Lecture 2 10.03.2011

M9302 Mathematical Models in Economics

2.1.Dynamic Games of Complete and Perfect Information

Instructor: Georgi Burlakov

Fast Revision on Lecture 1

- Strategic Games of Complete Information:
- Description
- Normal Form Representation
- Solution Concepts – IESDS vs. NE

How to solve the GT problem?

Solution Concepts:

- Strategic Dominance
- Nash Equilibrium (NE)

in static games of complete information

- Backwards Induction
- Subgame-Perfect Nash Equilibrium (SPNE)

in dynamic games of complete information

- Bayesian Nash Equilibrium (BNE)

in static games of incomplete information

- Perfect Bayesian Equilibrium (PBNE)

in dynamic games of incomplete information

Revision: Students' Dilemma -2 (simultaneous-move solution)

- Nash Equilibrium Solution:

OTHERS

		Easy	Hard
YOU	Easy	0,0	-3,-1
	Hard	-1,-3	-2,-2

Two Nash Equilibria:

{EASY/EASY; HARD/HARD}

Dynamic (sequential-move) games

- Informally, the games of this class could be described as follows:
- First, only one of the players chooses a move (action).
- Then, the other player(s) moves.
- Finally, based on the resulting combination of actions chosen in total, each player receives a given payoff.

Example 1: Students' Dilemma -2 (sequential version)

- Strategic behaviour of students taking a course:
- First, only YOU are forced to choose between studying HARD or taking it EASY.
- Then, the OTHERS observe what YOU have chosen and make their choice.
- Finally, both You and OTHERS do exam and get a GRADE.

Will the simultaneous-move prediction be defined?

The dynamic (sequential-move) games

- The aim of the first lecture is to show:

1. How to describe a dynamic game?

2. How to solve the simplest class of dynamic games with complete and perfect information?

How to describe a dynamic game?

- The extensive form representation of a game specifies:

1. Who are the PLAYERS.

2.1. When each player has the MOVE.

2.2. What each player KNOWS when she is on a move.

2.3. What ACTIONS each player can take.

3. What is the PAYOFF received by each player.

Example 1: Students' Dilemma (Sequential Version)

Extensive Form Representation:

1. Reduce the players to 2 – YOU vs. OTHERS

2.1. First YOU move, then – OTHERS.

2.2. OTHERS know what YOU have chosen when they are on a move but YOU don't.

2.3. Both YOU and OTHERS choose an ACTION from the set $A_i = \{Easy, Hard\}$, for *i = 1,…,n*

3. *Payoffs:*

$$u_i = u_i\left(a_i, a_{-i}\right) = LEISURE(a_i) - GRADE_i(a_i, a_{-i})$$

Example 1: Students' Dilemma -2 (Sequential Version)

<u>Grading Policy:</u>

the students over the average have a STRONG PASS (Grade A, or 1),

the ones with average performance get a PASS (Grade B, or 2) and

who is under the average

FAIL (Grade F, or 5).

Example 1: Students' Dilemma – 2 (Sequential Version)

<u>Leisure Rule:</u> HARD study schedule devotes all the time (leisure = 0) to studying distinct from the EASY one (leisure = 2).

Player i's choice	Others' choice	LEISURE	GRADE	Player i' payoff
Easy	All Easy	2	-2	0
	At least one Hard	2	-5	-3
Hard	At least one Easy	0	-1	-1
	All Hard	0	-2	-2

Dynamic Games of Complete and Perfect Information

- The simple class of dynamic games of complete and prefect information has the following general description:

1. Player 1 chooses and action a_1 from the feasible set A_1.
2. Player 2 OBSERVES a_1 and then chooses an action a_2 from the feasible set A_2.
3. Payoffs are $u_1(a_1,a_2)$ and $u_2(a_1,a_2)$.

Dynamic Games of Complete and Perfect Information

Standard assumptions:

- Players move at different, sequential moments

– it is DYNAMIC

- The players' payoff functions are common knowledge

– it is COMPLETE INFORMATION

- At each move of the game the player with the move knows the full history how the game was played thus far

– it is PERFECT INFORMATION

Example 1: Students' Dilemma -2 (Sequential Version)

Standard assumptions:

