

M9302 Mathematical Models in Economics

3.1. Dynamic Games of Complete but Imperfect Information

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INVESTMENTS IN EDUCATION DEVELOPMENT

How to solve the GT problem?

Solution Concepts:

- Strategic Dominance
 - Nash Equilibrium (NE)
in **static games of complete information**
 - Backwards Induction
 - Subgame-Perfect Nash Equilibrium (SPNE)
in **dynamic games of complete information**
 - Bayesian Nash Equilibrium (BNE)
in **static games of incomplete information**
 - Perfect Bayesian Equilibrium (PBNE)
in **dynamic games of incomplete information**
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Revision

What is information set?

An information set for a player is a collection of decision nodes satisfying:

- the player has the move at every node in the information set, and
 - when the play of the game reaches a node in the information set, the player with the move does not know which node in the information set is reached
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Revision

- What does the extensive form representation of a game specifies?
 1. Who are the PLAYERS.
 - 2.1. When each player has the MOVE.
 - 2.2. What each player KNOWS when she is on a move.
 - 2.3. What ACTIONS each player can take.
 3. What is the PAYOFF received by each player.
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Dynamic games of complete but imperfect information

- Informally, the games of this class could be described as follows:
- First, Players 1 and 2 **simultaneously** choose actions a_1 and a_2 from feasible sets A_1 and A_2 , respectively
- Second, players 3 and 4 observe the outcome of the first stage, (a_1, a_2) , and then *simultaneously choose actions a_3 and a_4 from feasible sets A_3 and A_4 , respectively.*
- Finally, based on the resulting combination of actions chosen in total, each player receives a given payoff $u_i(a_1, a_2, a_3, a_4)$ for $i=1,2,3,4$

Dynamic Games of Complete and Imperfect Information

Standard assumptions:

- Players move at different, sequential moments
 - it is **DYNAMIC**
 - The players' payoff functions are common knowledge
 - it is **COMPLETE INFORMATION**
 - At each stage of the game players move simultaneously
 - it is **IMPERFECT INFORMATION**
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Dynamic games of complete but imperfect information

- The aim of the third lecture is to show:
 1. What is the difference between perfect and imperfect information?
 2. How to solve games of complete but imperfect information?
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Perfect vs. Imperfect Information

What is perfect information?

- when at each stage the player with the move knows the full history of the game thus far
- When each information set is a singleton

Then, what is imperfect information?

- When there is at least one non-singleton information set
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How to solve dynamic games of imperfect information?

In a game of complete and perfect information BI eliminates noncredible threats. Why?

- Because each decision node represents a contingency in which a player might be called on to act.
 - The process of working backwards thus amounts to forcing each player to consider carrying out each threat
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How to solve dynamic games of imperfect information?

In a game of imperfect information BI does not work so simply. Why?

- Because working backwards would eventually lead us to a decision node in a non-singleton information set
 - Then the player does not know whether or not that decision node is reached
 - The player is forced to consider what it would eventually do if a node is really reached not in a contingency in which she is called on to act
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How to solve dynamic games of imperfect information?

How to deal with the problem of nonsingleton information sets in BI?

Work backwards until a nonsingleton information set is encountered, then:

- Skip over it and proceed the tree until a singleton information set is found and solve for the subgame emanating from it - SGPNE
 - Force the player with the move at the information set to consider what she would do if that information set was reached – Bayesian NE
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Dynamic games of Complete but Imperfect Information – key terms

- Subgame – a piece of a game that remains to be played beginning at any point at which the complete history of the game thus far is common knowledge among the players, i.e.:
 - begins at a singleton information set
 - includes all the decision and terminal nodes following but not preceding the starting singleton decision node
 - does not cut any (non-singleton) information sets.
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Dynamic games of Complete but Imperfect Information – key terms

- Strategy – a complete plan of action – it specifies a feasible action which the player will take in each stage, for every possible history of play through the previous stage.
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Dynamic games of Complete but Imperfect Information – SGPNE

- (Selten 1965) Subgame-perfect Nash Equilibrium (SGPNE)– a Nash equilibrium is subgame perfect if the players' strategies constitute a Nash equilibrium in every subgame.
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Dynamic games of Complete but Imperfect Information – Summary

- BI fails to eliminate noncredible threats in the games of imperfect information because of the non-singleton information sets.
 - Therefore a stronger solution concept called subgame-perfect N.E. is applied.
 - SGPNE includes not only the best response to the unique action played in the first stage but full plan of action (strategy) how it would be best to respond to any possible action in the unobserved part of the game (subgame).
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