PLYN × HVĚZDY

$$\frac{\partial \rho}{\partial t} + \rho \operatorname{div} \vec{v} = 0 \qquad p = \rho c^2$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\rho}\nabla p - \nabla \Phi \qquad \nabla^2 \Phi = 4\pi G \rho$$

BOLTZMANNOVA ROVNICE

$$\frac{\partial f}{\partial t} + \nabla f \cdot \vec{v} - \nabla_v f \cdot \nabla \Phi = 0, \qquad f = f(\vec{x}, \vec{v}, t)$$

$$\nabla^2 \Phi = 4\pi G \int f \, d^3 v$$

MOMENTY BOLTZ. ROVNICE

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \langle \vec{v} \rangle = 0$$

$$\frac{\partial \langle v_i \rangle}{\partial t} + (\langle \vec{v} \rangle \cdot \nabla)\langle v_i \rangle = -\frac{1}{\rho}\frac{\partial \rho \sigma_{ij}^2}{\partial x_i} - \frac{\partial \Phi}{\partial x_i}$$

Self-gravitující disk s nulovou disperzí rychlostí

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R}\left\{\frac{\partial R\Sigma v_R}{\partial R} + \frac{\partial \Sigma v_\theta}{\partial \theta}\right\} = 0$$

$$\frac{\partial v_R}{\partial t} + v_R \frac{\partial v_R}{\partial R} + \frac{v_\theta}{R}\frac{\partial v_R}{\partial \theta} - \frac{v_\theta^2}{R} = -\frac{\partial \Phi}{\partial R}$$

$$\frac{\partial v_\theta}{\partial t} + v_R \frac{\partial v_\theta}{\partial R} + \frac{v_\theta}{R}\frac{\partial v_\theta}{\partial \theta} + \frac{v_R v_\theta}{R} = -\frac{1}{R}\frac{\partial \Phi}{\partial \theta}$$

$$\frac{\partial^2 \Phi}{\partial R^2} + \frac{1}{R}\frac{\partial \Phi}{\partial R} + \frac{1}{R^2}\frac{\partial^2 \Phi}{\partial \theta} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G\, \Sigma(R, \theta, t)\, \delta(z)$$

Počáteční rovnovážný stav:

$\Sigma = \Sigma_0 (R)$
$\Phi = \Phi_0(R,z)$
$v_R = 0$
$v_\theta = V(R) = R\,\Omega(R) > 0$

Lineární porucha:

$\Sigma = \Sigma_0 (R) + \Sigma_1 (R,\theta,t)$
$\Phi = \Phi_0(R,z) + \Phi_1(R,\theta,z,t)$
$v_R = v_{R1}(R,\theta,t)$
$v_\theta = V(R) + v_{\theta 1}(R,\theta,t)$

Linearizace rovnic s poruchou

$$\frac{\partial \Sigma_1}{\partial t} + \Omega \frac{\partial \Sigma_1}{\partial \theta} + \frac{1}{R} \frac{\partial R \Sigma_0 v_{R1}}{\partial R} + \frac{\Sigma_0}{R} \frac{\partial v_{\theta 1}}{\partial \theta} = 0$$

$$\frac{\partial v_{R1}}{\partial t} + \Omega \frac{\partial v_{R1}}{\partial \theta} - 2\,\Omega\, v_{\theta 1} = - \frac{\partial \Phi_1}{\partial R}$$

$$\frac{\partial v_{\theta 1}}{\partial t} + \Omega \frac{\partial v_{\theta 1}}{\partial \theta} + 2\, B\, v_{R1} = - \frac{1}{R} \frac{\partial \Phi_1}{\partial \theta}$$

$$\frac{\partial^2 \Phi_1}{\partial R^2} + \frac{1}{R} \frac{\partial \Phi_1}{\partial R} + \frac{1}{R^2} \frac{\partial^2 \Phi_1}{\partial \theta} + \frac{\partial^2 \Phi_1}{\partial z^2} = 4\,\pi\, G\, \Sigma_1\, \delta(z)$$

$$B = \frac{\varkappa^2}{4\Omega}$$

$$\varkappa^2 = \frac{\partial^2 \Phi}{\partial R^2} + \frac{3}{R} \frac{\partial \Phi}{\partial R}$$