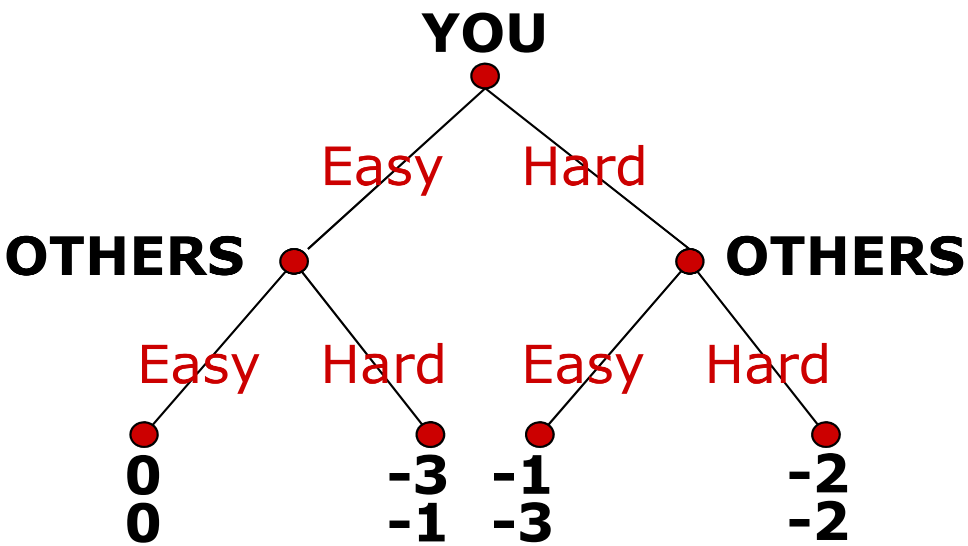
- Students choose between HARD and EASY SEQUENTIALLY.
- Grading is announced in advance, so it is COMMON KNOWLEDGE to all the students.
- Before making a choice in the second stage, OTHERS observe the choice of YOU in the first stage.

Simplification assumptions:

- Performance depends on CHOICE.
- EQUAL EFFICIENCY of studies.

Example 1: Students' Dilemma – 2 (Sequential Version)

Game Tree VS. Normal-Form



	(HARD, HARD)	(HARD, EASY)	(EASY, HARD)	(EASY, EASY)
HARD	-2,-2 (NE)	-2,-2	-1,-3	-1,-3
EASY	-3,-1	0,0 (NE)	-3,-1	0,0 (NE)

Backwards Induction

Solve the game from the last to the first stage:

- Suppose a unique solution to the second stage payoff-maximization:

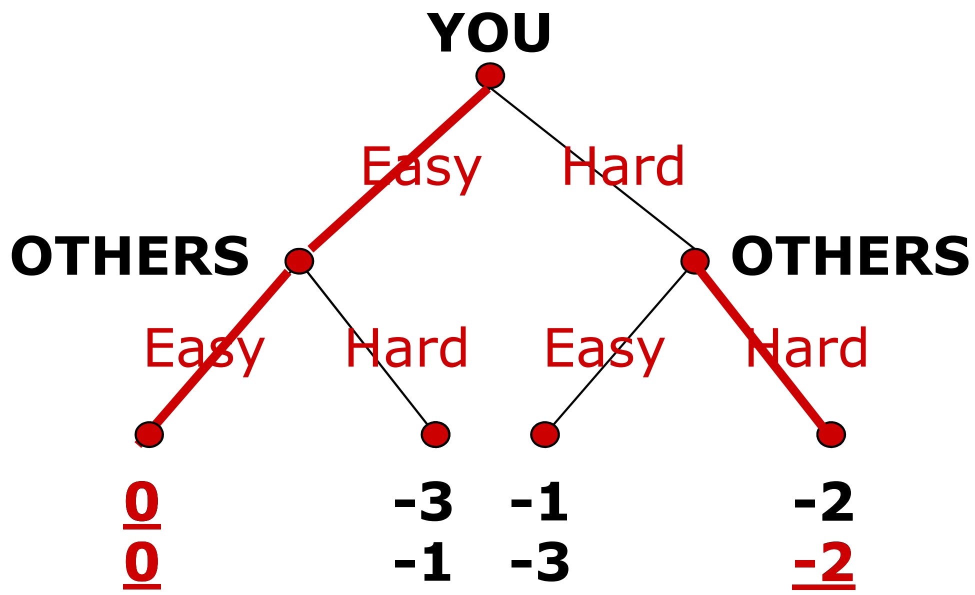
$$R_2(a_1) = \arg\max_{a_2 \in A_2} u_2(a_1, a_2)$$

- Then assume a unique solution to the first stage payoff-maximization:

$$a_1^* = \arg\max_{a_1 \in A_1} u_1(a_1, R_2(a_1))$$

- Call $(a_1^*, R_2(a_1^*))$ a *backwards-induction outcome.*

Example 1: Students' Dilemma – 2 (Sequential Version)



	(HARD, HARD)	(HARD, EASY)	(EASY, HARD)	(EASY, EASY)
HARD	-2,-2 (NE)	-2,-2	-1,-3	-1,-3
EASY	-3,-1	0,0 **(SPNE)**	-3,-1	0,0 (NE)

Example 2: Students' Dilemma -2 (with non-credible threat)

- Strategic behaviour of students taking a course:

- First, only YOU are forced to choose between studying HARD or taking it EASY.

