
0.5	-6	0
-1	0	0

$$\frac{1}{2}(x_1^2 + x_2^2) - x_1 - 2x_2 \rightarrow \min$$

$$2x_1 + 3x_2 \leq 6$$

$$x_1 + 4x_2 \leq 5$$

$$-x_1 + 4x_2 \leq 4$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$d = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ -1 & 4 \end{bmatrix}$$

$$C^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 4 \end{bmatrix} = \begin{bmatrix} | \\ | \\ | \end{bmatrix}$$

$$h = \begin{bmatrix} -8 \\ -9 \\ -7 \end{bmatrix} + \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ -3 \end{bmatrix}$$

	y_1	y_2	y_3
	0	0	0
	0.153846154	0	0
	0.153846154	0.108597285	0
	0.153846154	0.108597285	0
	0.036895231	0.108597285	0
	0.036895231	0.204909809	0
	0.036895231	0.204909809	0
	0	0.204909809	0
	0	0.235294118	0
	0	0.235294118	0
	0	0.235294118	0
	0	0.235294118	0
	0	0.235294118	0

$$A^T y^* = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 0.235294118 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.235294118 \\ 0.941176471 \end{bmatrix}$$

x_1^*	0.764705882
x_2^*	1.058823529

3
4
4

$$\mathbf{b} = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

13	14	10
14	17	15
10	15	17

$$A^T \mathbf{y}^* + \mathbf{d} = \begin{bmatrix} -0.76470588 \\ -1.05882353 \end{bmatrix}$$

$$(x_1+x_2)/(2x_1+x_2) \rightarrow \min$$

$$x_1+3x_2=5$$

$$x_1 \geq 0, x_2 \geq 0$$

	Σw	y	z_1	z_2	z_3	w_1	w_2	
w_1	0	0	1	3	-5	1	0	0
w_2	0	0	2	1	0	0	1	1
y	0	1	-1	-1	0	0	0	0
Σw	1	0	3	4	-5	0	0	1
z_2	0	0	0.333333	1	-1.66667	0.333333	0	0
w_2	0	0	1.666667	0	1.666667	-0.333333	1	1
y	0	1	-0.66667	0	-1.66667	0.333333	0	0
Σw	1	0	1.666667	0	1.666667	-1.333333	0	1
z_1	0	0	1	3	-5	1	0	0
w_2	0	0	0	-5	10	-2	1	1
y	0	1	0	2	-5	1	0	0
Σw	1	0	0	-5	10	-3	0	1
z_1	0	0	1	0.5	0	0	0.5	0.5
z_3	0	0	0	-0.5	1	-0.2	0.1	0.1
y	0	1	0	-0.5	0	0	0.5	0.5
Σw	1	0	0	0	0	-1	-1	0

x_1	5
x_2	0

0
1

0
0.6