Elementární řešení (vlnové módy):

Hustotní vlna spirálního tvaru

$$\Sigma_1 = \Sigma^*(R) \exp[i(\omega t - m\theta)] = \Sigma'(R) \exp[i(\omega t - m\theta + F(R))]$$

$\Sigma^*(R)$ komplexní: $\Sigma^*(R) = \Sigma'(R) \exp[\, i\, F(R)\,]$

**F(R): „fázový faktor“ (*phase factor*),
„tvarová funkce“ (*shape function*)**

podobně pro Φ_1, v_{R1} a $v_{\theta 1}$:

$$\Phi_1 = \Phi^*\,(R)\ \exp[i(\omega t - m\theta)]$$
$$v_{R1} = v_{R1}^*(R)\ \exp[i(\omega t - m\theta)]$$
$$v_{\theta 1} = v_{\theta 1}^*\,(R)\ \exp[i(\omega t - m\theta)]$$

Frekvence vlny:

$$\omega = \omega_R + i\omega_I$$

oscilující módy: $|\,\omega_R\,| \gg |\,\omega_I\,|$

- **tlumené:** $\omega_I > 0$
- **rostoucí:** $\omega_I < 0$ (*overstability*)
- **neutrální:** $\omega_I = 0$

nestabilní módy: $-\ \omega_I \gg |\,\omega_R\,|$

SPIRÁLNÍ STRUKTURA

$$\Phi_1(R,\theta,t) = \underline{\Phi}_1' e^{i(\omega t - m\theta + F(R))}$$

amplituda   frekv. oscilací   počet ramen

$$\Omega_p = \frac{\omega}{m}$$ "pattern speed"

pevné $\underline{t}$: $\Phi_1 = \underline{\Phi}_1' \cos\left[m\theta - F(R) - \theta_0(t)\right]$

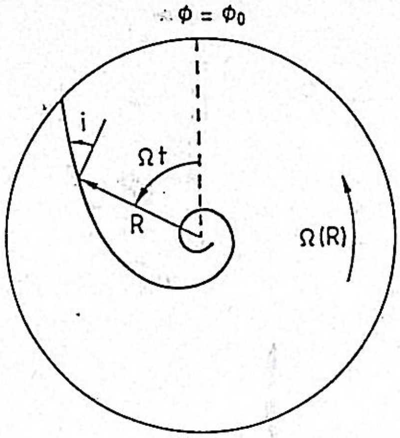
minimum Φ_1: $m\theta - F(R) - \theta_0(t) = 180^\circ$

$$\boxed{\theta_{min} = \frac{1}{m}\left[F(R) + \mathrm{const}\right]}$$

$$= \frac{1}{m} F(R) + \Omega_p t + 90^\circ$$

pro $\underline{m}$ ramen: $\theta_{min} = \frac{1}{m}\left[F(R) - 2\pi(k-1) + \mathrm{const}\right]$

$k = 1, \dots m$



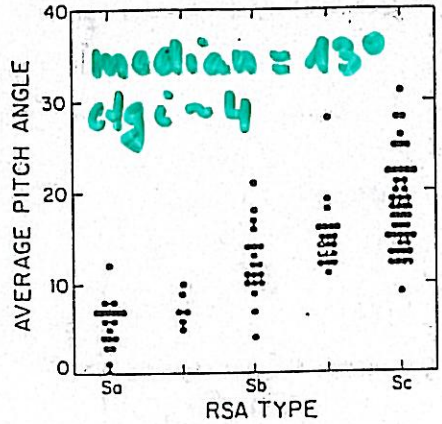


Figure 6-12. Measured pitch angle as a function of Hubble type for 113 galaxies (Kennicutt 1981). Reprinted by permission of *The Astronomical Journal.*

