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M9302 Mathematical Models in Economics

0.Game Theory – Brief Introduction

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What is Game Theory?

- We do not live in vacuum.
- Whether we like it or not, all of us are strategists.
- ST is art but its foundations consist of some simple basic principles.
- The science of strategic thinking is called Game Theory.

Where is Game Theory coming from?

- Game Theory was created by

  Von Neumann and Morgenstern (1944)

  in their classic book

  The Theory of Games and Economic Behavior

- Two distinct approaches to the theory of games:

1. Strategic/Non-cooperative Approach
2. Coalition/Cooperative Approach

Where is Game Theory coming from?

- The key contributions of John Nash:

1. The notion of Nash equilibrium
2. Arguments for determining the two-person bargaining problems

- Other significant names:

N-Nash, A-Aumann, S-Shapley&Selten, H-Harsanyi

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1.1.Static Games of Complete Information

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The static (simultaneous-move) games

- Informally, the games of this class could be described as follows:
- First, players simultaneously choose a move (action).
- Then, based on the resulting combination of actions chosen in total, each player receives a given payoff.

Example: Students' Dilemma

- Strategic behaviour of students taking a course:

- First, each of you is forced to choose between studying HARD or taking it EASY.

- Then, you do your exam and get a GRADE.

Static Games of Complete Information

Standard assumptions:

- Players move (take an action or make a choice) simultaneously at a moment

– it is STATIC

- Each player knows what her payoff and the payoff of the other players will be at any combination of chosen actions

– it is COMPLETE INFORMATION

Example: Students' Dilemma

Standard assumptions:

- Students choose between HARD and EASY SIMULTANEOUSLY.
- Grading policy is announced in advance, so it is known by all the students.

Simplification assumptions:

- Performance depends on CHOICE.
- EQUAL EFFICIENCY of studies.

The static (simultaneous-move) games

- Game theory answers two standard questions:

1. How to describe a type of a game?

2. How to solve the resulting game-theoretic problem?

How to describe a game?

- The normal form representation of a game contains the following elements:

1. PLAYERS – generally of number *n*
2. STRATEGIES – $s_i \in S_i$, for *i = 1,…,n*
3. PAYOFFS – $u_i = u_i(s_1,\ldots,s_n)$, for *i = 1,…,n*

- We denote the game of n-players by

$$G = \{S_1,\ldots,S_n;u_1,\ldots,u_n\}$$

Example: Students' Dilemma

Normal Form Representation:

1. Reduce the players to 2 – YOU vs. OTHERS

2. Single choice symmetric strategies

$$S_i = \{Easy, Hard\}$$, for $i = 1,\ldots,n$

3. Payoff function:

$$u_i = u_i(s_i, s_{-i}) = LEISURE_i(s_i) - GRADE_i(s_i, s_{-i})$$

Example: Students' Dilemma

Grading Policy:

the students over the average have a
    STRONG PASS (Grade A, or 1),

the ones with average performance get a
    WEAK PASS (Grade C, or 3) and

who is under the average

    FAIL (Grade F, or 5).

Example: Students' Dilemma

Leisure Rule: HARD study schedule devotes twice more time (leisure = 1) to studying than the EASY one (leisure = 2).

Player i's choice	Others' choice	LEISURE	GRADE	Player i' payoff
Easy	All Easy	2	3	**-1**
	At least one Hard	2	5	**-3**
Hard	At least one Easy	1	1	**0**
	All Hard	1	3	**-2**

Example: Students' Dilemma

Bi-matrix of payoffs:

		OTHERS	
		Easy	Hard
YOU	Easy	-1,-1	-3,0
	Hard	0,-3	-2,-2

How to solve the GT problem?

Solution Concepts:

- Strategic Dominance
- Nash Equilibrium (NE)

in static games of complete information

- Subgame-Perfect Nash Equilibrium (SPNE)

in dynamic games of complete information

- Bayesian Nash Equilibrium (BNE)

in static games of incomplete information

- Perfect Bayesian Equilibrium (PBNE)

in dynamic games of incomplete information

Strategic Dominance

Definition of a *strictly dominated strategy:*

- *Consider the normal-form game G =* $\{S_1,\ldots,S_n;u_1,\ldots,u_n\}$
- *Feasible strategy* s_i' *is strictly dominated by strategy* s_i''

if i's payoff from playing s_i' *is strictly less than i's payoff from playing* s_i''*:*

$$u_i\left(s_1,\ldots,s_{i-1},s_i',s_{i+1},\ldots,s_n\right) < u_i\left(s_1,\ldots,s_{i-1},s_i'',s_{i+1},\ldots,s_n\right)$$

for each feasible combination $\left(s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_n\right)$ *that can be constructed from the other players' strategy spaces* $S_1,\ldots,S_{i-1},S_{i+1},\ldots,S_n$.