- Then, the course instructor warns you:
- if YOU choose to study HARD in the first stage, all students get a WEAK PASS (C or 3)
- But if YOU choose to take it EASY, OTHERS still have a choice and YOU are on a threat to FAIL (F or 5)

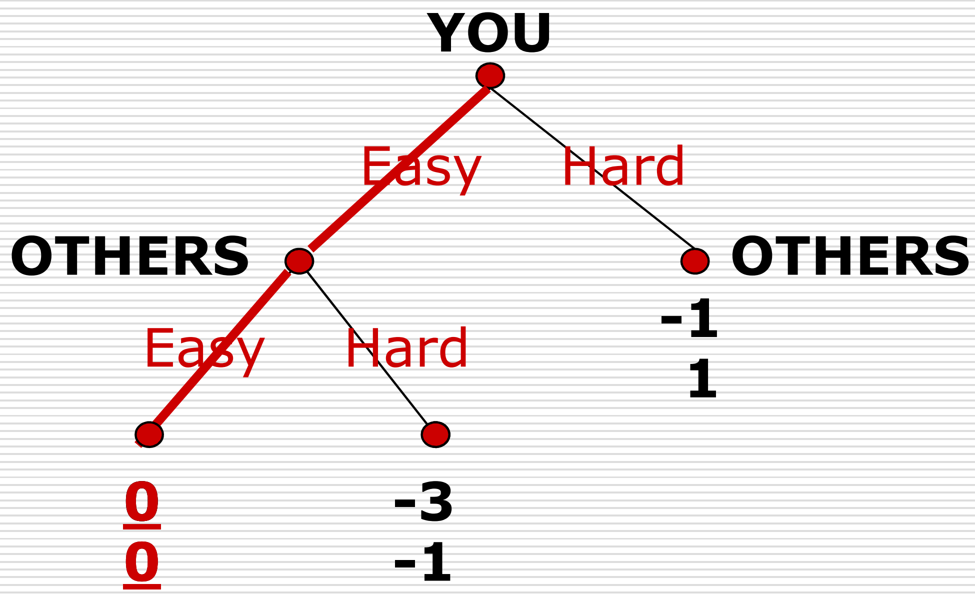
Is instructor's threat credible? Should YOU take it seriously?

Example 2: Students' Dilemma – 2 (with non-credible threat)

Leisure Rule: HARD study schedule devotes all the time (leisure = 0) to studying distinct from the EASY one (leisure = 2).

Player i's choice	Others' choice	LEISURE	GRADE	Player i' payoff
Easy	All Easy	2,2	-2,-2	0,0
	At least one Hard	2,0	-5,-1	-3,-1
Hard	No Choice	0,2	-1,-1	-1,1

Example 2: Students' Dilemma – 2 (with non-credible threat)



Subgame Perfect Nash Equilibrium

Informal Definition:

- The only subgame-perfect Nash equilibrium is the backwards-induction outcome.
- The backwards-induction outcome does not involve non-credible threats.

Summary

- Dynamic (sequential-move) games represent strategic situations where one of the players moves before the other(s) allowing them to observe her move before making a decision how to move themselves.
- To represent a dynamic game it is more suitable to use extensive form in which in addition to players, their strategy spaces and payoffs, it is also shown **when each player moves** and **what she knows before moving**.

Summary

- Graphically a dynamic game could be represented by the so called "game tree".
- the number of the subgames is equal to the number of decision nodes in the tree minus 1.
- Distinct from the static games of complete information, here the strategy set of the second player does not coincide with its set of feasible actions.
- Strategy in a dynamic game is a complete plan of action – it specifies a feasible action for each contingency (other player's preceding move) in which given player might be called to act.

Summary

- Dynamic games of complete information are solved by backwards induction i.e. first the optimal outcome in the last stage of the game is defined to reduce the possible moves in the previous stages.
- Backwards induction outcome does not involve non-credible threats – it corresponds to the subgame-perfect Nash equilibrium as a refinement of the pure-strategy NE concept.