ÚHEL SKLONU SPIRÁLNÍHO RAMENA

"PITCH ANGLE"

$$\mathrm{ctg}\, i = R \left| \frac{\partial \theta}{\partial R} \right|$$

$$0^\circ \leq i \leq 90^\circ$$

- obecně $i = i(R)$
- vztah mezi $\underline{i}$ a $F(R)$:

$$\mathrm{ctg}\, i = \frac{R}{m} \left| \frac{\partial F}{\partial R} \right|$$

SPIRÁLY $\theta = \frac{1}{m}\left[F(R) - 2\pi(k-1) + C\right]$

$$\operatorname{ctg} i = R\left|\frac{\partial\theta}{\partial R}\right|$$

ARCHIMEDOVA

$$R = a\theta \Rightarrow F(R) = \frac{Rm}{a}$$

$$\operatorname{ctg} i = \frac{R}{a}$$

HYPERBOLICKÁ

$$R = \frac{a}{\theta} \Rightarrow F(R) = \frac{am}{R}$$

$$\operatorname{ctg} i = \frac{a}{R}$$

LOGARITMICKÁ

$$R = b e^{a\theta}$$

$$\ln R = \text{const} + a\theta \qquad F(R) = \frac{m \ln R}{a}$$

$$\operatorname{ctg} i = \frac{1}{a}$$

- RADIÁLNÍ SEPARACE SOUSEDNÍCH RAMEN

$$\left.\begin{aligned} m\theta_0 + F(R) &= \mathrm{const} \\ m\theta_0 + F(R+\Delta R) + 2\pi &= \mathrm{const} \end{aligned}\right\} \Rightarrow$$

$$|F(R+\Delta R) - F(R)| = 2\pi$$

$$\left| \frac{\partial F}{\partial R} \Delta R \right| = 2\pi \qquad (\text{pro } \Delta R \text{ malé})$$

- RADIÁLNÍ VLNOVÁ DÉLKA

$$\lambda \equiv \frac{2\pi}{\left|\frac{\partial F}{\partial R}\right|} > 0, \qquad \lambda(R,t)$$

- RADIÁLNÍ VLNOVÉ ČÍSLO

$$k \equiv \frac{\partial F}{\partial R} \qquad k(R,t)$$

$$\mathrm{ctg}\, i = \frac{R}{m} |k|$$

KONKÁVNÍ A KONVEXNÍ RAMENA

"TRAILING"

$$\frac{d\theta}{dR} < 0$$

$$k = \frac{dF}{dR} < 0$$

"LEADING"

$$\frac{d\theta}{dR} > 0$$

$$k = \frac{dF}{dR} > 0$$

from Binney & Tremaine

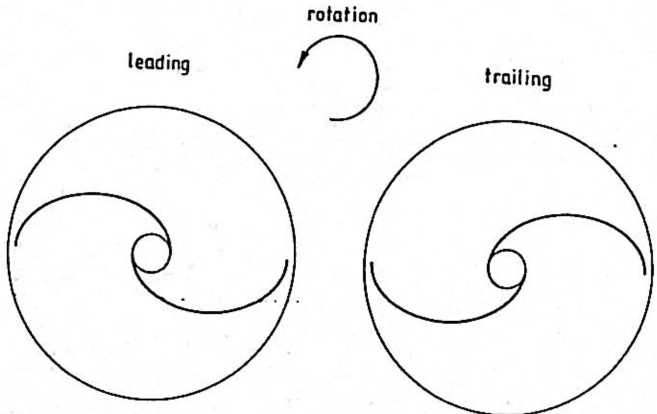


Figure 6-5. Leading and trailing arms.