Strategic Dominance

Solution Principle: Rational players do not play strictly dominated strategies.

The solution process is called "*iterated elimination of strictly dominated strategies*".

Example: Students' Dilemma

- Solution by iterated elimination of strictly dominated strategies:

		OTHERS: Easy	OTHERS: Hard
YOU	Easy	-1,-1	-3,0
	Hard	0,-3	-2,-2

After elimination a single strategy combination remains:

Easy is strictly dominated by Hard for YOU.

Easy is strictly dominated by Hard for OTHERS.

{HARD; HARD}

Weaknesses of IESDS

- Each step of elimination requires a further assumption about what the players know about each other's rationality
- The process often produces a very imprecise predictions about the play of the game

Example: Students' Dilemma -2

- Leisure Rule: HARD study schedule devotes all their time (leisure = 0) to studying.

Player i's choice	Others' choice	LEISURE	GRADE	Player i' payoff
Easy	All Easy	2	3	**-1**
	At least one Hard	2	5	**-3**
Hard	At least one Easy	0	1	**-1**
	All Hard	0	3	**-3**

Example: Students' Dilemma -2

- Solution by iterated elimination of strictly dominated strategies:

		OTHERS: Easy	OTHERS: Hard
YOU	Easy	-1,-1	-3,-1
	Hard	-1,-3	-3,-3

No single strategy could be eliminated:

{EASY/HARD; EASY/HARD}

Nash Equilibrium

- Definition (NE): In the n-player normal form game

G = $\{S_1,\ldots,S_n;u_1,\ldots,u_n\}$

the strategies $(s_1^*,\ldots,s_n^*)$ are a Nash equilibrium if,

for each player *i*,

s_i^* is (at least tied for) player *i*'s best response to the strategies specified for the n-1 other players:

$$u_i(s_1^*,\ldots,s_{i-1}^*,s_i^*,s_{i+1}^*\ldots,s_n^*)\geq u_i(s_1^*,\ldots,s_{i-1}^*,s_i,s_{i+1}^*\ldots,s_n^*)$$

for every feasible strategy s_i in S_i; that is, s_i^* solves

$$\max_{s_i\in S_i} u_i(s_1^*,\ldots,s_{i-1}^*,s_i,s_{i+1}^*\ldots,s_n^*)$$

Relation between Strategic Dominance and Nash Equilibrium

- If a single solution is derived through iterated elimination of strictly dominated strategies it is also a unique NE.
- The players' strategies in a Nash equilibrium always survive iterated elimination of strictly dominated strategies.

Example: Students' Dilemma - 2

Grading Policy:

the students over the average have a
STRONG PASS (Grade A, or 1),
the ones with average performance get a
PASS (Grade B, or 2) and
who is under the average
FAIL (Grade F, or 5).

Example: Students'Dilemma - 2

- Leisure Rule: HARD study schedule devotes all their time (leisure = 0) to studying than the EASY one (leisure = 2).

Player i's choice	Others' choice	LEISURE	GRADE	Player i' payoff
Easy	All Easy	2	3	-1
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Hard	At least one Easy	0	1	-1
	All Hard	0	3	-3

Example: Students' Dilemma -2

- Solution by iterated elimination of strictly dominated strategies:

		OTHERS: Easy	OTHERS: Hard
YOU	Easy	0,0	-3,-1
	Hard	-1,-3	-2,-2

No single strategy could be eliminated:

{EASY/HARD; EASY/HARD}

Example: Students' Dilemma -2

- Nash Equilibrium Solution:

		OTHERS: Easy	OTHERS: Hard
YOU	Easy	0, 0	-3, -1
	Hard	-1, -3	-2, -2

Two Nash Equilibria:

{EASY/EASY; HARD/HARD}

Example: Students' Dilemma - 2

Some useful policy implications:

- Harsh grading of the mediocre behavior would motivate the rational students to study hard.
- Extremely time-consuming studies discourage rational students and make them hesitant between taking it easy and studying hard.

Summary

- The simplest class of games is the class of Static Games of Complete Information.
- By ‘static’ it is meant that players choose their strategies simultaneously without observing each other’s choices.
- ‘Complete information’ implies that the payoffs of each combination of strategies available are known to all the players.
- Static games of complete information are usually represented in normal form consisting of bi-matrix of player’s payoffs.

Summary

- A strategy is strictly dominated if it yields lower payoff than another strategy available to a player irrespective of the strategic choice of the rest of the players.
- The weakest solution concept in game theory is the iterated elimination of strictly dominated strategies. It requires too strong assumptions for player's rationality and often gives imprecise predictions.
- Nash Equilibrium is a stronger solution concept that produces much tighter predictions in a very broad class of games.