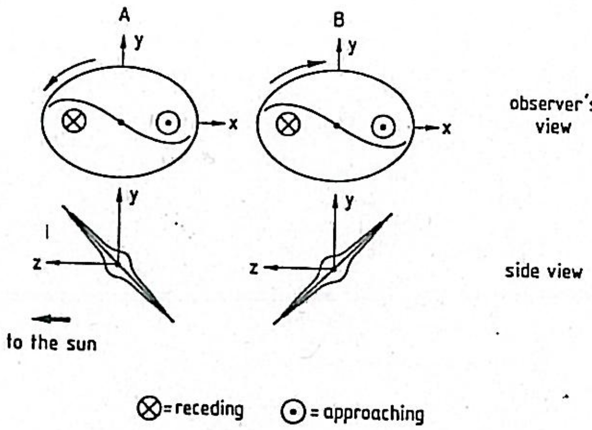


Figure 6-6. The appearance of leading and trailing arms. Galaxy A has leading arms, while galaxy B has trailing arms, but both exhibit the same pattern on the sky and the same radial velocity field.

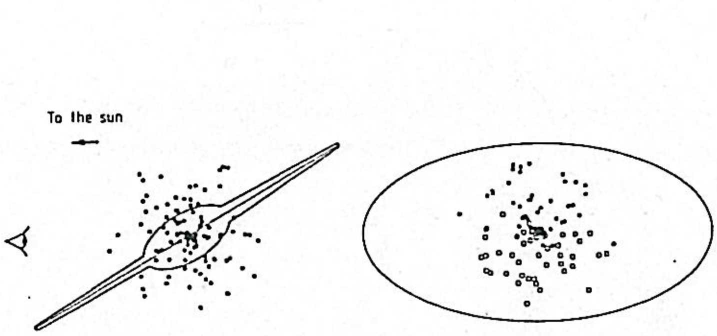


Figure 6-7. Distinguishing near and far sides of a disk galaxy. The dots represent objects such as novae or globular clusters. There is an obscuring dust layer in the central plane of the disk which is shown as a line in the side view at left. In the observer's view, at right, objects behind the dust layer are fainter and are shown as open circles.

LIN & SHU (1964+)

"TIGHT WINDING" / WKB APPROXIMACE:

úhel sklonu i je malý:

$$\operatorname{ctg} i = \frac{R|k|}{m} \gg 1$$

$\Rightarrow$ LOKÁLNÍ "ODPOVĚĎ" POTENCIÁLU NA HUSTOTU:

$$\Phi_1(R,\theta,t) = -2G\Sigma_1(R,\theta,t)$$

Φ_1 a Σ_1 ve fázi

- DISPERZNÍ RELACE:

$$(\omega - m\Omega)^2 = \varkappa^2 - 2\pi G \Sigma_0 |k|$$

$\uparrow$

$m\Omega_p$

$$\nu \equiv \frac{\omega - m\Omega}{\varkappa} = \frac{m(\Omega_p - \Omega)}{\varkappa}$$

$\Rightarrow$ TVAR SPIRÁLNÍCH RAMEN

- PORUCHY v_{R1} a $v_{\Theta 1}$

v_{R1} : v antifázi se Σ_1

$v_{\Theta 1}$: posunuta vzhledem k Σ_1 o $90°$

- $$\frac{\Sigma_1'}{\Sigma_0} : \frac{v_{R1}'}{V} : \frac{v_{\Theta 1}'}{V} = 1 : \left| \frac{\lambda}{\pi R}\left(1 - \frac{\Omega_p}{\Omega}\right) \right| : \left| \frac{\lambda \varkappa^2}{4\pi R \Omega^2} \right|$$

$$\frac{\Sigma_1'}{\Sigma_0} : \frac{v_{R1}'}{V} : \frac{v_{\theta 1}'}{V} = 1 : \left| \frac{\lambda}{\pi R}\left(1 - \frac{\Omega_p}{\Omega}\right) \right| : \left| \frac{\lambda \varkappa^2}{4\pi R \Omega^2} \right|$$

PŘÍKLAD

$$\frac{\Omega_p}{\Omega} = \frac{1}{2}, \left(\frac{\varkappa}{\Omega}\right)^2 = 1.6, \quad \lambda = 4\,\mathrm{kpc}$$

$$\Rightarrow \quad 1 : 0.06 : 0.05$$

$\Rightarrow$ pro $\frac{\Sigma_1^*}{\Sigma_0^*} = 0.05$: $v_{R1} \doteq 0.8\,\mathrm{km/s}$, $v_{\theta 1} \doteq 0.6\,\mathrm{km/s}$

pro $\frac{\Sigma_1^{\mathrm{PLYN}}}{\Sigma_0^{\mathrm{PLYN}}} = 0.5$: $v_{R1} = 8\,\mathrm{km/s}$, $v_{\theta 1} = 6\,\mathrm{km/s}$

$F^* = 0.45$ (PLYN)

$F = 0.066$ (HVĚZDY)

DISK S NENULOVOU DISPERZÍ RYCHLOSTÍ

PLYN: $(\omega - m\Omega)^2 = \kappa^2 - 2\pi G \Sigma_0 |k| + k^2 c_s^2$

HVĚZDY:

$$(\omega - m\Omega)^2 = \kappa^2 - 2\pi G \Sigma_0 |k| \mathcal{F}\left(\frac{\omega - m\Omega}{\kappa}, \frac{k^2 \sigma_R^2}{\kappa^2}\right)$$

$$\nu \equiv \frac{\omega - m\Omega}{\varkappa} = \frac{m(\Omega_p - \Omega)}{\varkappa}$$

$$\Rightarrow \quad \nu^2 = 1 - \frac{2\pi G \Sigma_0}{\varkappa^2} |k| \, \mathcal{F} \quad (\text{hvězdy})$$

$$\nu^2 = 1 - \frac{2\pi G \Sigma_0}{\varkappa^2} |k| + \frac{k^2 c^2}{\varkappa^2} \quad (\text{plyn})$$

$$k_{crit} \equiv \frac{\varkappa^2}{2\pi G \Sigma_0} \qquad \lambda_{crit} = \frac{4\pi^2 G \Sigma_0}{\varkappa^2}$$

$$\Rightarrow \quad \nu^2 = 1 - \frac{|k|}{k_{crit}} \, \mathcal{F} \quad (\text{hvězdy})$$

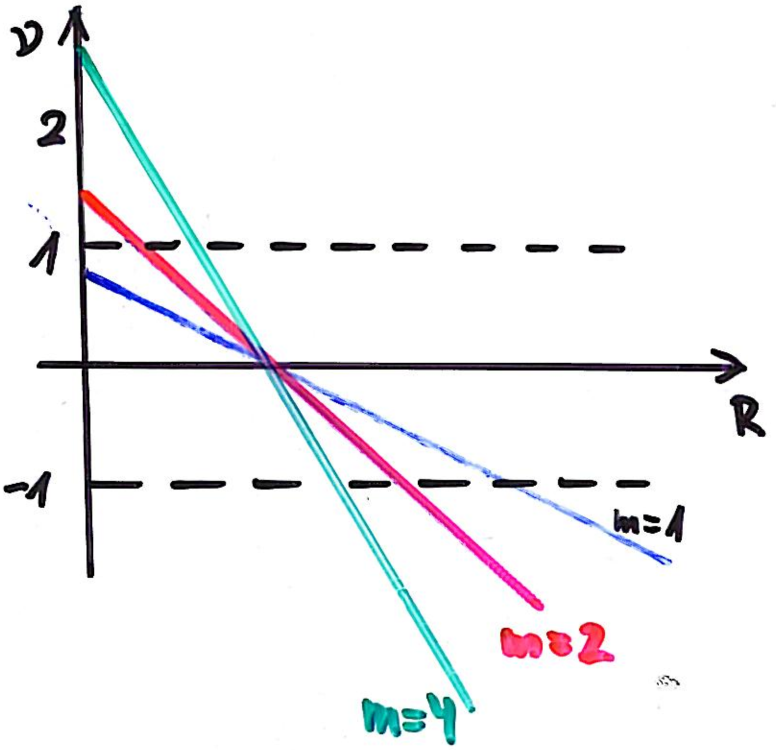
$$\nu^2 = 1 - \frac{|k|}{k_{crit}} + \left(\frac{k}{k_{crit}}\right)^2 \frac{Q^2}{4} \quad (\text{plyn})$$

PŘÍKLAD: MESTELŮV DISK

$$\Sigma(R) = \frac{\Sigma_e R_e}{R}, \quad v_c \equiv V = \sqrt{2\pi G \Sigma_e R_e} = \text{konst}$$

$$\Omega(R) = \frac{v_c}{R}, \quad \varkappa = \sqrt{2}\,\Omega(R)$$

$$\nu \equiv \frac{m(\Omega - \Omega_p)}{\varkappa} = \frac{m}{\sqrt{2}}\left(1 - \frac{\Omega_p}{v_c} R\right) = \frac{m}{\sqrt{2}}\left(1 - \frac{\Omega_p}{\Omega}\right)$$



$$\text{ILR: } \Omega_p = \left(1 - \frac{\sqrt{2}}{m}\right)\Omega$$

$$\text{OLR: } \Omega_p = \left(1 + \frac{\sqrt{2}}{m}\right)\Omega$$

POLOHY REZONANCÍ:

$$R_{CR} = \frac{v_c}{\Omega_p} \qquad R_{ILR} = \frac{v_c}{\Omega_p}\left(1 - \frac{\sqrt{2}}{m}\right) \qquad R_{OLR} = \frac{v_c}{\Omega_p}\left(1 + \frac{\sqrt{2}}{m}\right)$$

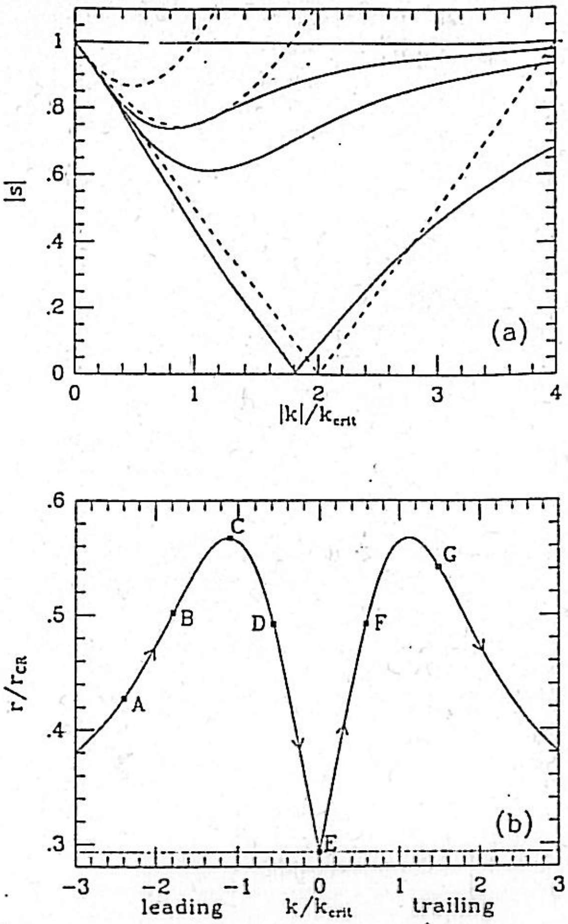


Figure 6-14. (a) The dispersion relation for tightly wound disturbances in gaseous [eq. (6-40), dashed lines] and stellar [eq. (6-46), solid lines] disks. The curves shown are (bottom to top) $Q = 1$, 1.5, and 2. Since only $|s|$ and $|k|$ are shown, there is no distinction between leading and trailing waves, or waves inside and outside corotation. (b) Dispersion relation in the form of wavenumber versus radius for an $m = 2$ tightly wound wave in a stellar Mestel disk with $Q = 1.5$. The radial scale is in units of the corotation radius r_{CR}, and only waves inside corotation are shown. The inner Lindblad resonance is at $r = 0.293 r_{CR}$. The direction of the group velocity is shown by arrows.

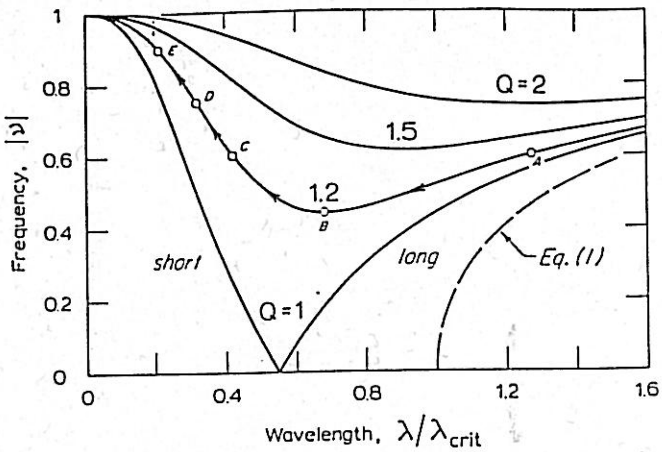


Figure 4 The Lin-Shu-Kalnajs dispersion relation for axisymmetric density waves of lo frequency $\omega = \nu\kappa$ and modest radial wavelength λ in a thin, rotating disk of stars endov with Q times the minimum random motions required by Equation (3).

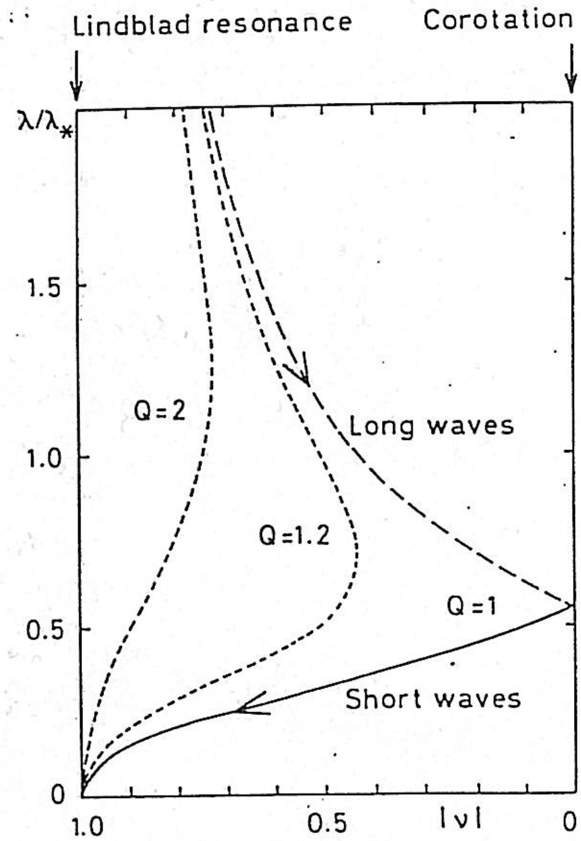
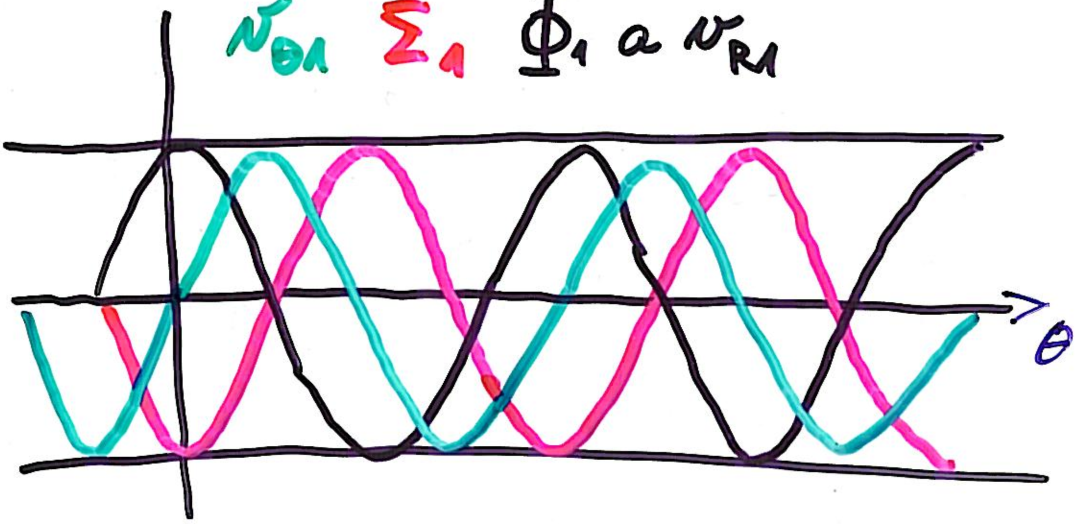
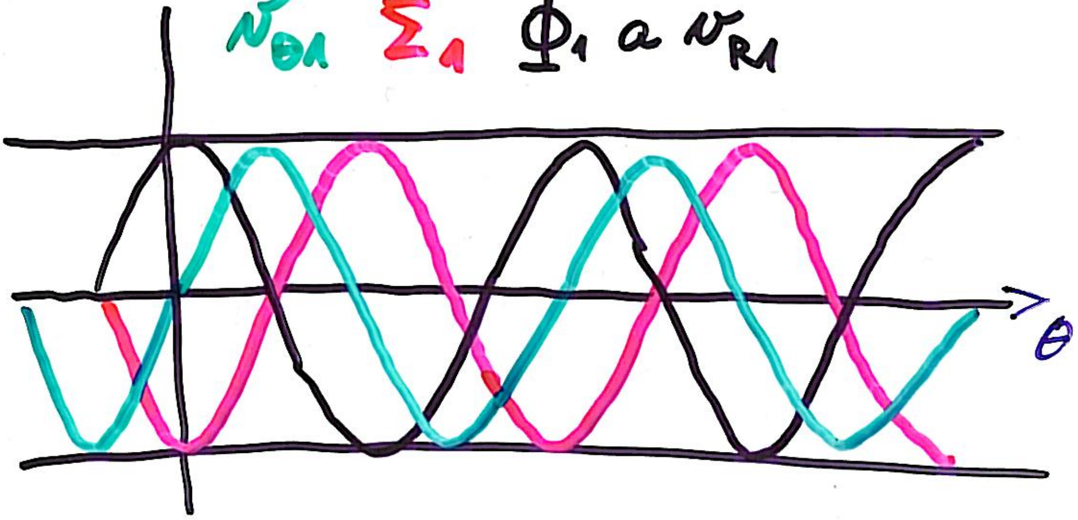


FIG. 6 — Dispersion relation $\lambda(\nu)$ for density waves (Lin and Shu). The arrows show the direction of propagation of the waves. Q measures the velocity dispersion.



Phase shifts between

Φ_1, v_{R1}, $v_{\theta 1}$ & Σ_1



Phase shifts between

Φ_1, v_{R1}, $v_{\Theta 1}$ & Σ_1